Departamento de Produção e Sistemas
Universidade do Minho

Decomposition Methods for Integer Programming **(PDMA, PDEIS)**

Exercise Block Angular Structure

1. Consider the following linear programming (LP) problem:

$$
\begin{array}{lrrrrcl}
\max & 3\,x_1 & +5\,x_2 & +1\,y_1 & +2\,y_2 & & \\
\text{suj. a} & 1\,x_1 & +2\,x_2 & +2\,y_1 & +1\,y_2 & \leq & 6 \\
 & 3\,x_1 & +2\,x_2 & +1\,y_1 & +1\,y_2 & \leq & 8 \\
 & 1\,x_1 & +2\,x_2 & & & \leq & 4 \\
 & 1\,x_1 & & & & \leq & 2 \\
 & & & 3\,y_1 & +1\,y_2 & \leq & 3 \\
 & x_1,\ x_2,\ y_1,\ y_2 \geq 0 & & & & &
\end{array}
$$

   which has a block angular structure. Consider the decomposition where the following two bounded sets X and Y are considered in different subproblems:

$$
\begin{aligned}
X &= \{(x_1,x_2)^T : x_1 + 2x_2 \leq 4, x_1 \leq 2,\ \ x_1, x_2 \geq 0\} \\
Y &= \{(y_1,y_2)^T : 3y_1 + y_2 \leq 3,\ \ y_1, y_2 \geq 0\},
\end{aligned}
$$

   a) Recognize that the problem has a block angular structure and identify the matrix blocks.

   b) Apply the suggested DW decomposition to obtain the reformulated model.

   c) Draw the solution space of the two sets X and Y.

   d) Solve the reformulated Dantzig-Wolfe model using column generation (in each iteration, solve the master problem and the two subproblems with an LP package).

VC, 2010