# Esercizio

$$\begin{cases} y' = \dfrac{x^2}{\operatorname{sen} y \cos y} \\ y(0) = -\dfrac{\pi}{4} \end{cases}$$

$$\Rightarrow f(x) \cdot \underbrace{g(y)}$$

$g(y) = 0$

$y = y_0$

no soluzioni costanti

$$\int \operatorname{sen} y \cos y \, dy = \int x^2 dx$$

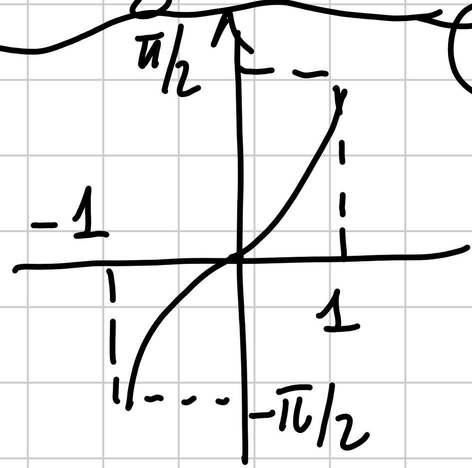
$$\frac{1}{2} \operatorname{sen}^2 y = \frac{x^3}{3} + C \qquad y = \dots$$

$$\operatorname{sen}^2 y = \frac{2}{3}x^3 + C$$

$$0 < \frac{2}{3}x^3 + C \le 1 \quad (2)$$

$$\operatorname{sen} y = \pm\sqrt{\frac{2}{3}x^3 + C}$$

$$\boxed{y = \operatorname{arcsen}\left(\pm\sqrt{\frac{2}{3}x^3 + C}\right)}$$



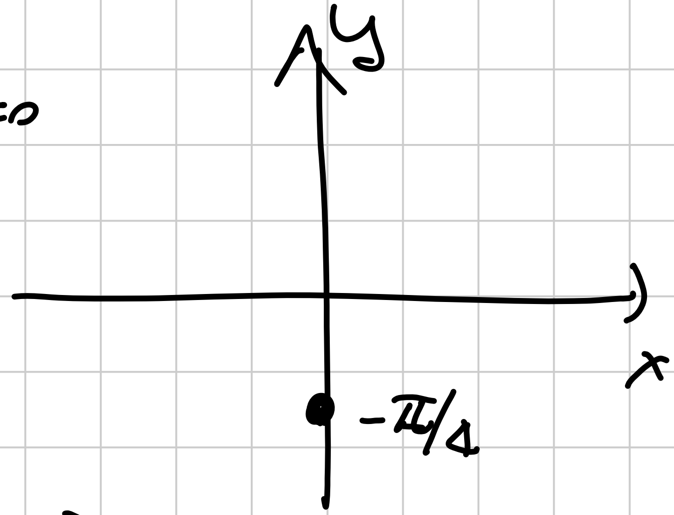

arcsen( )

(1) $|y(x)| \le \frac{\pi}{2}$

$y(0) = -\pi/4$

Pb di Cauchy :

Poiché $y = -\pi/4$ in $x=0$




$$\operatorname{sen}\left(-\frac{\pi}{4}\right) < 0$$

quindi scelgo

$$\operatorname{sen} y = -\sqrt{\frac{2}{3}x^3 + C} \Longrightarrow$$

sen

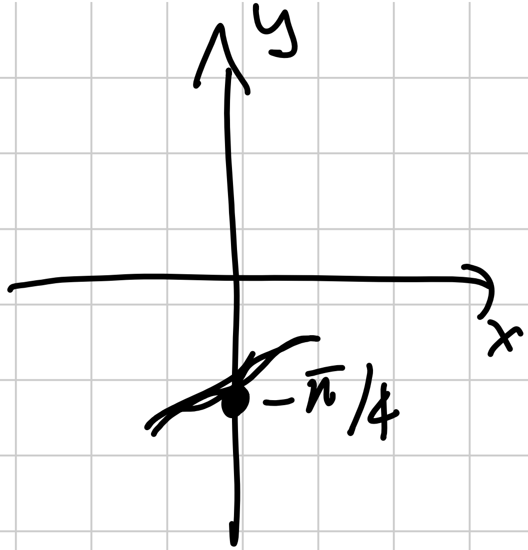
$$y = \arcsin\left(-\sqrt{\frac{2}{3}x^3 + C}\right)$$

$$y = -\pi/4$$
$$x = 0$$

$$\operatorname{sen}\left(-\frac{\pi}{4}\right) = -\sqrt{C}$$

$$\left( -\frac{\sqrt{2}}{2} = -\sqrt{C} \qquad C = 1/2 \right.$$

$$y = \arcsin\left(-\sqrt{\frac{2}{3}x^3 + \frac{1}{2}}\right)$$




è definita $\forall x$ : $\quad 0 \le \frac{2}{3}x^3 + \frac{1}{2} \le 1$

$$\Downarrow$$

$$a \le x \le b$$

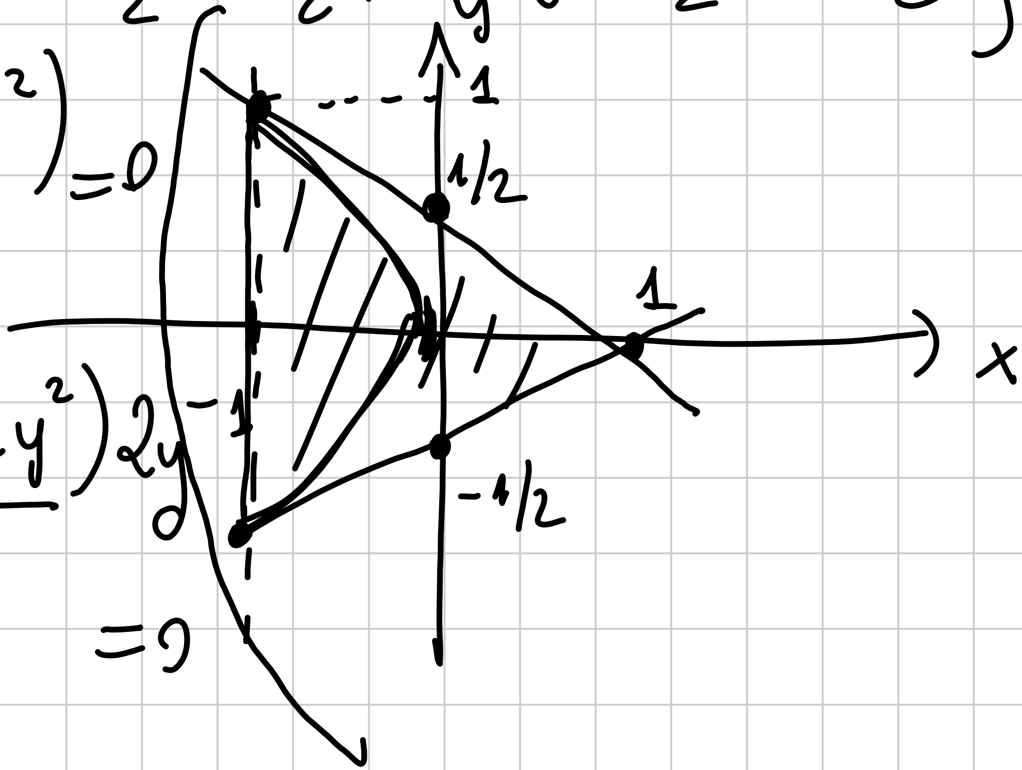
**Esercizio** Trovare max e min assoluti

di $f(x,y) = \operatorname{arctg}\,(x+y^2)^2$ in $E$

$$E = \left\{ x \geq -1,\ \ y + \frac{x}{2} \leq \frac{1}{2},\ \ y - \frac{x}{2} \geq -\frac{1}{2} \right\}$$

$$f_x = \frac{1 \cdot 2(x+y^2)}{1+(x+y^2)^4} = 0$$

$$f_y = \frac{1 \cdot 2(x+y^2)\,2y}{1+(x+y^2)^4} = 0$$



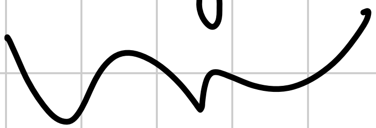
$x+y^2=0$ $\quad f_x = 0 = f_y$

$x=-y^2$ curva di punti critici

$y=0 \Rightarrow x=0$

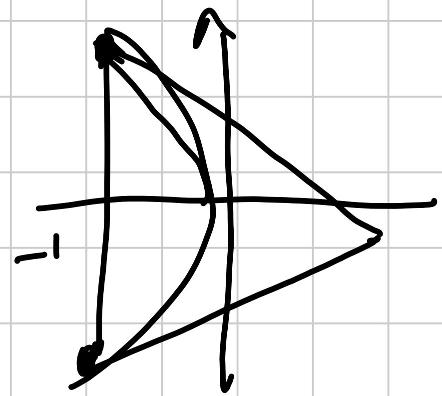
Cosa succede sulla curva $\gamma$ di p.ti critici

$x+y^2=0$ $\quad f(x,y) = $ [illegible] $(x+y^2)^2 \geq 0$



$f|_\gamma = 0$

i punti di $\gamma$ sono p.ti di min. assoluto



Cerchiamo i max assoluti

in $x = -1$ $\quad f\big|_{\{x=-1\}} = \operatorname{arctg}\left(y^2-1\right)^2 = g(y)$

$$g'(y) = \frac{1}{1+(\quad)^2} \cdot 2\left(y^2-1\right) \cdot 2y$$

$y = \pm 1 \longrightarrow$

$y = 0$ $\quad g(0) = \operatorname{arctg} 1 = \pi/4 \longrightarrow ?$

Su $y = \frac{1}{2} - \frac{x}{2}$

$$f/_{\text{retta}} = \operatorname{arctg}\left(x + \frac{1}{4} + \frac{x^2}{4} - \frac{x}{2}\right)^2 =$$

$$= \operatorname{arctg}\left(\frac{x^2}{4} + \frac{x}{2} + \frac{1}{4}\right)^2 = g(x)$$

$$g'(x) = \frac{1}{1+(\ )^4} \cdot 2\left(\frac{x^2}{4} + \frac{x}{2} + \frac{1}{4}\right)\left(\frac{x}{2} + \frac{1}{2}\right)$$

$$= \frac{1}{1+(\ )^4} \cdot \frac{2}{4}(x+1)^2 \underbrace{\frac{(x+1)}{2}}_{\geq 0} = 0$$

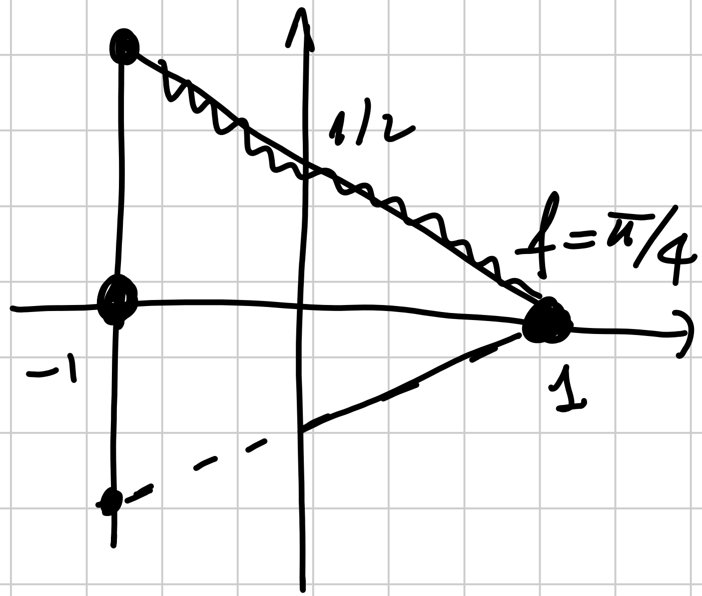
$x = -1 \Rightarrow$ minimo assoluto

$g'(x)$

$x = 1$

$y = 0$

$f(1,0) = \operatorname{arctg} 1 = \frac{\pi}{4}$




- Analog. su

$y = \frac{x}{2} - \frac{1}{2}$

due punti di max assoluto in

$(-1, 0)$ e $(1, 0)$ $\quad \max_E f = \pi/4$

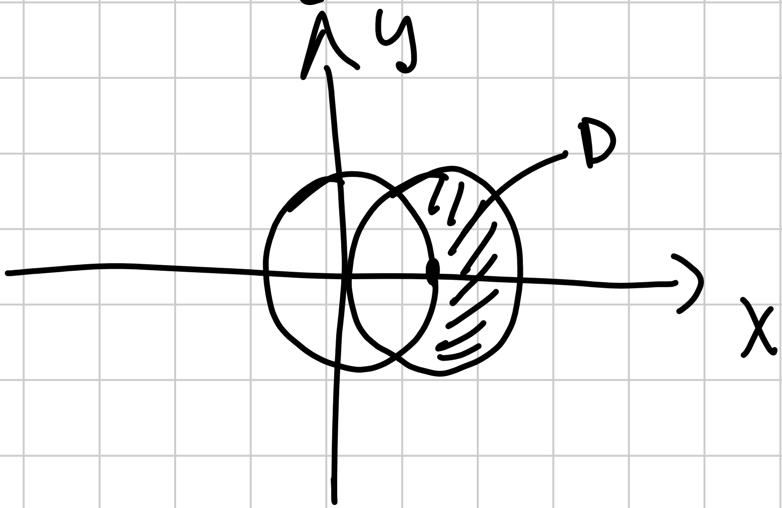
Curva di punti di min. assoluto $\quad y^2 + x = 0$

$$\min_E f = 0$$

Esercizio Calcolare

$$\iiint_{\Omega} \frac{y^2}{x^2+y^2}\,dx\,dy\,dz$$

$$\Omega = \left\{ (x,y,z) \in \mathbb{R}^3 : \quad 1 < x^2+y^2 < 2x \ ; \quad 0 \le z \le \frac{x^2+y^2}{x^2} \right\}$$



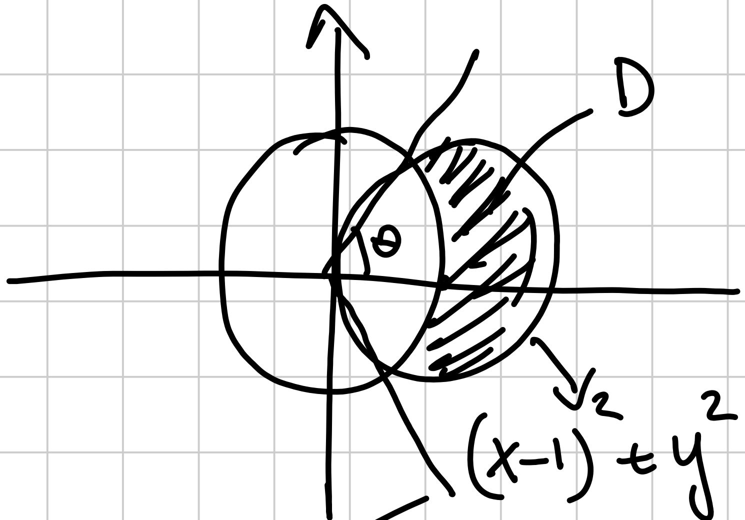

$$(x-1)^2+y^2<1$$

Integrazione con le formule di riduzione

$$I = \iint_D \left( \int_0^{\frac{x^2+y^2}{x^2}} \frac{y^2}{x^2+y^2} \, dz \right) dx\,dy =$$

$$= \iint_D \frac{y^2}{\cancel{x^2+y^2}} \cdot \frac{\cancel{x^2+y^2}}{x^2} \, dx\,dy =$$

$$= \iint_D \frac{y^2}{x^2}\, dx\, dy$$




$$\begin{cases} x = \rho\cos\theta \\ y = \rho\,\mathrm{sen}\,\theta \end{cases}$$

$x^2+y^2>1$

$\rho > 1$

$$x^2-2x+y^2 \le 0$$

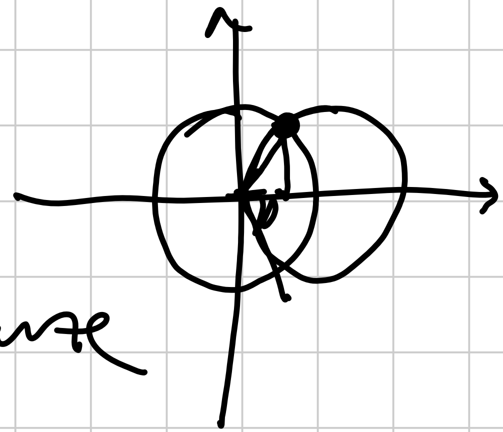
$$\rho^2\cos^2\theta - 2\rho\cos\theta + \rho^2\mathrm{sen}^2\theta \le 0$$

$$\rho^2 - 2\rho\cos\theta \le 0$$

$$\rho \le 2\cos\theta$$

$$1 \leq \rho \leq 2\cos\theta$$

$$\theta \in ?$$



Intersezione delle due circonferenze

$$\begin{cases} x^2+y^2=1 \\ x^2+y^2=2x \end{cases} \qquad 2x=1 \qquad \begin{cases} x=1/2 \\ y=\frac{\sqrt{3}}{2} \end{cases}$$

$$\begin{cases} \frac{1}{2}=\cos\theta \\ \frac{\sqrt{3}}{2}=\operatorname{sen}\theta \end{cases} \Rightarrow \theta=\pi/3$$

$$\theta \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$$

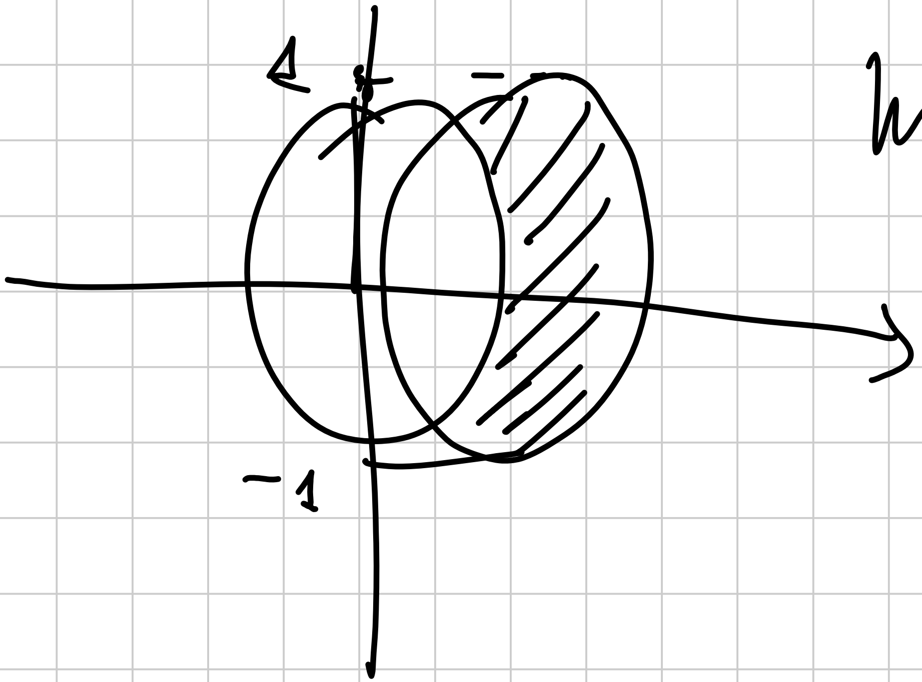
$$\iint_D \frac{y^2}{x^2}\, dx\, dy = \int_{-\pi/3}^{\pi/3} \int_{1}^{2\cos\theta} \frac{\cancel{\rho^2} \operatorname{sen}^2\theta}{\cancel{\rho^2} \cos^2\theta}\, \rho\, d\rho\, d\theta$$

$$= \int_{-\pi/3}^{\pi/3} \frac{\operatorname{sen}^2\theta}{\cos^2\theta} \left. \frac{\rho^2}{2} \right|_{\rho=1}^{\rho=2\cos\theta} d\theta = \frac{1}{2} \int_{-\pi/3}^{\pi/3} \frac{\operatorname{sen}^2\theta}{\cos^2\theta} \left(4\cos^2\theta - 1\right) d\theta$$

$$= \frac{1}{2} \int_{-\pi/3}^{\pi/3} 4 \operatorname{sen}^2\theta\, d\theta - \frac{1}{2} \int_{-\pi/3}^{\pi/3} \operatorname{tg}^2\theta\, d\theta =$$

$= \dots\dots$

$$R = \pi - \frac{3}{2}\sqrt{3}$$

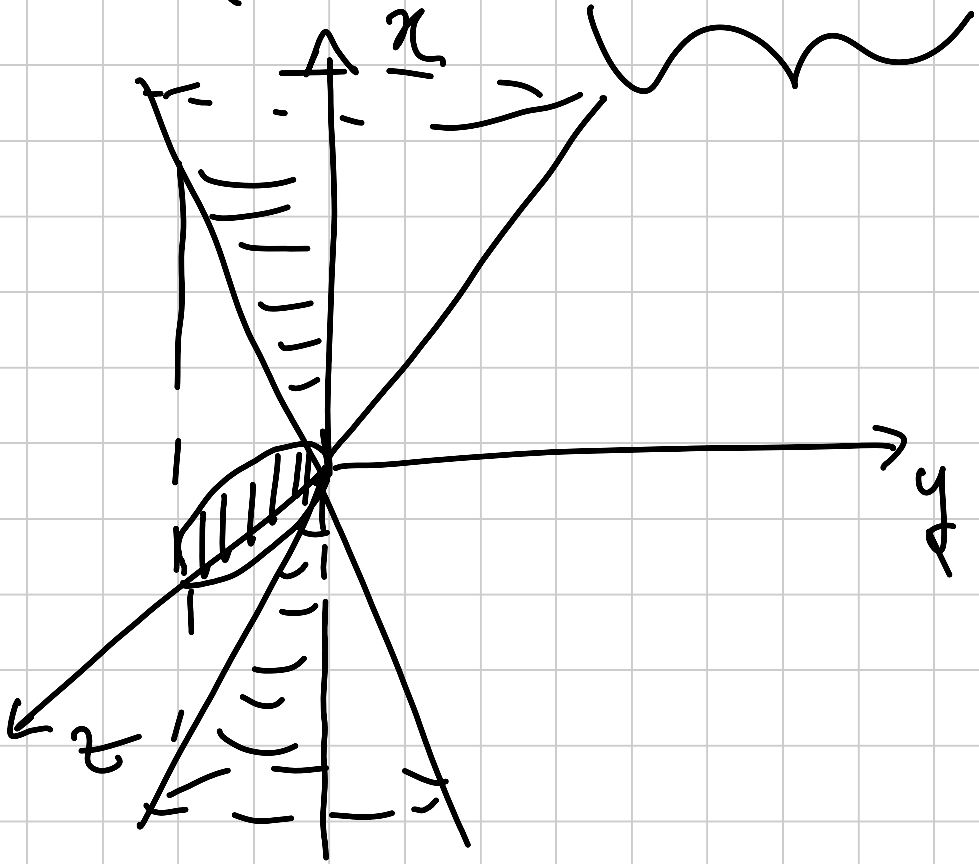



ho!

## Esercizio

Calcolare l'area di

$$S = \left\{ (x,y,z) : \quad y^2+z^2 = x^2 , \quad y^2+z^2 \le 4z \right\}$$




$$y^2 + (z-2)^2 \le 4$$

$(0,2)$

$$A(S) = 2A(S^+)$$ $S^+$ quello con $x > 0$

$$A(S^+) = \iint_{(u,v)\in D} \|\sigma_u \wedge \sigma_v\| \, du\, dv$$ $u, v$

$$y^2 + z^2 = x^2$$

$$\|\sigma_u \wedge \sigma_v\| = \sqrt{1 + f_y^2 + f_z^2}$$

$$x = \sqrt{y^2 + z^2} = f(y, z)$$

è superficie in forma cartesiana

$$(y, z) \; : \; y^2 + (z-2)^2 \leq 4$$

$$f_y = \frac{1}{\cancel{2}\sqrt{y^2+z^2}}\cancel{2}y \qquad f_z = \frac{\cancel{2}z}{\cancel{2}\sqrt{y^2+z^2}}$$

$$\|\sigma_u \wedge \sigma_v\| = \sqrt{1 + \frac{y^2}{y^2+z^2} + \frac{z^2}{y^2+z^2}} = \sqrt{2}$$

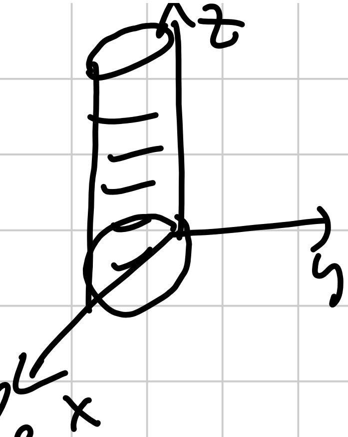
$$A(S^+) = \iint_D \sqrt{2}\, dy\, dz \qquad y^2 + (z-2)^2 \leq 4$$

$$= \sqrt{2}\,(\text{Area } D) = \sqrt{2}\,\pi\, 4$$

Esercizio Flusso del campo

$$F = \left( y, \ x, \ z\sqrt{x^2+y^2} \right)$$

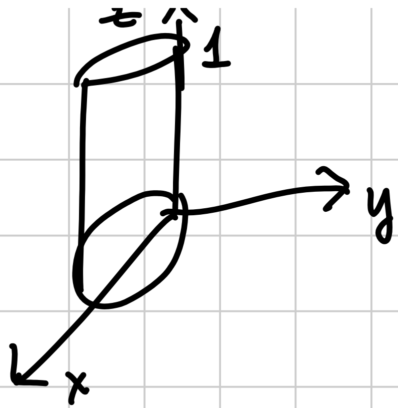
uscente dalla superficie laterale del cilindro




$$S = \left\{ x^2 - 2x + y^2 \leq 0, \quad 0 < z < 1 \right\}$$

$$(x-1)^2 + y^2 \leq 1$$

**1° modo** $\quad \text{Flusso} = \displaystyle\int_{\text{superficie laterale cilindro}} F \cdot N \, dS$




$$\begin{cases} (x-1)^2 + y^2 = 1 \\ z \in (0,1) \end{cases}$$

$$\theta \in [0, 2\pi] \qquad z \in (0,1) \qquad \begin{cases} x - 1 = \cos\theta \\ y = \operatorname{sen}\theta \\ z = z \end{cases}$$

N

$$\sigma_u \wedge \sigma_v = \begin{pmatrix} -\operatorname{sen}\theta & 0 \\ \cos\theta & 0 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ -\operatorname{sen}\theta & 0 \end{pmatrix}$$

vettore normale $= (\cos\theta, \ \operatorname{sen}\theta, \ 0)$

$$\text{Flusso} = \int_0^{2\pi}\int_0^1 F \cdot (\cos\theta, \operatorname{sen}\theta, 0)\, d\theta\, dz$$

$$F = \left(y, x, \ z\sqrt{x^2+y^2}\right)$$

$x^2 = \operatorname{sen}^2\theta$

$y^2 = 1 + \cos^2\theta + 2\cos\theta$

$x^2 + y^2 = 2 + 2\cos\theta$

$$= \left(\operatorname{sen}\theta, \ 1+\cos\theta, \ z\sqrt{2+2\cos\theta}\right)$$

$\Rightarrow$

Finire $\Rightarrow$ Flusso $= 0$

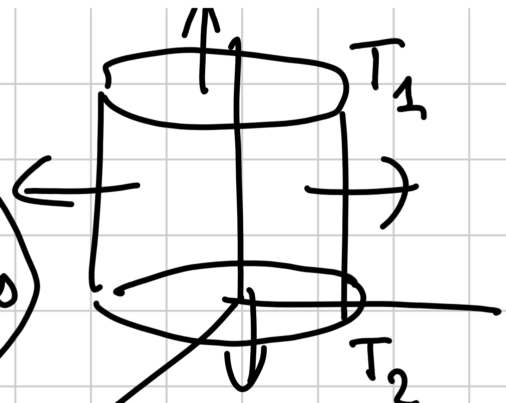
2° modo: si poteva usare il Teo. della divergenza?

$$\iiint_{\Omega} \operatorname{div} F = \int_{\partial\Omega} F \cdot N = \text{flusso attraverso } \Omega$$

$\partial\Omega$ frontiera di $\Omega$

$\Omega$ = cilindro "pieno" <u>chiuso</u> (con i tappi)

$$\iiint_{\text{Cilindro}} \operatorname{div} F = \text{Flusso attraverso la superficie laterale} + \text{Flusso attraverso tappo } T_1 + \text{Flusso attraverso tappo } T_2$$




$$\text{Flusso attraverso la sup. laterale} = \iiint \operatorname{div} F - \text{Flusso attraverso } T_1 - \text{Flusso } T_2$$