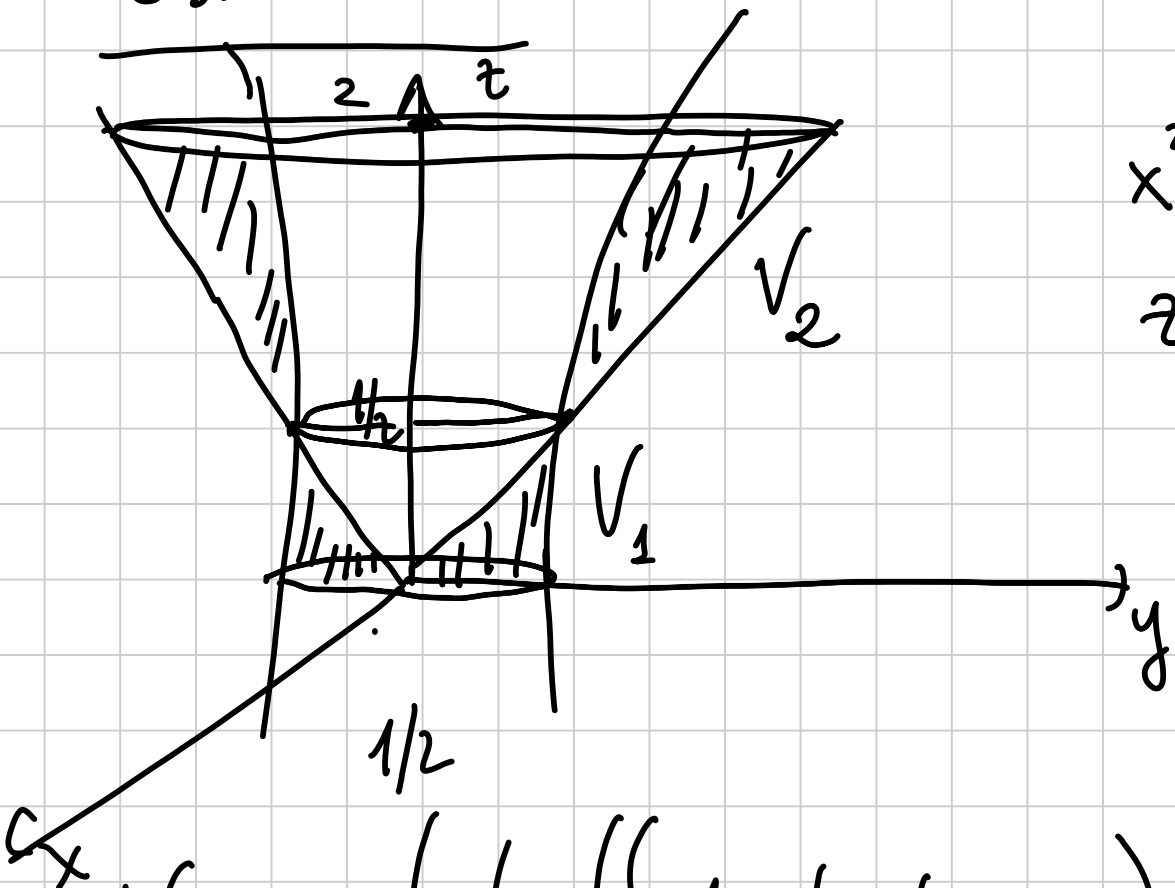
Es. di ieri



$$\frac{x^2+y^2}{5} = z^2$$

$$x^2+y^2 = 1+z^2$$

$$z \in [0,2]$$

$$V(S) = ?$$
$$= V_1 + V_2$$

$$V_1 = \int_0 \left(\iint_{5z^2 \le x^2+y^2 \le 1+z^2} 1 \, dx\, dy \right) dz =$$

$$\begin{cases} x = \rho\cos\varphi \\ y = \rho\sin\varphi \\ \varphi \in [0, 2\pi] \end{cases}$$

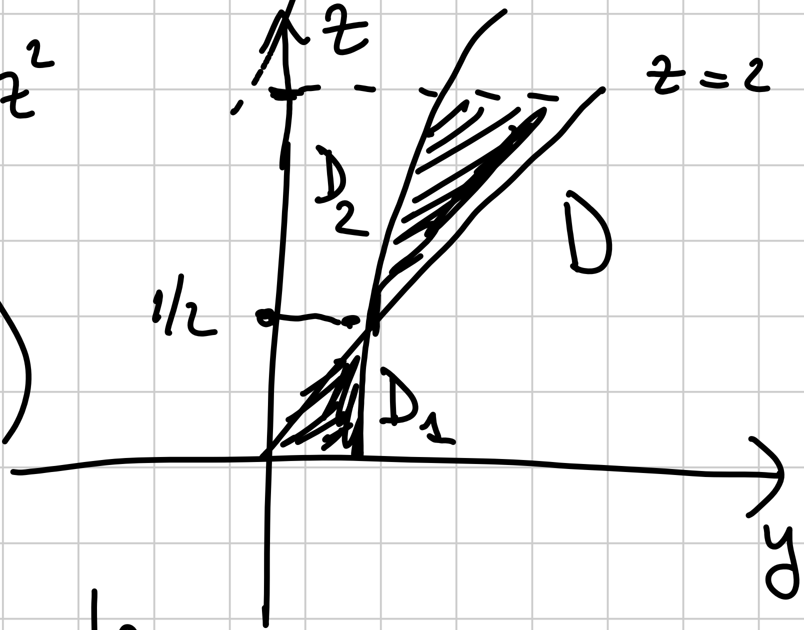
$$\sqrt{5}\,z \le \rho \le \sqrt{1+z^2}$$

$$= \int_0^{1/2} \left(\int_0^{2\pi} \int_{\sqrt{5}z}^{\sqrt{1+z^2}} \rho \, d\rho \, d\varphi \right) dz =$$

$$= 2\pi \int_0^{1/2} \left(\frac{1}{2} \rho^2 \Big|_{\rho=\sqrt{5}z}^{\rho=\sqrt{1+z^2}} \right) dz =$$

$$= \frac{2\pi}{\cancel{2}} \int_0^{1/2} \left(1 + z^2 - 5z^2 \right) dz = \pi \left(z - \frac{4}{3} z^3 \right) \Big|_{z=0}^{z=1/2} = \cdot$$

$$V_2 = \int_{1/2}^{2} \left(\iint_{1+z^2 \le x^2+y^2 \le 5z^2} 1 \, dx\, dy \right)$$

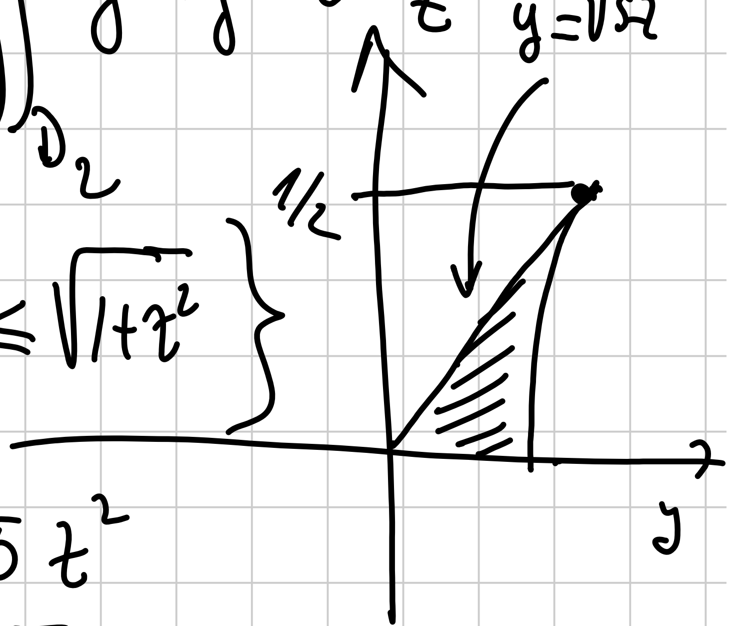


Con Guldino (provare)

$$V(S) = 2\pi \iint_D y \, dy\, dz$$

$$\iint_D y\,dy\,dz = \iint_{D_1} y\,dy\,dz + \iint_{D_2} y\,dy\,dz$$

$$D_1 = \left\{0 \le z \le 1/2,\ \sqrt{5z} \le y \le \sqrt{1+z^2}\right\}$$



$$\frac{x^2+y^2}{5} = z^2 \qquad x=0 \qquad y^2 = 5z^2$$

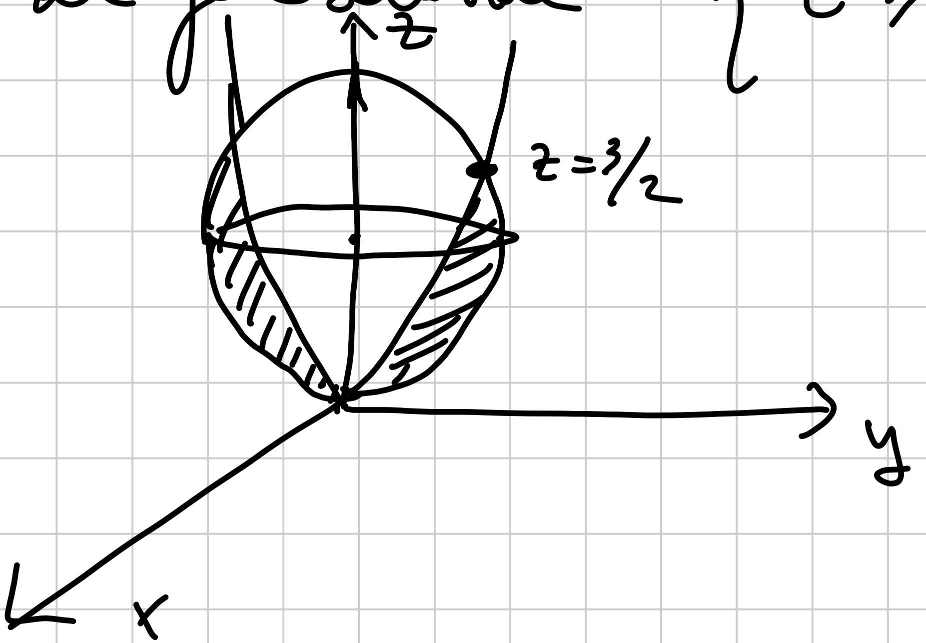
$$y = \sqrt{5}\,z$$

$$x^2+y^2 = 1+z^2 \qquad x=0 \qquad y^2 = 1+z^2$$

$$y = \sqrt{1+z^2}$$

Es. per casa

$V(E)$ dove E regione contenuta nella sfera $\{x^2+y^2+z^2 \le 2z\}$ ~~ed~~ esterna al paraboloide $\{z \geq 2(x^2+y^2)\}$



$$x^2+y^2+(z-1)^2 \le 1$$

centro in $(0,0,1)$

$$R = \frac{9}{16}\pi$$

Equazioni differenziali

Es. Voglio trovare $y(x)$ t.c.

$$y' = y$$

$$e^x = e^x$$

$$y(x) = e^x$$

$$y'(x) = e^x$$

$$y(x) = Ce^x$$

$$C \in \mathbb{R}$$

$$y' = Ce^x$$

$$y(x) = 5e^x$$

$$y'(x) = 5e^x$$

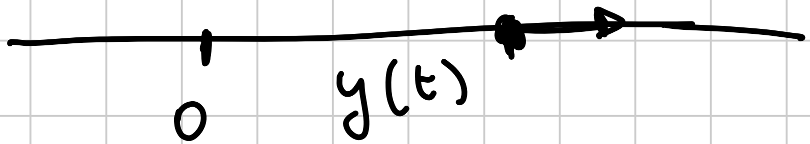
$$y' = y$$ NO

$$y(x) = \operatorname{sen} x$$ non è soluzione

$$y'(x) = \cos x$$

$$\cos x = \operatorname{sen} x \quad \forall x$$

Es.



$y(t)$ posizione del punto

$$F = ma \quad / \quad F(t, y(t), \dot{y}(t)) = m\,\ddot{y}(t)$$

$$\cancel{m}\,\ddot{y} = -\cancel{m}\,g \qquad g \text{ costante}$$

$$\ddot{y} = -g$$

In generale un'equazione differenziale è un'equazione del tipo $\quad y = y(x)$

$$f(x, y, y', y'', \ldots\, y^{(n)}) = 0$$

di ordine $\underline{\underline{n}}$.

$y = y(x)$ è la funzione incognita.

Eq. differenziali del 1° ordine

$$F\left(x, y(x), y'(x)\right) = 0 \qquad (E)$$

Def. $y(x)$ è soluzione dell'eq. (E) se $y(x)$ è derivabile in I e se $F\left(x, y(x), y'(x)\right) = 0$, $\forall x \in I$.

es. $y' = 3y$

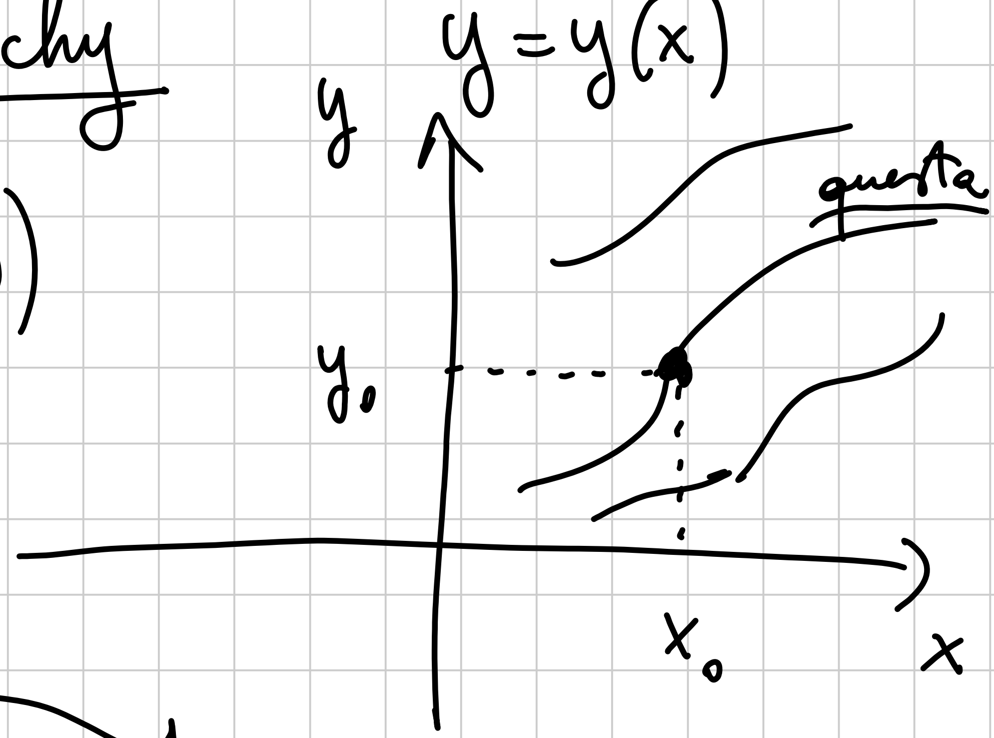
$y(x) = -e^{3x}$ è soluzione?

$y'(x) = -3e^{3x}$

$-3e^{3x} = 3\left(-e^{3x}\right) \quad \forall x$

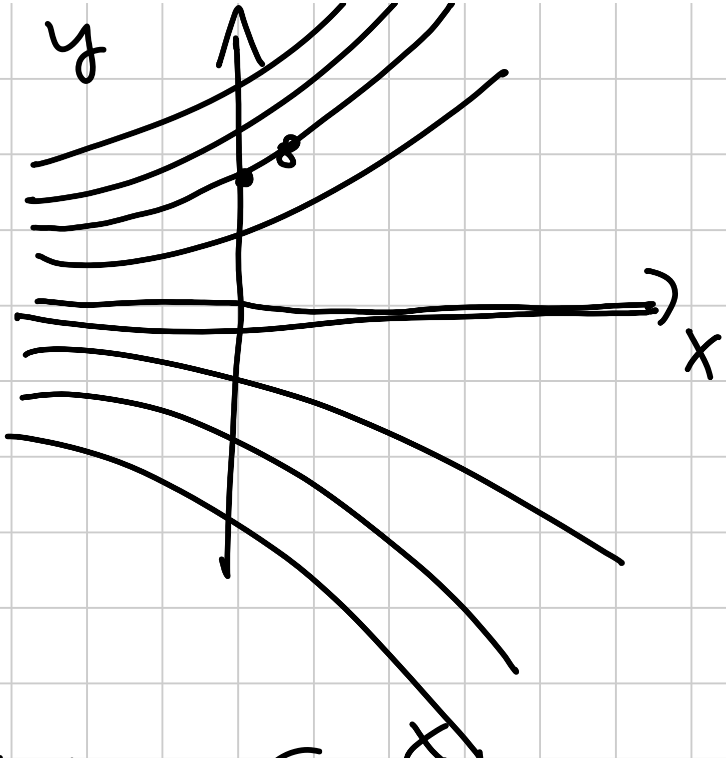
Problema di Cauchy

$$\begin{cases} y' = f(x,\ y(x)) \\ y(x_0) = y_0 \end{cases}$$



es. $\begin{cases} y' = y \\ y(0) = 8 \end{cases} \longrightarrow y(x) = Ce^{x}, \quad C \in \mathbb{R}$

soluzioni dell'eq. diff.



$\forall C \in \mathbb{R}$ tutte soluzioni

$y' = y$

$y(x) = Ce^{x}$

$y(0) = C = 8$ $\Longrightarrow$ $y(x) = 8e^{x}$

è la soluzione del pb. di Cauchy

Es.

g assegnata

$$y' = g(x)$$ fun es.

$$y' = \operatorname{sen} x \implies y(x) = \int y'(x)\,dx + C$$

$$= \int \operatorname{sen} x \, dx + C$$

$$y(x) = -\cos x + C$$

$$y' = g(x) \implies y(x) = \int g(x)\,dx + C$$

Due classi di eq. del 1° ordine

- a variabili separabili
- lineari

Eq. a variabili separabili

$$y' = f(x) \cdot g(y)$$

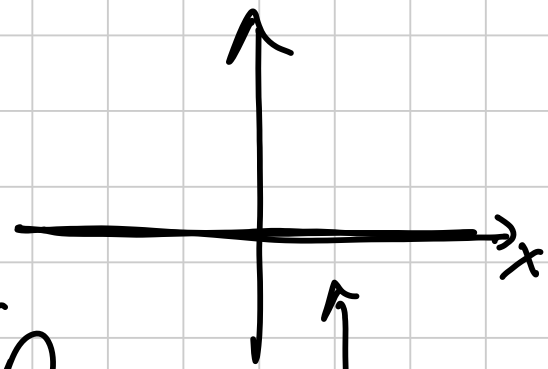
es.

$y' = x y^2$

$y' = \log y \, e^x$

$y' = xy + \operatorname{sen} y \rightarrow$ NO

$y' = y^2$



$y(x) \equiv 0$

Voglio $y(x) \neq 0$, $\forall x \in I$

$\frac{y'}{y^2} = 1 \quad \forall x \quad \Rightarrow \quad \int \frac{y'(x)}{y^2}\,dx = \int 1\,dx$

$$\int \frac{dt}{t^2} = x + C$$

$$y(x) = t$$

$$y'(x)\,dx = dt$$

$$-\frac{1}{t} = x + C \implies -\frac{1}{y} = x + C$$

$$\frac{1}{y} = -(x+C)$$

$$y(x) = -\frac{1}{x+C}$$

tutte le soluzioni $\neq 0$ di $y' = y^2$ $\quad C \in \mathbb{R}$

$+ \; y \equiv 0$.

$$y' = y^2 \qquad y(x) \not\equiv 0, \ \forall x$$

$$\frac{y'}{y^2} = 1 \qquad \int \frac{dy}{y^2} = \int 1 \, dx$$

es.

$$y' = x\, y^2 \qquad y \equiv 0$$

$$\frac{y'}{y^2} = x \qquad \int \frac{dy}{y^2} = \int x \, dx$$

$$-\frac{1}{y} = \frac{x^2}{2} + C$$

$$\frac{1}{y} = -\left(\frac{x^2}{2} + C\right)$$

$$y(x) = \frac{-1}{\frac{x^2}{2} + C}, \quad C \in \mathbb{R}$$

$$y' = x y^2$$

risolve $\forall x \in \mathbb{R}$

es.

$C = -1$

$|x| = 2$

In <u>generale</u>

$$y'(x) = f(x) \cdot g(y)$$

1) $g(y_0) = 0$ $\quad y(x) = y_0$ è soluzione dell'equazione

$y' = 0$

2) Cerco y t.c. $g(y) \neq 0$

$$\frac{y'}{g(y)} = f(x)$$

$$\underbrace{\int \frac{dy}{g(y)}}_{\substack{\text{primitiva di } \frac{1}{g} \\ \text{(funzione di } y)}} = \underbrace{\int f(x)\,dx}_{\substack{\text{primitiva di } f \\ \text{(funzione di } x)}}$$

e poi si cerca di esplicitare in y

e quindi $y = y(x)$.

s. $y' = \underbrace{\log x}_{f} \underbrace{e^{y}}_{g}$

1) $g(y) = 0$? $e^{y} \neq 0$

2) $\dfrac{y'}{e^{y}} = \log x$ $\displaystyle\int \frac{dy}{e^{y}} = \int \log x \, dx$

$-e^{-y}$ $\displaystyle\int 1 \log x \, dx = x \log x - \int \cancel{x} \frac{1}{\cancel{x}} dx$

$-e^{-y} = x \log x - x + C$

$$e^{-y} = -x \lg x + x + C$$

$$-y = \log(-x \lg x + x + C)$$

$$\boxed{y = -\log(-x \lg x + x + C)}$$

<u>Pb. di Cauchy</u>

$$\begin{cases} y' = \lg x \, e^{y} \\ y(1) = -5 \end{cases}$$

$x = 1$
$y = -5$

$-5 = -\log(1+C)$

$5 = \log(1+C) \qquad e^5 = 1+C$

$C = e^5 - 1$

la soluzione del pb di Cauchy

è $\boxed{y(x) = -\log\left(x\log x + x + e^5 - 1\right)}$