REGULARITY AT THE BOUNDARY AND TANGENTIAL REGULARITY OF SOLUTIONS OF THE CAUCHY-RIEMANN SYSTEM

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Abstract. For a pseudoconvex domain $D \subset \mathbb{C}^n$, we prove the equivalence of the local hypoellipticity of the system $(\bar\partial, \bar\partial^*)$ with the system $(\bar\partial_b, \bar\partial_b^*)$ induced at the boundary. This develops a former result of our's in which the theory of the "harmonic" extension by Kohn was used. This technique is inadequate for the purpose of the present paper and must be replaced by that of the "holomorphic" extension.

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Let D be a pseudoconvex domain of $\mathbb{C}^n$ defined by $r<0$ with C^∞ boundary bD. We use the standard notations $\Box = \bar\partial\bar\partial^* + \bar\partial^*\bar\partial$ for the complex Laplacian and $Q(u,u) = ||\bar\partial u||^2 + ||\bar\partial^* u||^2$ for the energy form and some variants as, for an operator Op, $Q_{\text{Op}}(u,u) = ||\text{Op}\bar\partial u||^2 + ||\text{Op}\bar\partial^* u||^2$. Here u is a $(0,k)$ form belonging to $D_{\bar\partial^*}$. We similarly define the tangential version of these objects, that is, $\Box_b$, $\bar\partial_b$, $\bar\partial_b^*$, Q^b_{Op}. We take local coordinates (x,r) in $\mathbb{C}^n$ with $x\in\mathbb{R}^{2n-1}$ being the tangential coordinates and r, the equation of bD, serving as the last coordinate. We define the tangential s-Sobolev norm by $|||u|||_s := ||\Lambda^s u||_0$ where Λ^s is the standard tangential pseudodifferential operator with symbol $\Lambda^s_\xi = (1+|\xi|^2)^{\frac{s}{2}}$. We note that

$$
\begin{cases}
||\bar\partial u||^2_s + ||\bar\partial^* u||^2_s = \sum\limits_{j\le s} Q_{\Lambda^{s-j}\partial_r^j}(u,u),\\
|||\bar\partial u|||^2_s + |||\bar\partial^* u|||^2_s = Q_{\Lambda^s}(u,u),\\
||\bar\partial_b u_b||^2_s + ||\bar\partial_b^* u_b||^2_s = Q^b_{\Lambda^s}(u_b,u_b).
\end{cases}
\tag{1.1}
$$

We decompose u in tangential and normal component, that is

$$u = u^\tau + u^\nu,$$

and further decompose in microlocal components (cf. [11])

$$u^\tau = u^{\tau +} + u^{\tau -} + u^{\tau 0}.$$

We similarly decompose $u_b = u_b^+ + u_b^- + u_b^0$. We use the notation $\bar{L}_n$ for the "normal" $(0,1)$-vector field and $\bar{L}_1, ..., \bar{L}_{n-1}$ for the tangential ones. We have therefore the description for the totally real tangential, resp. normal, vector field T, resp ∂_r:

$$\begin{cases} T = i(L_n - \bar{L}_n), \\ \partial_r = L_n + \bar{L}_n. \end{cases}$$

From this, we get back $\bar{L}_n = \frac{1}{2}(\partial_r + iT)$. We denote by σ the symbol of a (pseudo)differential operator and by $\tilde{u}$ the partial tangential Fourier transform of u. We define a "holomorphic" extension (cf. [8]) $u^{\tau+(H)}$ of $u^{\tau+}|_{bD}$ by

$$u^{\tau+(H)} = (2\pi)^{-2n+1} \int_{\mathbb{R}^{2n-1}} e^{ix\xi} e^{r\sigma(\dot{T})} \psi^+(\xi)\tilde{u}(\xi, 0) d\xi, \tag{1.2}$$

where $\dot{T} := T(x, 0)$. Note that $\sigma(T) \gtrsim (1 + |\xi|^2)^{\frac{1}{2}}$ for ξ in $\operatorname{supp} \psi^+$ and (x, r) in a local patch; thus in the integral, the exponential is dominated by $e^{-|r|(1+|\xi|^2)^{\frac{1}{2}}}$ for $r < 0$. Differently from the harmonic extension by Kohn, the present one is well defined only in positive microlocalization. We can think of $u^{\tau+(H)}$ in two different ways: either as a modification of $u^{\tau+}$ or as an extension of u_b^+. The property which motivates the terminology of "holomorphic extension" is

$$||\bar{L}_n u^{\tau+(H)}|| \leq ||u_b^{\tau+}||^b_{-\frac{1}{2}}. \tag{1.3}$$

This follows from the relation $\bar{L}_n = \frac{1}{2}(\partial_r + iT)$ and $T - \dot{T} = rTan$; thus $||\bar{L}_n v|| = ||rTanv|| \leq ||v_b||_{-\frac{1}{2}}$. We have a first relation ([11] p. 241), between a trace v_b and a general extension v: for any ϵ and suitable c_ϵ

$$||v_b||_s \lesssim c_\epsilon |||v|||_{s+\frac{1}{2}} + \epsilon |||\partial_r v|||_{s-\frac{1}{2}}. \tag{1.4}$$

This can been seen in [11] p. 241 and [8] as for the small/large constant argument. As a specific property of our extension we have the reciprocal relation to (1.4), that is

$$||r^k u^{\tau+(H)}||_s \lesssim ||u_b^+||_{s-k-\frac{1}{2}}. \tag{1.5}$$

This is readily checked ([8] (1.12)).

Combination of (1.3) **and** (1.4) **shows that** $\bar{L}_n$ **acts on** $u^{\tau+(H)}$ **as an operator of order** 0**. On the other hand, on the straightening of** $b\Omega$ **in which** $r = x_n$**, we have that** $J\partial_r$**, i.e.** T**, coincides with** ∂_{y_n} **and therefore** $\bar{L}_n$ **is the Cauchy-Riemann operator** $\partial_{\bar{z}_n}$**. A reference to the related literature is in order. The extension of generalized functions to half-spaces or wedges of**

$\mathbb{C}^n$ using the decomposition of the δ-function in plane waves as in (1.2), was introduced by Sato, Kashiwara and Kawai in [15] as a general method of microlocal decomposition of the singularities. In particular, in the study of the singularities of the Szegö and Bergman kernels, it has been used among others by Boutet de Monvel and Sjöstarnd in [1] and by Hsiao in [4].

We denote by $\bar\partial^\tau$ the extension of $\bar\partial_b$ from $b\Omega$ to Ω which stays tangential to the level surfaces $r \equiv$ const. It acts on tangential forms u^τ and its action is $\bar\partial^\tau u^\tau = (\bar\partial u^\tau)^\tau$. We denote by $\bar\partial^{\tau*}$ its adjoint; thus $\bar\partial^{\tau*} u^\tau = \bar\partial^*(u^\tau)$. We use the notations $\Box^\tau$ and Q^τ for the corresponding Laplacian and energy form. We notice that

$$Q(u^{\tau+(H)}, u^{\tau+(H)}) = Q^\tau(u^{\tau+(H)}, u^{\tau+(H)}) + ||\bar L_n u^{\tau+(H)}||^2_0. \tag{1.6}$$

We have to describe how (1.4) and (1.5) are affected by $\bar\partial$ and $\bar\partial^*$.

Proposition 1.1. *We have for any extension v of v_b*

$$Q^b(v_b, v_b) \underset{\sim}{<} Q^\tau_{\Lambda^{\frac12}}(v,v) + Q^\tau_{\partial_r \Lambda^{-\frac12}}(v,v), \tag{1.7}$$

and, specifically for $u^{\tau+(H)}$

$$Q^\tau(u^{\tau+(H)}, u^{\tau+(H)}) \underset{\sim}{<} Q^b_{\Lambda^{-\frac12}}(u_b^+, u_b^+) + ||u_b^+||^2_{-\frac12}. \tag{1.8}$$

Proof. We have

$$\bar\partial^\tau v|_{bD} = \bar\partial_b v_b, \qquad \bar\partial^{\tau*} v|_{bD} = \bar\partial_b^* v_b.$$

Then, (1.7) follows from (1.4).

We pass to prove (1.8). We have $\bar\partial^\tau = \bar\partial_b + r\mathrm{Tan}$, $\bar\partial^{\tau*} = \bar\partial_b^* + r\mathrm{Tan}$ which yields

$$\begin{cases} \bar\partial^\tau u^{\tau+(H)} = (\bar\partial_b u_b)^{\tau+(H)} + r\mathrm{Tan}\, u^{\tau+(H)}, \\ \bar\partial^{\tau*} u^{\tau+(H)} = (\bar\partial_b^* u_b)^{\tau+(H)} + r\mathrm{Tan}\, u^{\tau+(H)}. \end{cases} \tag{1.9}$$

Application of (1.5) yields

$$\begin{aligned} ||\bar\partial^\tau u^{\tau+(H)}||^2 + ||\bar\partial^{\tau*} u^{\tau+(H)}||^2 &= ||(\bar\partial_b u_b)^{\tau+(H)}||^2 + ||(\bar\partial_b^* u_b)^{\tau+(H)}||^2 + ||r\mathrm{Tan}\, u^{\tau+(H)}||^2 \\ &\underset{\sim}{<} ||\bar\partial_b u_b^+||^2_{-\frac12} + ||\bar\partial_b^* u_b^+||^2_{-\frac12} + ||u_b^+||^2_{-\frac12}. \end{aligned}$$

$\square$

We decompose $u^{\tau+} = u^{\tau+(H)} + u^{\tau+(0)}$ which also serves as a definition of $u^{\tau+(0)}$. Let ζ and ζ' be a couple of cut-off with $\zeta \prec \zeta'$ in the sense that $\zeta'|_{\mathrm{supp}\,\zeta} \equiv 1$.

Proposition 1.2. *Each of the forms* $u^\# = u^\nu,\ u^{\tau\,-},\ u^{\tau\,0},\ u^{\tau+(0)},\ u_b^-,\ u_b^0$ *enjoys elliptic estimates, that is*

$$||\zeta u^\#||_s \underset{\sim}{<} ||\zeta'\bar\partial u^\#||_{s-1} + ||\zeta'\bar\partial^* u^\#||_{s-1} + ||u^\#||_0 \qquad s \geq 2. \tag{1.10}$$

Proof. Estimate (1.10) follows, by iteration, from

$$||\zeta u^\#||_s \underset{\sim}{<} ||\zeta\bar\partial u^\#||_{s-1} + ||\zeta\bar\partial^* u^\#||_{s-1} + ||\zeta' u^\#||_{s-1}. \tag{1.11}$$

As for u^ν and $u^{\tau+(0)}$ this latter follows from $u^\nu|_{bD} \equiv 0$ and $u^{\tau+(0)}|_{bD} \equiv 0$. For the terms with $-$ and 0, this follows from the fact that $|\sigma(T)| \underset{\sim}{<} |\sigma(\bar\partial)|$ in the region of 0-micolocalization and from $\sigma[\bar\partial, \bar\partial^*] \leq 0$ and $\sigma(T) < 0$ in the negative microlocalization. We refer to formula (1) in the Main Theorem of [3] as a general reference but also give an outline of the proof. We start from

$$|||\zeta u^\#|||_1^2 \underset{\sim}{<} Q(\zeta u^\#, \zeta u^\#) + ||\zeta' u^\#||_0^2; \tag{1.12}$$

this is the basic estimate for u^ν and $u^{\tau+(0)}$ (which vanish at bD) whereas it is [11] Lemma 8.6 for $u^{\tau\,-}$, $u^{\tau\,0}$ and u_b^-, u_b^0. Applying (1.12) to $\zeta\Lambda^{s-1}\zeta u^\#$ one gets the estimate of tangential norms for any s, that is, (1.11) with the usual norm replaced by the "triplet" norm. Finally, by non-characteristicity of $(\bar\partial, \bar\partial^*)$ one passes from tangential to full norms along the guidelines of [16] Theorem 1.9.7. The version of this argument for $\Box$ can be found in [11] second part of p. 245.

$\Box$

Let s and l be a pair of indices.

Theorem 1.3. *Consider the estimates*

(1.13)
$$||\zeta u_b||_s \underset{\sim}{<} ||\zeta'\bar\partial_b u_b||_{s+l} + ||\zeta'\bar\partial_b^* u_b||_{s+l} + ||u_b||_0 \quad \text{for any } u_b \in C^\infty(b\Omega),$$

(1.14)
$$||\zeta u||_s \underset{\sim}{<} ||\zeta'\bar\partial u||_{s+l} + ||\zeta'\bar\partial^* u||_{s+l} + ||u||_0 \quad \text{for any } u \in D_{\bar\partial^*} \cap C^\infty(\bar\Omega),$$

(1.15)
$$||\zeta u||_s \leq \epsilon(||\zeta\bar\partial u||_s + ||\zeta\bar\partial^* u||_s) + c_\epsilon ||u|| \quad \text{for any } \epsilon, \text{ for suitable } c_\epsilon \text{ and for any } u \in D_{\bar\partial^*} \cap C^\infty(\bar\Omega).$$

Then (1.13) *implies* (1.14) *and* (1.15) *implies* (1.13) *for* $l = 0$.

Remark 1.4. (i) The above estimates (1.13) and (1.14) for any s, ζ, ζ' and for suitable l, characterize the local hypoellipticity of the system $(\bar\partial_b, \bar\partial_b^*)$ and $(\bar\partial, \bar\partial^*)$ respectively (cf. [12]). When $l > 0$, one says that

the system has a "loss" of l derivatives; when $l<0$, one says that it has a "gain" of $-l$ derivatives.
(ii) The point in (1.15), differently from (1.13) and (1.14) is that we have the same cut-off ζ in both sides and also that there is a factor ϵ of compactness. Though (1.15) is stronger than (1.14), there are wide classes of domains Ω for which it holds including all domains of infra-exponential type, for which a superlogarithmic estimates holds (see [7]). In fact, let R^s be the pseudodifferential operator defined by $\widetilde{R^s u}=\Lambda_\xi^{s\sigma(x)}\tilde{u}$ (cf. Kohn [11] p. 234). On the one hand have $R^s\sim\Lambda^s$ modulo operators of order $-\infty$ over u such that $\sigma|_{\mathrm{supp}\,u}\equiv 1$. On the other, we have that $[R^s,\zeta']$ has order $-\infty$ if $\zeta'|_{\mathrm{supp}\,\sigma}\equiv 1$ and hence the supports of σ and $\dot{\zeta}'$ are disjoint. Finally, we have $|\zeta''[\bar\partial,R^s]\zeta'|\underset{\sim}{<}\log\Lambda R^s\zeta'$ in the sense of operators, when $\sigma\prec\zeta'\prec\zeta''$. Using R^s as a substitute of Λ^s, we can prove (1.15) whenever a superlogarithmic estimate holds (cf. [11] Section 7).

Proof. First, it is clearly not restrictive that u and u_b have compact support. Because of Proposition 1.2, it suffices to prove (1.13) for u_b^+ and (1.14) for $u^{\tau+}$. It is also obviuos that we can consider cut-off functions ζ and ζ' in the only tangential coordinates, not in r. We start by proving that (1.13) implies (1.14). We recall the decomposition $u^{\tau+}=u^{\tau+(H)}+u^{\tau+(0)}$ and begin by estimating $u^{\tau+(H)}$. We have

(1.16)
$$\begin{aligned}|||\zeta u^{\tau+(H)}|||_s^2 &\underset{(1.5)}{\underset{\sim}{<}} ||\zeta u_b^+||^2_{s-\frac12}\\ &\underset{(1.13)}{\underset{\sim}{<}} Q^b_{\Lambda^{s+l-\frac12}\zeta'}(u_b^+,u_b^+)+||u_b^+||^2_{-\frac12}\\ &\underset{(1.7)}{\underset{\sim}{<}} Q^\tau_{\Lambda^{s+l}\zeta'}(u^{\tau+},u^{\tau+})+Q^\tau_{\partial_r\Lambda^{s+l-1}\zeta'}(u^{\tau+},u^{\tau+})+||u^{\tau+}||_0^2.\end{aligned}$$

It remains to estimate $u^{\tau+(0)}$; since $u^{\tau+(0)}|_{bD}\equiv 0$, then by 1-elliptic estimates

(1.17)
$$\begin{aligned}|||\zeta u^{\tau+(0)}|||_s &\underset{(1.11)}{\underset{\sim}{<}} Q_{\Lambda^{s-1}\zeta}(u^{\tau+(0)},u^{\tau+(0)})+|||\zeta'u^{\tau+(0)}|||^2_{s-1}\\ &\underset{\sim}{<} Q_{\Lambda^{s-1}\zeta}(u^{\tau+},u^{\tau+})+Q^\tau_{\Lambda^{s-1}\zeta}(u^{\tau+(H)},u^{\tau+(H)})+|||r\zeta u^{\tau+(H)}|||^2_s+|||\zeta'u^{\tau+(0)}|||^2_{s-1}\\ &\underset{\sim}{<} Q_{\Lambda^{s-1}\zeta}(u^{\tau+},u^{\tau+})+|||\zeta u^{\tau+(H)}|||^2_s+|||\zeta'u^{\tau+(H)}|||^2_{s-1}+|||\zeta'u^{\tau+(0)}|||^2_{s-1},\end{aligned}$$

where we have used that $Q = Q^\tau + O(r)\Lambda$ over $u^{\tau+(H)}$, that is (1.6) in addition to the second of (1.8), in the second inequality together with the estimate $Q^\tau_{\Lambda^{s-1}} \underset{\sim}{<} \Lambda^s$ in the third. We estimate terms in the last line. First, the term $|||\zeta u^{\tau+(H)}|||^2_s$ is estimated by means of (1.16). Next, the terms in $(s-1)$-norm can be brought to 0-norm by combined inductive use of (1.16) and (1.17) and eventually their sum is controlled by $||u^{\tau+}||^2_0$. We put together (1.16) and (1.17) (with the above further reductions), recall the first of (1.1) in order to estimate $Q^\tau\Lambda^s\zeta' + Q^\tau_{\partial_r\Lambda^{s-1}\zeta'}$ in the right of (1.16) and end up with

$$|||\zeta u^{\tau+}|||_s \underset{\sim}{<} ||\zeta'\bar\partial u^{\tau+}||_s + ||\zeta'\bar\partial^* u^{\tau+}||_s + ||u^{\tau+}||_0. \tag{1.18}$$

Finally, by non-characteristicity of $(\bar\partial, \bar\partial^*)$ one passes from tangential to full norms in the left side of (1.18) along the guidelines of [16] Theorem 1.9.7. The version of this argument for $\Box$ can be found in [11] second part of p. 245. Thus we get (1.14).

We prove that (1.15) implies (1.13) for $l = 0$. Thanks to $\partial_r = \bar L_n +$ Tan and to the second of (1.8), we have $\partial_r u^{\tau+(H)} = \operatorname{Tan} u^{\tau+(H)}$ and $\bar L_n u^{\tau+(H)} = rTanu^{\tau+(H)}$. It follows

(1.19)

$$\begin{aligned}
||\zeta u_b^+||^2_s &\underset{\underset{(1.4)}{\sim}}{<} |||\zeta u^{\tau+(H)}|||^2_{s+\frac12} + |||\partial_r\zeta u^{\tau+(H)}|||^2_{s-\frac12}\\
&\underset{\sim}{<} |||\zeta u^{\tau+(H)}|||^2_{s+\frac12} + ||\bar L_n\zeta u^{\tau+(H)}||^2_{s-\frac12}\\
&\underset{\underset{(1.15)}{\sim}}{<} \epsilon\Big(Q^\tau_{\Lambda^{s+\frac12}\zeta}(u^{\tau+(H)}, u^{\tau+(H)}) + |||\zeta\bar L_n u^{\tau+(H)}|||^2_{s+\frac12}\Big)\\
&\qquad + c_\epsilon\Big(|||\zeta' u^{\tau+(H)}|||^2_{s-\frac12} + |||u^{\tau+(H)}|||^2_0\Big)\\
&\underset{\underset{(1.8)}{\sim}}{<} \epsilon(Q^b_{\Lambda^s\zeta}(u_b^+, u_b^+) + ||\zeta u_b^+||^2_s) + (Q^b_{\Lambda^{s-1}\zeta'}(u_b^+, u_b^+) + ||\zeta' u_b^+||^2_{s-1}) + ||u^{\tau}{}^+||^{b\,b}_{-\frac12}\\
&\underset{\sim}{<} Q^b_{\Lambda^s\zeta'}(u_b^+, u_b^+) + \epsilon||\zeta u_b^+||^2_s + ||\zeta' u_b^+||^2_{s-1} + ||u_b^{\tau}{}^+||_{-\frac12},
\end{aligned}$$

where in the second line from the bottom we have calculated $[\zeta, \#^{(H)}]$ which yields $|||\zeta u^{\tau+(H)}|||_{s+\frac12} \le ||\zeta u_b^+||_s + ||\zeta' u_b^+||_{s-1}$ (and similarly as for $[\zeta, Q^{(H)}]$. We absorb the term with ϵ and get (1.13).

$\Box$

Since on a pseudoconvex domain the H^0-ranges of $\Box$ and $\Box_b$ are closed by basic estimate and by [9] respectively, then there are well defined the H^0-inverses denoted by N and G and named the Neumann and Green operator respectively.

Remark 1.5. (1.13) and (1.14) imply local regularity in degree ≥ 2 of G and N respectively. We first prove for N. We start from remarking that

$$\begin{cases} \bar\partial^* N_q \text{ is regular over Ker}\bar\partial & q \geq 2, \\ \bar\partial N_q \text{ is regular over Ker}\bar\partial^* & q \geq 0. \end{cases} \tag{1.20}$$

As for the first, we put $u = \bar\partial^* Nf$ for $f \in \text{Ker}\,\bar\partial$. We have ($\bar\partial u = f$, $\bar\partial^* u = 0$) and hence by (1.14) $||\zeta u||_s \underset{\sim}{<} ||\zeta' f||_{s+l} + ||u||_0$. To prove the second, we have just to put $u = \bar\partial N f$ for $f \in \text{Ker}\,\bar\partial^*$ and reason likewise. It follows from (1.20), that the Bergman projection B_q is regular in any degree $q \geq 0$. (Notice that even if one started from exact regularity by assuming (1.15), this is perhaps lost by taking the additional $\bar\partial$ in $B := \text{Id} - \bar\partial^* N \bar\partial$.) Finally, we exploit formula (5.36) in [14] in unweighted norms, that is, for $t = 0$:

$$\begin{aligned} N_q = {} & B_q(N_q\bar\partial)(Id - B_{q-1})(\bar\partial^* N_q)B_q \\ & + (Id - B_q)(\bar\partial^* N_{q+1})B_{q+1}(N_{q+1}\bar\partial)(Id - B_q). \end{aligned} \tag{1.21}$$

Now, in the right side, the $\bar\partial N$'s and $\bar\partial^* N$'s are evaluated over Ker$\bar\partial^*$ and Ker$\,\bar\partial$ respectively; thus they are regular for $q \geq 2$. The B's are also regular and therefore such is N. This concludes the proof of the regularity of N. The proof of the regularity of G is similar, apart from replacing (1.21) by its version for the Green operator G stated in Section 5 of [6].

$\square$

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