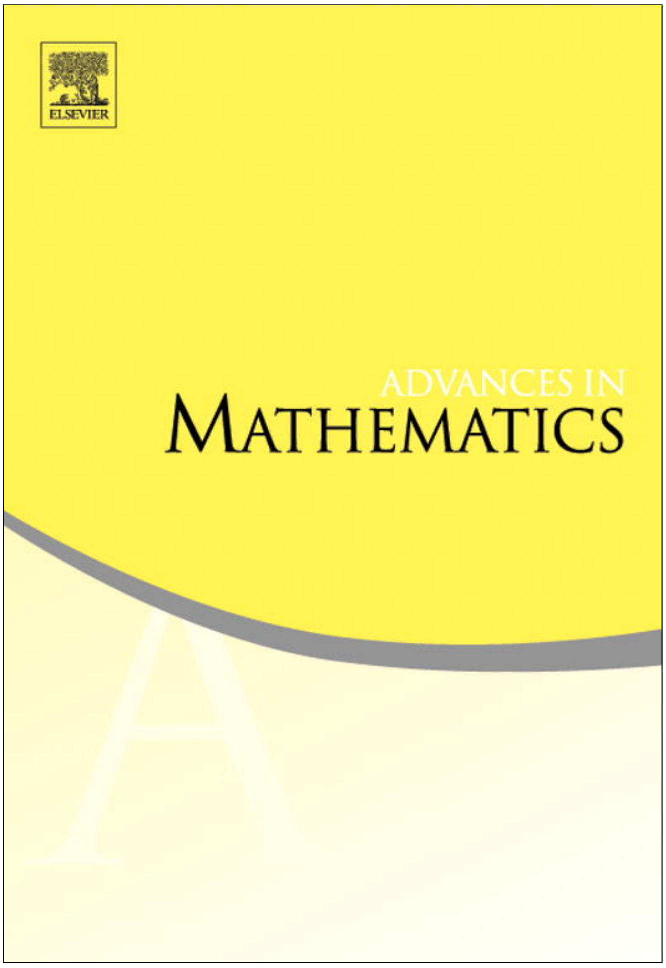
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Propagation of holomorphic extendibility and non-hypoellipticity of the $\bar{\partial}$-Neumann problem in an exponentially degenerate boundary

Luca Baracco, Tran Vu Khanh, Giuseppe Zampieri *

Dipartimento di Matematica, Università di Padova, via Trieste 63, 35121 Padova, Italy

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Abstract

We prove that CR lines in an exponentially degenerate boundary are propagators of holomorphic extendibility. This explains, in the context of the CR geometry, why in this situation the induced Kohn–Laplacian $\Box_b$ is not hypoelliptic (Christ (2000) [2]).

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Keywords: $\bar{\partial}$-Neumann problem; Propagation of holomorphic extendibility; Analytic discs with singular boundary

1. Introduction

Kohn noticed in [8] that analytic discs in the boundary of a pseudoconvex domain $\Omega \subset \mathbb{C}^n$ prevent from the C^∞-hypoellipticity of the $\bar{\partial}$-Neumann problem: the canonical solution is not smooth exactly at the boundary points where the datum is. On the other hand, it has been explained by Hanges and Treves in [5] that discs sitting in $\partial\Omega$ are propagators of holomorphic extendibility from Ω across $\partial\Omega$. Thus, propagation and hypoellipticity appear to be in contrast one to another. Christ proved in [3] that on the hypersurface in $\mathbb{C}^2$ defined by

* Corresponding author.
E-mail addresses: baracco@math.unipd.it (L. Baracco), khanh@math.unipd.it (T.V. Khanh), zampieri@math.unipd.it (G. Zampieri).

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$$x_2 = e^{-\frac{1}{|y_1|^s}}, \tag{1.1}$$

one does not have hypoellipticity for the induced Kohn–Laplacian $\Box_b$ when $s \geq 1$. Note that for $s < 1$ this is hypoelliptic as well as the complex Laplacian $\Box$: in fact, in this case, one has superlogarithmic estimates which are sufficient for hypoellipticity of $\Box_b$ and $\Box$ according to [10, Theorems 1.6 and 8.3 respectively] (cf. also [11]). (In [10], hypoellipticity of $\Box$ is deduced from microlocal hypoellipticity of $\Box_b$ but it could be proved in a direct way as in [7, Theorem 2.1].) It is worth remarking that superlogarithmicity does not entirily rule hypoellipticity. The pseudoconvex domain whose boundary is defined by the same equation as (1.1) but with y_1 replaced by z_1, that is, $x_2 = e^{-\frac{1}{|z_1|^s}}$, has the same range $s < 1$ for superlogarithmic estimates and, nonetheless, there always is hypoellipticity, for any value of s both for $\Box_b$ (Kohn [9, Main Theorem in Section 3]) and for $\Box$ [6, Theorem 1.1]. Here the matter is of a genuinely geometric type: there are no curves running in complex tangential directions along which the manifold is flat and which are, therefore, possible propagators (see also [4]). Coming back to the tubolar domain with boundary (1.1), we show here that the x_1-axis ia a propagator of holomorphic extendibility when $s \geq 1$. Our result, Theorem 2.6 below, applies in fact to more general domains, not necessarily rigid. More precisely, our argument consists in exhibiting a family of discs squeezed along the x_1-axis, singular at $x_1 = 0$ and with boundary in $\partial\Omega$ apart from $x_1 = 1$ where they enter in $x_2 < 0$. We show that they "point down" at $x_1 = 0$ if and only if $s \geq 1$. These discs propagate the extendibility down; as it has already been said, the proof does not go through when $s < 1$. There is no surprise about it because, for $s < 1$, there cannot be propagation of smoothness at the boundary. In fact, let $\chi = \chi(x_1)$ be C^∞ and satisfy $\chi \equiv 0$ at 1 and $\chi \equiv 1$ at 0, and consider the $\bar\partial$-closed form $f := \bar\partial\left(\frac{\chi(x_1)}{z_2}\right)$. If $s < 1$ the $\bar\partial$-Neumann problem is hypoelliptic and hence the equation $\bar\partial u = f$ has a solution u in Ω which is smooth at 0 and 1; thus the difference $u - \frac{\chi(x_1)}{z_2}$ is holomorphic in Ω, singular at $x_1 = 0$ but smooth at $x_1 = 1$.

2. Squeezing discs along lines

Analytic discs with Lipschitz boundary have been studied in several papers such as, for instance, [1,12–14]; we introduce here discs with logarithmic singularity at the boundary. In the standard disc Δ of the complex plane $\mathbb{C}$ with variable $\tau = re^{i\theta}$ for $\theta \in [0, 2\pi]$ or $\theta \in [-\pi, +\pi]$ according to the need, we consider the family of holomorphic mappings (= discs) depending on a small real parameter α:

$$\varphi_\alpha(\tau) = -\frac{1}{\log\left(\frac{1}{4}\left(\frac{1-\tau}{2}\right)^\alpha\right)}.$$

These discs are squeezed along the interval $\left(0, \left|\log\frac{1}{4}\right|^{-1}\right)$ as $\alpha \searrow 0$ with the points $+1$ and -1 interchanged with the left and right bounds respectively and they are singular at $\tau = 1$. Moreover, the most of their mass concentrates at $\tau = -1$. We have

$$\frac{1}{|\varphi_\alpha(\tau)|} \sim -\alpha \log|1-\tau|, \qquad \tau \in \Delta,\ \tau \text{ close to } 1.$$

With the notation $\tau = e^{i\theta} \in \partial\Delta$ we also have

$$-\arg\left(\frac{1-\tau}{2}\right) = \operatorname{arctg}\left(\frac{\sin\theta}{1-\cos\theta}\right)$$

$$= \operatorname{arctg}\left(\frac{\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}\right)$$
$$= \operatorname{arctg}\left(\operatorname{cotg}\frac{\theta}{2}\right) = \frac{\pi}{2} - \frac{\theta}{2}$$

and finally, at $\tau = 1$

$$\frac{1}{|\operatorname{Im}\varphi_\alpha|} \sim \frac{\log^2\left(\frac{1}{4}\frac{|1-\tau|^\alpha}{2^\alpha}\right) + \alpha^2\left(\frac{\pi}{2} - \frac{\theta}{2}\right)^2}{\alpha\left|\frac{\pi}{2} - \frac{\theta}{2}\right|}$$
$$\sim \frac{\log^2\left(\frac{1}{4}\frac{|1-\tau|^\alpha}{2^\alpha}\right)}{\alpha} + O(\alpha)$$
$$= \frac{1}{\alpha}\log^2\frac{1}{4} + 2\log\frac{1}{4}\log\frac{|1-\tau|}{2} + \alpha\log^2\frac{|1-\tau|}{2} + O(\alpha).$$

Thus, for α fixed as in next proposition, we have $\frac{1}{|\operatorname{Im}\varphi_\alpha|} \sim \alpha\log^2|1-\tau|$ at $\tau = 1$.

Proposition 2.1. *We have*

(i) $e^{-\frac{1}{|\varphi_\alpha|^s}} = O^\infty(1-\tau)$ *for* $s > 1$,
(ii) $e^{-\frac{1}{|\operatorname{Im}\varphi_\alpha|^s}} = O^\infty(1-\tau)$ *for* $s > \frac{1}{2}$.

Proof. As for (i), we have to notice that

$$e^{-\frac{1}{|\varphi_\alpha|^s}} \sim e^{-\alpha^s|\log|1-\tau\|^s}$$
$$= |1-\tau|^{\alpha^s|\log|1-\tau|^{s-1}|} = O^\infty(|1-\tau|) \quad \text{for } s > 1.$$

As for (ii), this follows from

$$e^{-\frac{1}{|\operatorname{Im}\varphi_\alpha|^s}} \sim e^{-\alpha^s|\log|1-\tau\|^{2s}}$$
$$= |1-\tau|^{\alpha^s|\log|1-\tau\|^{2s-1}} = O^\infty(|1-\tau|) \quad \text{for } s > \frac{1}{2}.$$

This concludes the proof of the proposition.

In particular, the two functions in the statement of the proposition are C^∞, and thus also $C^{1,\beta}$, at $\tau = 1$ for $s > 1$ and $s > \frac{1}{2}$ in the two respective cases. We have a basic result about composition of φ_α with flat functions more general than $e^{-\frac{1}{|z_1|^s}}$ or $e^{-\frac{1}{|y_1|^s}}$. For this, let $h_\eta(z_1, y_2)$, $(z_1, y_2) \in \mathbb{C}\times\mathbb{R}$, be a function sufficiently smooth depending on a parameter η.

Proposition 2.2. *Let* $\eta \mapsto h_\eta$, $\mathbb{R} \to C^3$ *be* C^k *and satisfy* $\partial_\eta h_\eta \equiv 0$ *in a neighborhood of* $z_1 = 0$. *Assume that all (mixed) derivatives up to order 2 in* τ *and* k *in* η *are* $O\left(e^{-\frac{1}{|y_1|^s}}\right)$ *for* $s \geq \frac{1}{2}$. *Then, the function* $(\eta, v) \mapsto h_\eta(\varphi_\alpha, v)$ *has the properties:*

(i) *it sends* $\mathbb{R}\times C^{1,\beta} \to C^{1,\beta}$,
(ii) *it is* C^k *with respect to* η,
(iii) *it is differentiable with respect to* v *at* $v = 0$ *and its differential has small norm.*

Proof. (i): For a function g of a real variable t, the assumptions

$$g = O\left(e^{-\frac{1}{t^s}}\right), \qquad g' = O\left(e^{-\frac{1}{t^s}}\right),$$

imply

$$g(|\mathrm{Im}\,\varphi_\alpha|) = O^\infty(|1-\tau|) \text{ when } s > \tfrac{1}{2} \text{ (Proposition 2.1)},$$
$$\partial_\tau |\mathrm{Im}\,\varphi_\alpha| \leq \frac{1}{|\log^3(1-\tau)|}\frac{1}{|1-\tau|},$$
$$\partial_\tau (g(|\mathrm{Im}\,\varphi_\alpha|)) = g'(\mathrm{Im}\,\varphi_\alpha)\frac{1}{|\log^3(1-\tau)||1-\tau|} = O^\infty(|1-\tau|) \text{ (again, Proposition 2.1)}.$$

This concludes the proof of (i).

(ii): Since $\partial_\eta h_\eta \equiv 0$ when φ_α is singular, then the C^k dependence of $h_\eta(\varphi_\alpha, v)$ on η is a standard fact: if $\eta \mapsto g_\eta,\ \mathbb{R} \to C^2(\mathbb{R})$ is C^2 and $\sigma \in C^{1,\beta}$, then $\eta \mapsto g_\eta(\sigma),\ \mathbb{R} \to C^{1,\beta}$ is C^k.

(iii): It is convenient to use a more general setting. Thus, let g_η be C^3. Then, the mapping $G_\eta : C^{1,\beta} \to C^{1,\beta},\ v \mapsto g_\eta(v)$ is C^1 at $v = 0$ and its differential G'_η satisfies

$$\|G'_\eta|_{v=0}\|_{L(C^{1,\beta},C^{1,\beta})} \lesssim \|g_\eta\|_{C^3}.$$

Note that, in our application, $g_\eta = h_\eta(\varphi_\alpha, \cdot)$; thus $\|g_\eta\|_{C^3}$ is small near $v = 0$.

Now, we can set up a Bishop's equation in the unknown $v \in C^{1,\beta}$

$$v - T_1(h_\eta(\varphi_\alpha, v)) = 0, \tag{2.1}$$

where T_1 is the Hilbert transform normalized by taking value 0 at $\tau = 1$. We rewrite the Eq. (2.1) in the functional space $C^{1,\beta}$ as $G_\eta(v) = 0$. By (iii) of Proposition 2.2, we have

$$\|G'_\eta\big|_{v=0} - \mathrm{id}\|_{L(C^{1,\beta},C^{1,\beta})} \lesssim \|h_\eta(\mathrm{Im}\,\varphi_\alpha, 0)\|_{C^3}. \tag{2.2}$$

By the implicit function theorem, we readily get

Corollary 2.3. *For small η, the Eq. (2.1) has a unique solution $v \in C^{1,\beta}$ and this depends in a C^k-fashion on η.*

We write $v = v_{\alpha,\eta}$ for the solution of (2.1) and also write $u = -T_1 v$ and $u = u_{\alpha,\eta}$. We also denote by $A = A_{\alpha\,\eta}$ the disc $A = (\varphi, u + iv)$. When only dependence on α is relevant, we write $v = v_\alpha$, $u = u_\alpha$ and $A = A_\alpha$. Note that under our assumption $h = O\left(e^{-\frac{1}{|y_1|^s}}\right)$ we have

$$u = O\left(e^{-\frac{1}{|y_1|^s}}\right).$$

For $\tau = re^{i\theta}$ and for a function in $C^0(\partial\Delta)$, such as u_α, the harmonic extension of u_α from $\partial\Delta$ to Δ, that we still denote by u_α, has a radial derivative which is given by

$$\partial_r u_\alpha|_{\tau=1} = -\int_{-\pi}^{\pi} \frac{u_\alpha}{1-\cos\theta}\,d\theta, \tag{2.3}$$

where the integral is taken in the sense of the principal value. We first show that the values of θ for which φ_α is not contained in the δ-neighborhood of $\left|\log\frac{1}{4}\right|^{-1}$ is very small.

Lemma 2.4. *We have the inclusion*

$$\left\{\theta : \left|\varphi_\alpha(\theta) - \left|\log\frac{1}{4}\right|^{-1}\right| > \delta\right\} \subset \left[-e^{-\frac{\delta}{2\alpha}}, e^{-\frac{\delta}{2\alpha}}\right]. \tag{2.4}$$

In other words, the whole circle $e^{i\theta}$, $\theta \in [-\pi, \pi]$, except from $\theta \in \left[-e^{-\frac{\delta}{2\alpha}}, e^{-\frac{\delta}{2\alpha}}\right]$, is mapped via φ_α into the δ-neighborhood of $\varphi_\alpha(-1)$.

Proof. We have

$$\left|-\frac{1}{\log\left(\frac{1}{4}\left(\frac{1-\tau}{2}\right)^\alpha\right)} + \frac{1}{\log\frac{1}{4}}\right| = \left|\frac{\log\left(\frac{1-\tau}{2}\right)^\alpha}{\log^2\frac{1}{4} + \log\frac{1}{4}\log\left(\frac{1-\tau}{2}\right)^\alpha}\right|.$$

Now, the denominator is ≥ 2. Hence, the set in the left of (2.4) is contained in

$$\left\{\tau : \left|\log\left(\frac{1-\tau}{2}\right)^\alpha\right| > 2\delta\right\},$$

which is in turn contained in $\left\{\tau = e^{i\theta} : |\theta| < e^{-\frac{\delta}{2\alpha}}\right\}$.

By renaming α, we neglect the irrelevant constant 2. Taking into account of Lemma 2.4, we decompose the integration in (2.3) as

$$\partial_r u_\alpha = -\int_{-\pi}^{\pi} \cdot = -\int_{-e^{-\frac{\delta}{\alpha}}}^{e^{-\frac{\delta}{\alpha}}} \cdot - \int_{-\pi}^{-e^{-\frac{\delta}{\alpha}}} \cdot - \int_{e^{-\frac{\delta}{\alpha}}}^{\pi} \cdot.$$

We approximate, near $\theta = 0$, $1 - \cos\theta$ by θ^2 and define

$$F_\alpha := \int_{-e^{-\frac{\delta}{\alpha}}}^{e^{-\frac{\delta}{\alpha}}} \frac{e^{-\frac{1}{|\operatorname{Im}\varphi_\alpha|^s}}}{\theta^2} d\theta.$$

Proposition 2.5. (i) *For $s \geq 1$, we have* $\lim_{\alpha\to 0} F_\alpha = 0$.
(ii) *For $s < 1$, we have* $\lim_{\alpha\to 0} F_\alpha = +\infty$.

Proof. (i): Note that $\left|\frac{1-\tau}{2}\right| \sim \theta$ on the unit circle near $\tau = 1$. We have

$$\begin{aligned}
F_\alpha &\leq \int_{-e^{-\frac{\delta}{\alpha}}}^{e^{-\frac{\delta}{\alpha}}} \frac{e^{-\frac{1}{|\operatorname{Im}\varphi_\alpha|}}}{\theta^2} d\theta \quad (\text{since } s \geq 1) \\
&\sim \int_{-e^{-\frac{\delta}{\alpha}}}^{e^{-\frac{\delta}{\alpha}}} \frac{e^{-\left[\frac{1}{\alpha}\log^2\frac{1}{4} + 2\log\frac{1}{4}\log\theta + \alpha\log^2\theta\right]}}{\theta^2} d\theta \\
&\leq \int_{-e^{-\frac{\delta}{\alpha}}}^{e^{-\frac{\delta}{\alpha}}} \frac{e^{-2\log\frac{1}{4}\log\theta}}{\theta^2} d\theta \\
&\leq \int_{-e^{-\frac{\delta}{\alpha}}}^{e^{-\frac{\delta}{\alpha}}} 1\, d\theta \leq 2e^{-\frac{\delta}{\alpha}}.
\end{aligned}$$

This proves (i).

(ii): We assume now $s < 1$ and also suppose, without loss of generality, $s > \frac{1}{2}$. By using the substitution $-\log\theta = t$, we get

$$\begin{aligned} F_\alpha &\geq \int_0^{e^{-\frac{\delta}{\alpha}}} e^{-\alpha^s \log^{2s}\theta - 2\log\theta} d\theta \\ &= \int_{\frac{\delta}{\alpha}}^{+\infty} e^{-\alpha^s t^{2s} + t} dt. \end{aligned}$$

Now, we remark that $-\alpha^s t^{2s} + t > 0$ if and only if $t < \left(\frac{1}{\alpha}\right)^{\frac{s}{2s-1}}$. Thus,

$$\begin{aligned} \int_{\frac{\delta}{\alpha}}^{+\infty} \cdot &\geq \int_{\frac{1}{\alpha}}^{\left(\frac{1}{\alpha}\right)^{\frac{s}{2s-1}}} 1 dt \\ &= \left(\left(\frac{1}{\alpha}\right)^{\frac{s}{2s-1}} - \frac{1}{\alpha}\right) \to +\infty, \end{aligned}$$

where the last conclusion follows from $\frac{s}{2s-1} > 1$.

Theorem 2.6. *Let $\Omega \subset \mathbb{C}^2$ be a domain defined by $x_2 > h(z_1, y_2)$ with h satisfying $\partial_{z_1}^j h = O\left(e^{-\frac{1}{|y_1|^s}}\right)$ for $j = 1, 2$. In particular, the boundary contains the real lines L defined by $y_1 = 0$, $x_2 = 0$ and $y_2 = const$. Assume $s \geq 1$; then each line L is a propagator of holomorphic extendibility. Namely, if $f \in hol(\Omega)$ extends to a full neighborhood of a point $z^1 \in L$, then it also extends to a neighborhood of any other point $z^o \in L$.*

Proof. We may assume that $z^o = (0, 0)$, $z^1 = \left(\left|\log\frac{1}{4}\right|^{-1}, 0\right)$ and that f extends to $B_{2\delta}(z^1)$, the 2δ-neighborhood of z^1. Recall that the points z^o and z^1 correspond to $\tau = 1$ and $\tau = -1$ respectively under the map φ_α. We also remark that

$$\varphi_\alpha\left([-\pi, +\pi] \setminus \left[-e^{-\frac{\delta}{2\alpha}}, e^{\frac{\delta}{2\alpha}}\right]\right) \subset B_\delta(z^1).$$

We deform h by allowing a δ-bump at z^1. Thus, we define

$$\tilde{h} = \begin{cases} h & \text{outside } B_{2\delta}(z^1), \\ -\delta & \text{on } B_\delta(z^1), \end{cases} \tag{2.5}$$

continued smoothly on $B_{2\delta}(z^1) \setminus B_\delta(z^1)$. Attach a disc $A_\alpha = (\varphi_\alpha, \tilde{u}_\alpha + i\tilde{v}_\alpha)$ over φ_α to the hypersurface defined by $x_2 = \tilde{h}$ according to Proposition 2.2; we have

$$\begin{aligned} \partial_r \tilde{u}_\alpha &= -\int_{-\pi}^{\pi} \frac{\tilde{u}_\alpha}{1 - \cos\theta} d\theta \\ &\geq -\int_{-e^{-\frac{\delta}{\alpha}}}^{e^{-\frac{\delta}{2\alpha}}} \frac{e^{-\frac{1}{|\mathrm{Im}\,\varphi_\alpha|^s}}}{1 - \cos\theta} d\theta + 2\int_{e^{-\frac{\delta}{2\alpha}}}^{\pi} \frac{\delta}{1 - \cos\theta} d\theta. \end{aligned}$$

Since $s \geq 1$, then $\int_{-e^{-\frac{\delta}{\alpha}}}^{e^{-\frac{\epsilon}{2\alpha}}} \frac{e^{-\frac{1}{|\mathrm{Im}\,\varphi_\alpha|^s}}}{1-\cos\theta} d\theta \to 0$ according to Proposition 2.5(i); thus

$$\partial_r \tilde{u}_\alpha > 0.$$

In other terms, $\tilde{u}_\alpha$ "points down" at $\tau = 1$; in particular,

$$\tilde{u}_\alpha(1-r) < 0 \quad \text{for } r < 1 \text{ close to } r = 1. \tag{2.6}$$

We fix α for which (2.6) is fulfilled and do not keep track of it in the notations which follow. If we replace φ by $-\epsilon + \varphi$, for a fixed ϵ, and substitute in (2.5) $-\delta$ by $-\eta\delta$ for any $\eta \in [-1, 1]$, we get a family of discs $\{A_\eta\}_\eta = \{(-\epsilon + \varphi, \tilde{u}_\eta + i\tilde{v}_\eta)\}_\eta$ such that

$$\begin{cases} \partial A_\eta \subset \partial\Omega \cup B_{2\delta}(z^1), \\ \Omega \bigcup \left(\bigcup_\eta A_\eta \right) \text{ contains a neighborhood of } 0. \end{cases} \tag{2.7}$$

At this point, we move up in x_2-direction our discs so that $\partial A_\eta \subset \Omega \cup B_{2\delta}(z^1)$ still keeping the second of (2.7). By Cauchy's formula, f extends from the boundaries ∂A_η to the full discs A_η whose union is a neighborhood of 0.

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