Methods and Models for Combinatorial Optimization

Introduction

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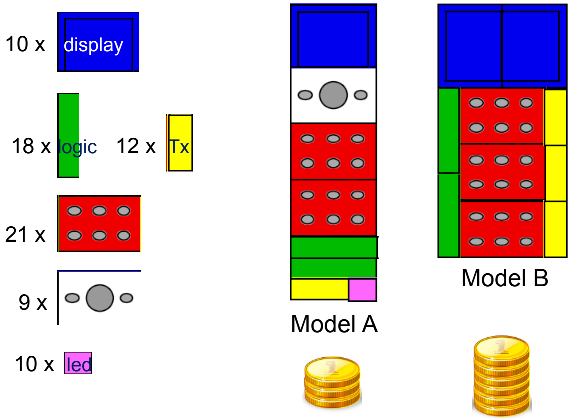
Course webpage

http://www.math.unipd.it/∼luigi/courses/metmodoc.html

Course goals

- Introduction to advanced modelling and solution techniques for combinatorial optimization problems in decision supporting.
- The course aims at providing mathematical and algorithmic tools to solve optimization problems of practical interest, also with the use of the most popular software packages or libraries.

Combinatorial optimization problem: example 1



Objective

Decision: How many A, B?

The space of feasible combinations

- "Easy" to find a feasible solution
- "Easy" to find the optimal solution if all the feasible combinations can be explored
- but, *what if the number of product models and/or resources is large?*

How to manage the combinatorial explosion of the size of the solution space?

- Quantum computing? Still "not operational..."
- In the (al least next) future: **MeMoC(O)**

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Combinatorial optimization problems: example 2

A farmer owns 12 hectares of land where he can grow potatoes or tomatoes. Beyond the land, the available resources are: 70 kg of tomato seeds, 18 tons of potato tubers, 160 tons of fertilizer. The farmer knows that all his production can be sold with a net gain of 3000 Euros per hectare of tomatoes and 5000 Euros per hectare of potatoes. Each hectare of tomatoes needs 7 kg seeds and 10 tons fertilizer. Each hectare of potatoes needs 3 tons tubers and 20 tons fertilizer. How does the farmer divide his land in order to gain as much as possible from the available resources?

Using a mathematical model: formulation

- Declare "what" is the solution, instead of stating "how" it is found
- What should we decide? **Decision variables**

  $x_T \geq 0,\ x_P \geq 0$

- What should be optimized? **Objective** as a function of the decision variables

  $\max 3000\, x_T + 5000\, x_P$

- What are the characteristics of the feasible combinations of values for the decisions variables? **Constraints** as mathematical relations among decision variables

$$\begin{array}{rcrcrl} x_T & + & x_P & \leq & 12 & \text{(land)} \\ 7\,x_T & & & \leq & 70 & \text{(tomato seeds)} \\ & & 3\,x_P & \leq & 18 & \text{(potato tubers)} \\ 10\,x_T & + & 20\,x_P & \leq & 160 & \text{(fertilizer)} \end{array}$$

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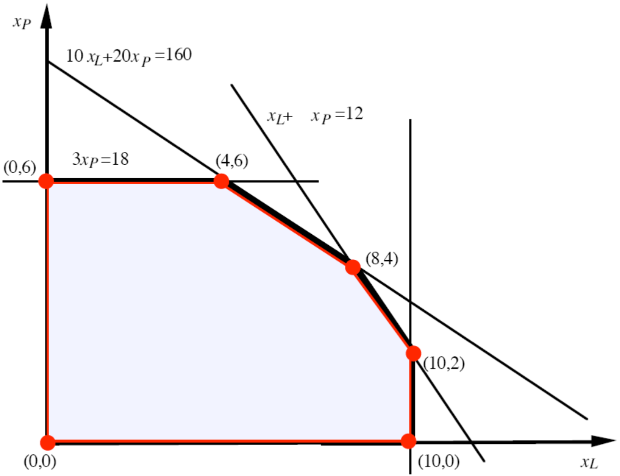
Using a mathematical model: solution!



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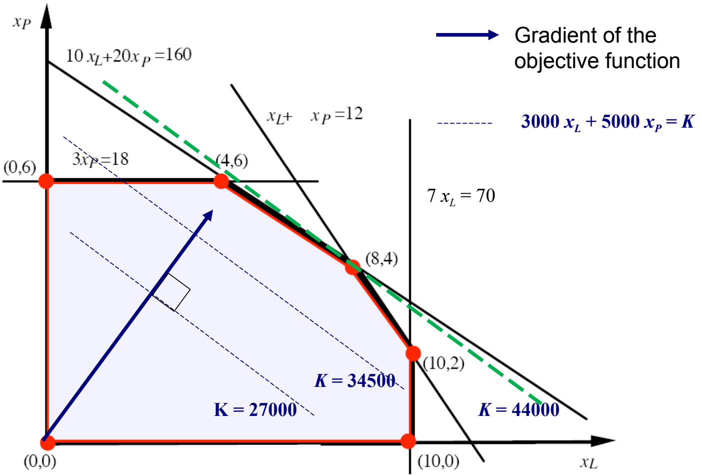

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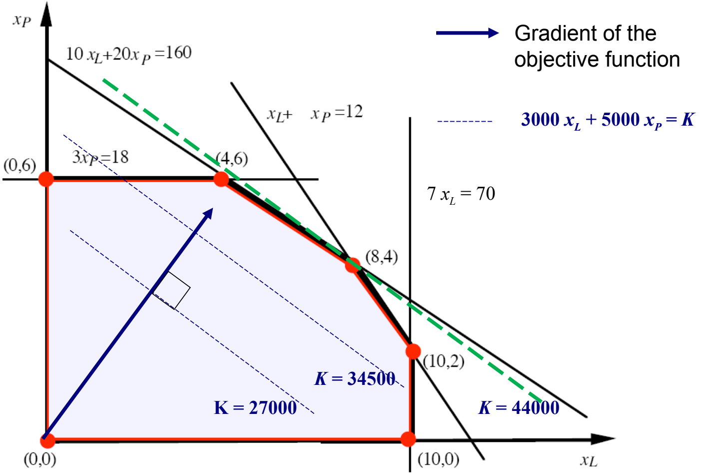
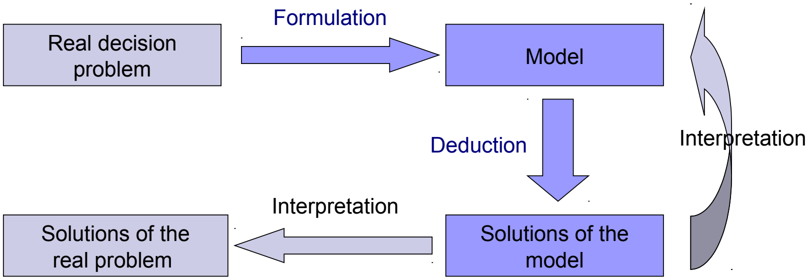


Linear relations: Linear Programming (LP) models

From decision problem to solution: the Operations Research approach



- **Formulation:** models (mathematical, graph, simulation, game theory), solution representation ...
- **Deduction:** quantitative methods, efficient algorithms

MeMoCO: Preliminary Programme

- Review, advanced topics and application of LP and Duality
  - LP models, simplex method, basic notions of duality theory
  - Column generation technique for large size linear programming models
  - Applications: production planning optimization, network flows
- Advanced methods for Mixed **Integer** Linear Programming (MILP)
  - Alternative formulations, Branch & Bound, Branch & Cut
  - Applications: TSP, Facility Location, Set Covering etc.
- **Meta-heuristics** for Combinatorial Optimization
  - Neighbourhood search and variants
  - Genetic Algorithms
- Network Optimization
  - Modelling optimization problems on graphs
- **Labs**
  - On-line optimization servers (e.g., NEOS)
  - Optimization software and Algebraic modelling languages
  - Optimization libraries (e.g. **Cplex**, Coin-OR, Scip)

Practical info

- **Schedule**:
  - Thursday, Friday 9:30 - 11:30
  - room 1BC50 **or** LabTA (**always check**!)
- **Textbooks and course material**
  - Lecture notes provided by the teacher + articles from scientific journals
  - Optimization software packages available on line and in labs
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  - **Two lab exercises**: implementation of 1) a MILP model and 2) a metaheuristic, to be delivered some days before the oral examination.
    Mandatory [1-10 /30, minimum 5]
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