Methods and Models for Combinatorial Optimization

Introduction

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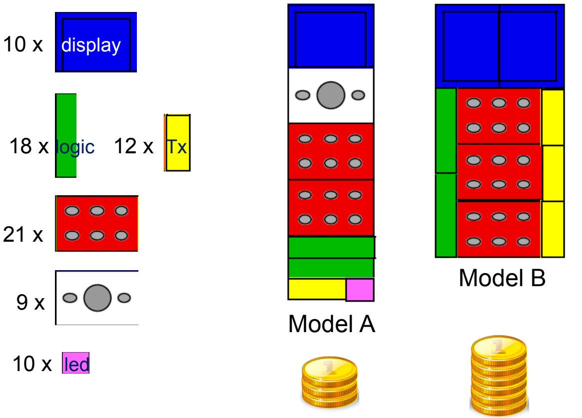
Course webpage

http://www.math.unipd.it/∼luigi/courses/metmodoc.html

Course goals

- Introduction to advanced modelling and solution techniques for combinatorial optimization problems in decision supporting.
- The course aims at providing mathematical and algorithmic tools to solve optimization problems of practical interest, also with the use of the most popular software packages or libraries.
- Find, understand, implement state-of-the-art approaches to solve real combinatorial optimization problems

Combinatorial optimization problem: example 1



Objective

Decision: How many A, B?

The space of feasible combinations

- "Easy" to find a feasible solution
- "Easy" to find the optimal solution if all the feasible combinations can be explored
- but, *what if the number of product models and/or resources is large?*

How to manage the combinatorial explosion of the size of the solution space?

- Quantum computing? Still "not operational..."
- Now and in the next future: **MeMoC(O)**

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Combinatorial optimization problems: example 2

A farmer owns 11 hectares of land where he can grow potatoes or tomatoes. Beyond the land, the available resources are: 70 kg of tomato seeds, 18 tons of potato tubers, 145 tons of fertilizer. The farmer knows that all his production can be sold with a net gain of 3000 Euros per hectare of tomatoes and 5000 Euros per hectare of potatoes. Each hectare of tomatoes needs 7 kg seeds and 10 tons fertilizer. Each hectare of potatoes needs 3 tons tubers and 20 tons fertilizer. How does the farmer divide his land in order to gain as much as possible from the available resources?

Using a mathematical model: formulation

- Declare "what" is the solution, instead of stating "how" it is found
- What should we decide? **Decision variables**

  $x_T \geq 0,\ x_P \geq 0$

- What should be optimized? **Objective** as a function of the decision variables

  $\max 3000\,x_T + 5000\,x_P$

- What are the characteristics of the feasible combinations of values for the decisions variables? **Constraints** as mathematical relations among decision variables

$$\begin{array}{rcrcrl} x_T & + & x_P & \leq & 12 & \text{(land)} \\ 7\,x_T & & & \leq & 70 & \text{(tomato seeds)} \\ & & 3\,x_P & \leq & 18 & \text{(potato tubers)} \\ 10\,x_T & + & 20\,x_P & \leq & 160 & \text{(fertilizer)} \end{array}$$

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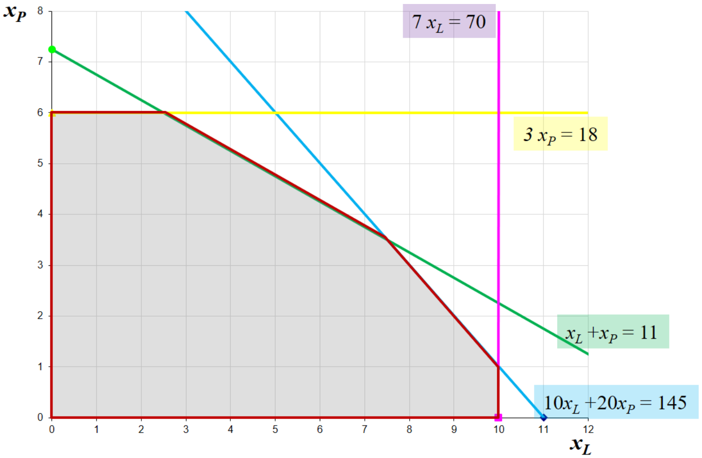

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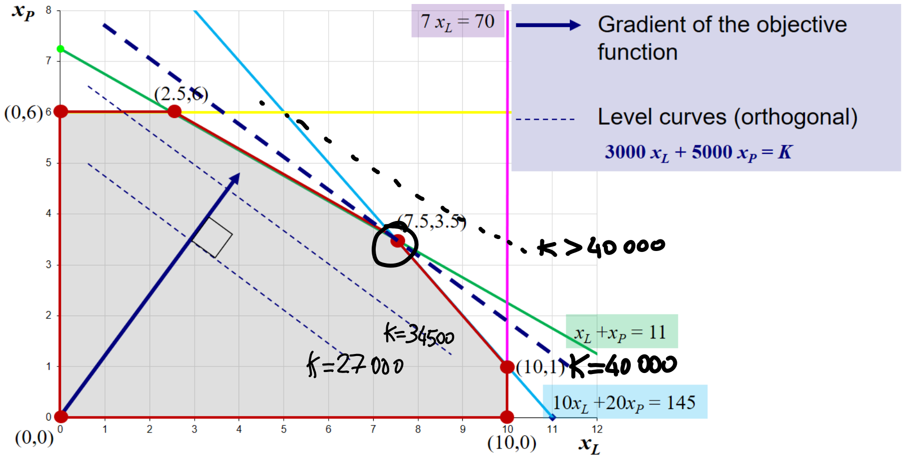

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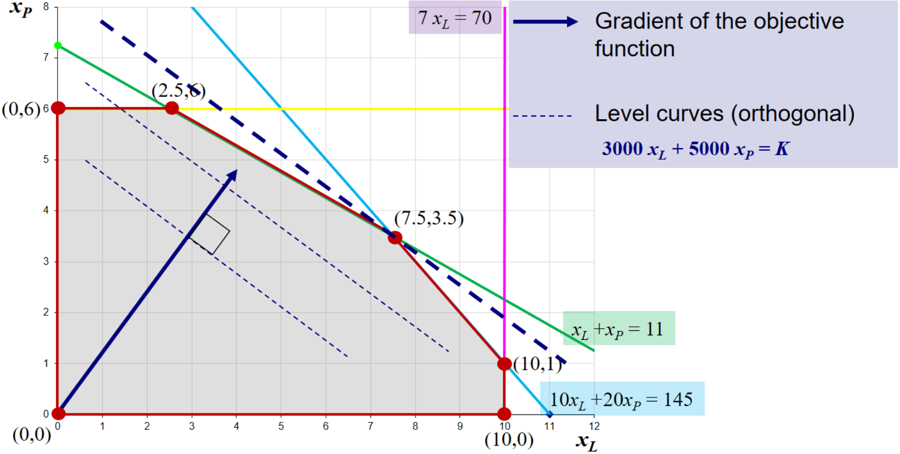
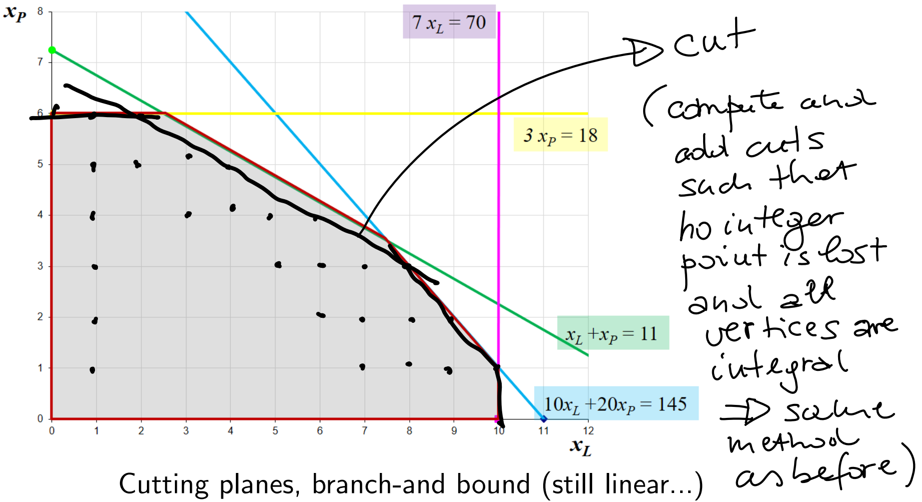


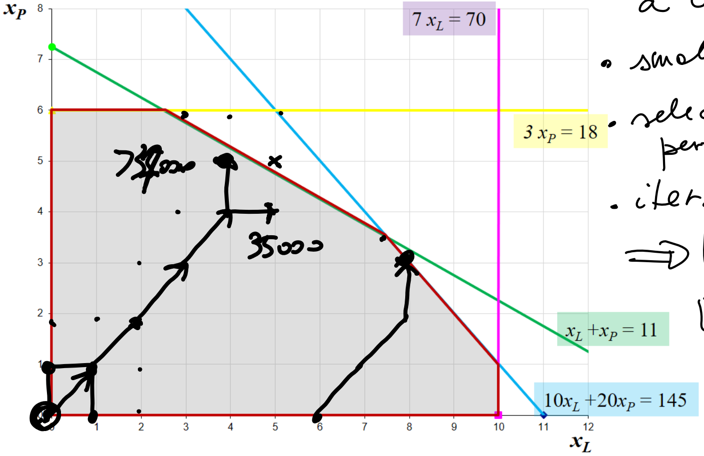
Linear relations: Linear Programming (LP) models

Example: integer variables - exact method



Cutting planes, branch-and bound (still linear...)

Example: integer variables - heuristic method



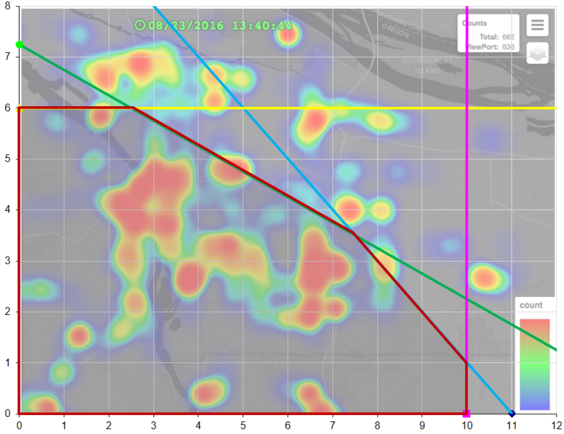
- start from a combination
- small perturbations
- select best perturbation
- iterate

⇒ local optimum

[start from different combinations ⇒ other (better) local optima]

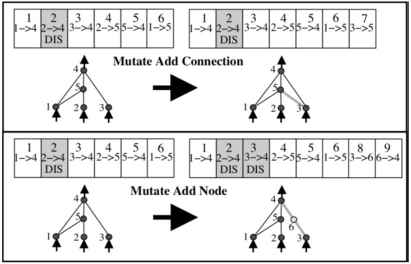
neighborhood search, evolutionary computation etc.

Example: a more general combinatorial optimization problem



exact methods (critical) heuristic (still works)

Example: applications



Stanley et al. 2002, Evolving Neural Networks through Augmenting Topologies, Evolutionary Computation Journal

Beccaro et al. 2018, Tabu searc approach, Master Thesis

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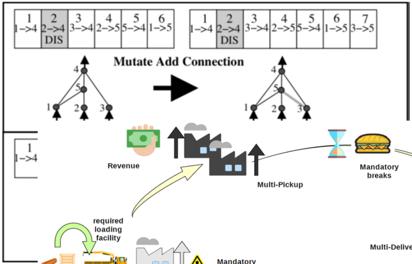

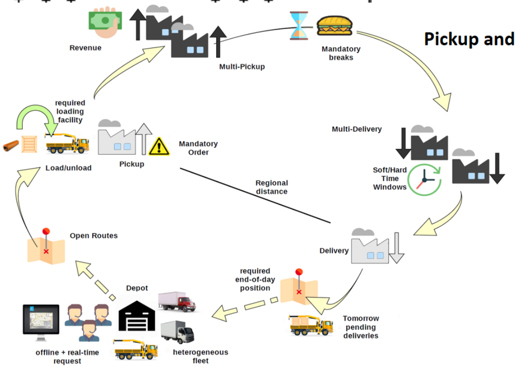

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Pickup and delivery routes in freight transportation

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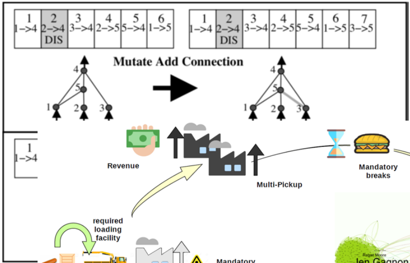

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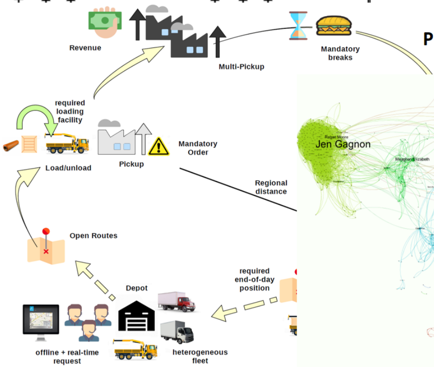
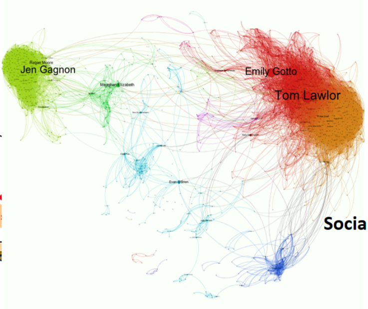
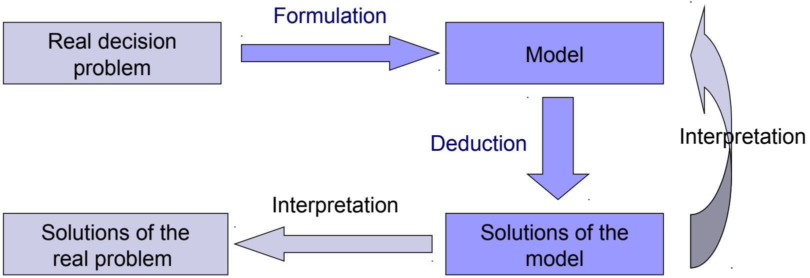




Social Network Analysis

From decision problem to solution: the Operations Research approach



- **Formulation:** models (mathematical, graph, simulation, game theory), solution representation/perturbation, data driven ...
- **Deduction:** quantitative methods, artificial intelligence, efficient algorithms

MeMoCO: Preliminary Programme

- Review, advanced topics and application of LP and Duality
  - LP models, simplex method, basic notions of duality theory
  - Column generation technique for large size linear programming models
  - Applications: production planning optimization, network flows
- Advanced methods for Mixed **Integer** Linear Programming (MILP)
  - Alternative formulations, Branch & Bound, Branch & Cut
  - Applications: TSP, Facility Location, Set Covering etc.
- **Meta-heuristics** for Combinatorial Optimization
  - Neighbourhood search and variants
  - Genetic Algorithms
- Relevant applications
  - Network Optimization: modelling optimization problems on graphs
  - Data driven optimization (Air Traffic Management)
  - ...
- **Labs**
  - On-line optimization servers (e.g., NEOS)
  - Optimization software and Algebraic modelling languages (e.g. AMPL, **IBM-OPL**)
  - Optimization libraries (e.g. **IBM Cplex**, Coin-OR, Scip)

Practical info

- **Schedule**:
  - Thursday, Friday 8:30 - 10:30
  - room 1BC50 **or** LabTA (**always check**!)
- **Textbooks and course material**
  - Lecture notes provided by the teacher + articles from scientific journals (available **before** the class: read them!)
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- **Examination methods**
  - **Two lab exercises**: implementation of 1) a MILP model and 2) a metaheuristic, to be delivered some days before the oral examination.
    Mandatory [1-10 /30, minimum 5]
  - **Oral examination** on course contents.
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