Lezione del 1 Ottobre
Canale 1

Programma di FAM 2

- Funzioni di più variabili

$$f : \mathbb{R}^n \longrightarrow \mathbb{R}$$

$$(x_1, \dots x_n) \longrightarrow f(x_1, \dots x_n)$$

$$f(x, y)$$
$$f(x, y, z)$$

In generale

$$f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$$

$$(x_1, \dots x_n) \longrightarrow (y_1, \dots y_m)$$

casi particolari

$n = 1$, $m = 1$

$$f : \mathbb{R} \to \mathbb{R}$$

$n \qquad\qquad m = 1$

$$f : \mathbb{R}^n \to \mathbb{R}$$

$f(x, y)$

$f(x, y, z)$

ricerca di
max e minimi di f "liberi"

e "vincolati"

$f(x) = x^2$

Sempre per funzioni di due e tre variabili, definiremo gli $\int$

$$\iint f(x,y) \qquad \iiint f(x,y,z)$$

$$f : \mathbb{R}^n \to \mathbb{R}^m$$

$$n = 1$$

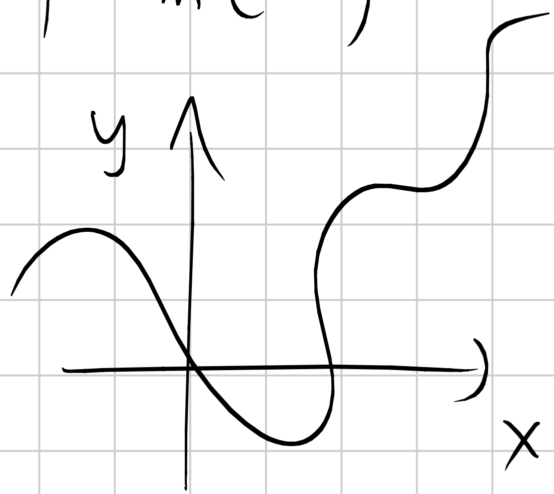
$$f : \mathbb{R} \to \mathbb{R}^m$$

CURVE

$$t \longrightarrow (x_1, \dots, x_m)$$

$$(x_1(t), \dots, x_m(t))$$

$m = 2$



$$f : \mathbb{R} \to \mathbb{R}^2$$

$$t \to (x(t), y(t))$$

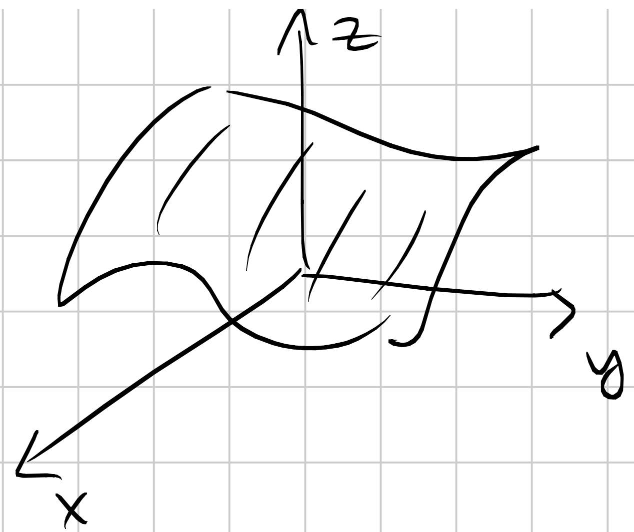
CURVA in $\mathbb{R}^2$

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$$

$$(u, v) \longrightarrow (x, y, z)$$

$\int$ su curve

$\int$ su superfici



- Equazioni differenziali

$$F(y, y', y'') = 0$$

$y = y(x)$
incognita

Ripasso funzioni in più variabili

$f : \mathbb{R}^n \to \mathbb{R}$

o meglio

$f : A \subseteq \mathbb{R}^n \to \mathbb{R}$ $\quad$ A dominio

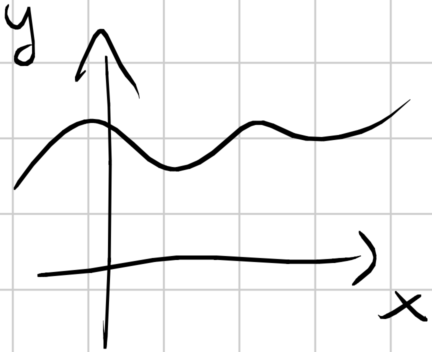
$(x_1, \dots x_n) \longrightarrow f(x_1, \dots x_n)$

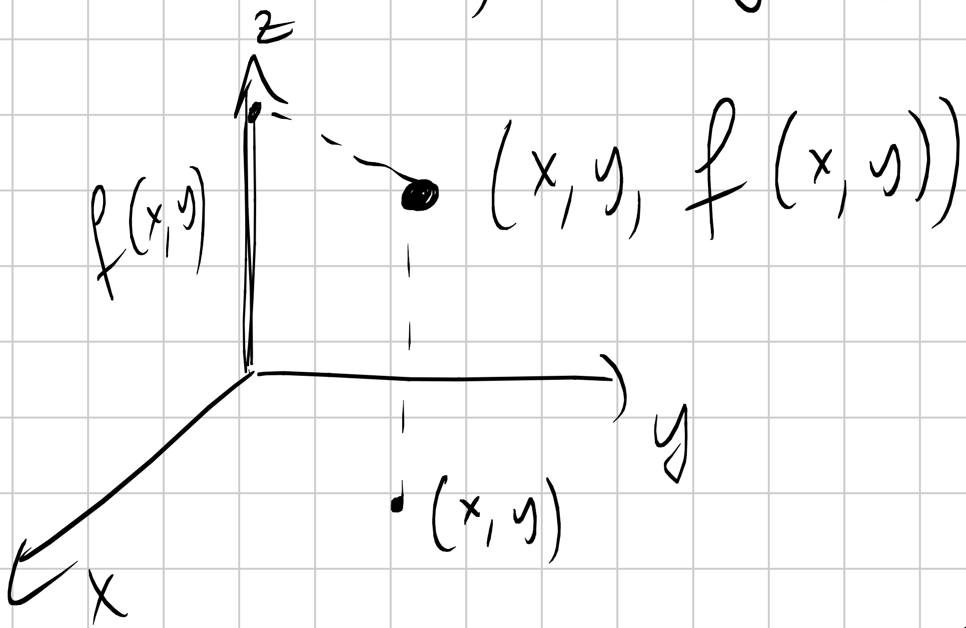
$n = 2 \qquad f(x, y)$

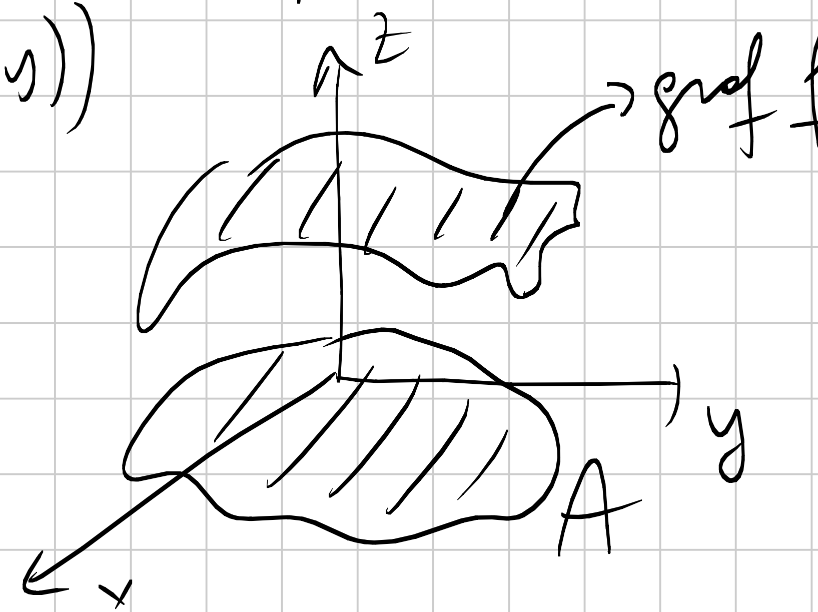
$n = 3 \qquad f(x, y, z)$

$n=2 \qquad \text{graf } f = \{ (x,y,z) : z = f(x,y) \}$

$n=1 \qquad (x,y) : y = f(x)$







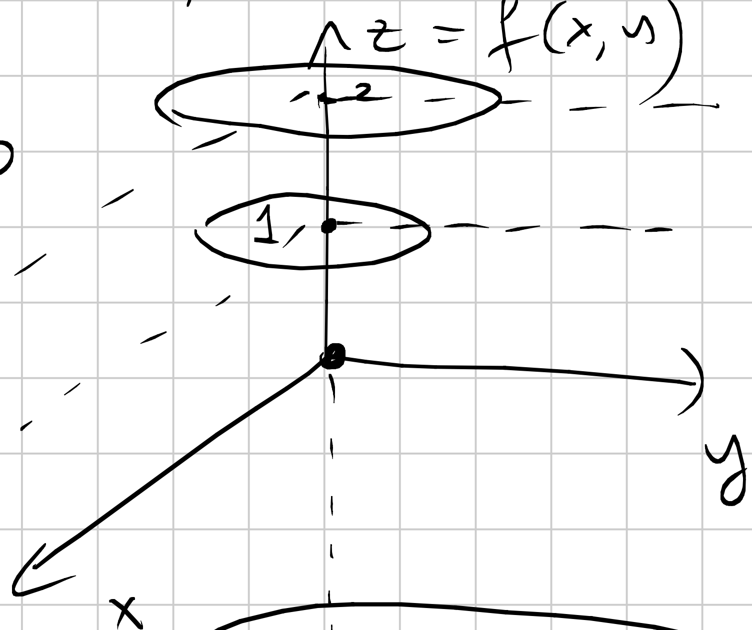
es. di grafico di f in 2 variabili

$$f(x,y) = x^2 + y^2 \qquad A = \mathbb{R}^2$$

$$(x,y) \longrightarrow x^2 + y^2 \geq 0$$

$$(0,0) \longrightarrow 0$$

$$(0,0,0)$$

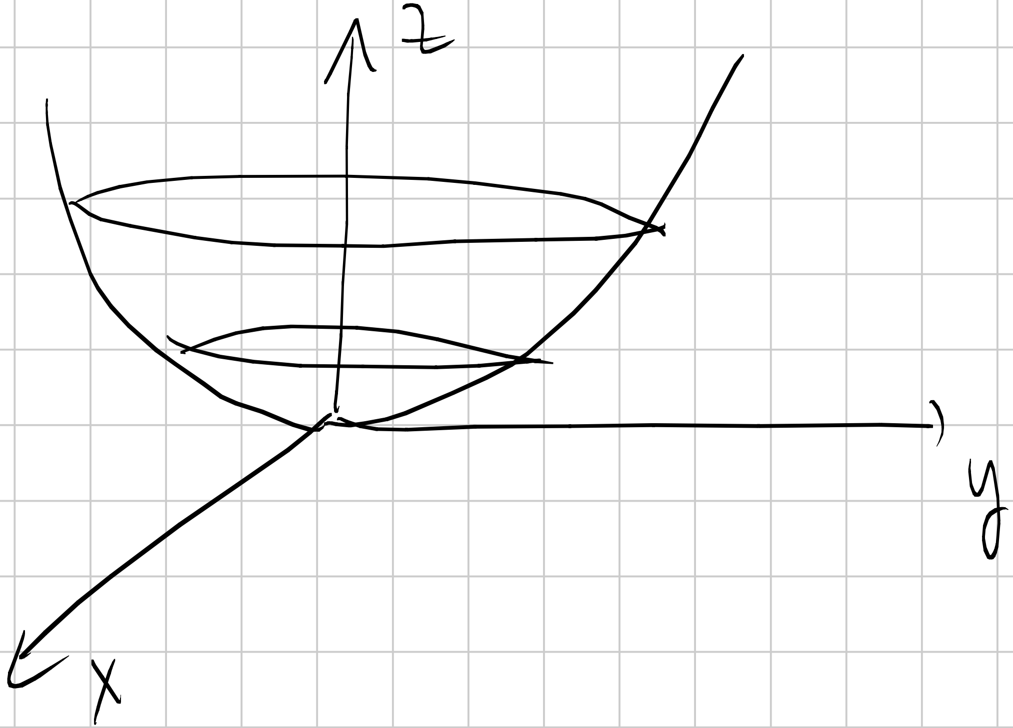


$$z = f(x,y) = 1$$

$z = 1$ $\qquad x^2 + y^2 = 1$

$z = 2$ $\qquad x^2 + y^2 = 2$

$$\boxed{x^2 + y^2 = z}$$



$$\boxed{z = x^2 + y^2}$$

$y = 0$

$z = x^2$

$x = 0$

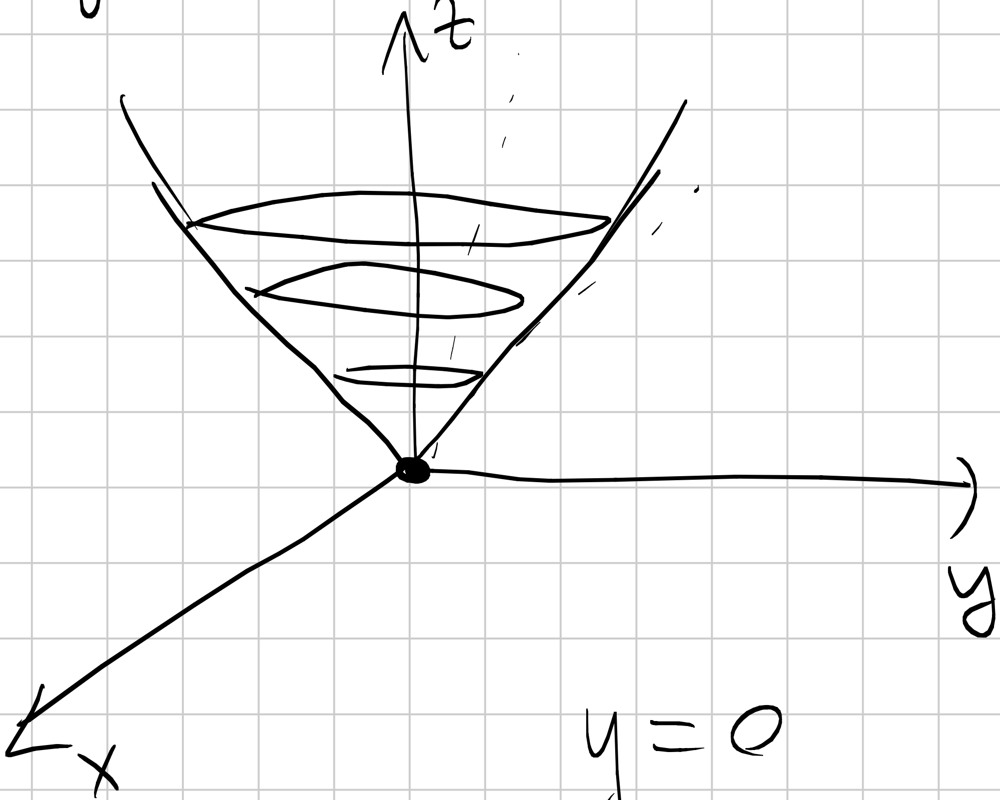
$z = y^2$

$f(x,y) = \sqrt{x^2+y^2}$ $\quad A = \mathbb{R}^2$

$f \geq 0$

$z = \sqrt{x^2+y^2}$

$(0,0,0) \in$ graf f



$x \quad z = c$

$x^2+y^2 = c^2$ circonferenze

$y = 0$

$z = \sqrt{x^2} = |x|$

es. $f(x,y) = 1 - (x^2 + y^2)$

per casa

$$f(x,y) = \sqrt{x+y}$$

$x + y \geq 0$

$y \geq -x$

$$f(x, y, z) = \sin(x+y) + \log z$$

$z > 0$

$A \subseteq \mathbb{R}^3$

$f : \mathbb{R}^n \to \mathbb{R}$

$\underline{x} = (x_1, \dots x_n)$

$\underline{x}_0 = (x_{10}, \dots x_{n0})$

$$\lim_{\underline{x} \to \underline{x}_0} f(x) = \underbrace{\ell}_{\in \mathbb{R}}$$

<u>modulo</u> di un vettore (norma)

$$|\underline{x}| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2} = (\|\underline{x}\|)$$

$$\underline{x} = (x_1, \dots x_n)$$

<u>es.</u> $\underline{x} = (1, 2, 3)$ $|\underline{x}| = \sqrt{1 + 4 + 9} =$

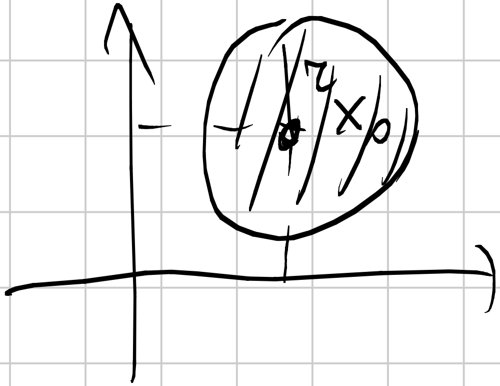
$$= \sqrt{14}$$

Intorno sferico di $\underline{x}_0 \in \mathbb{R}^n$

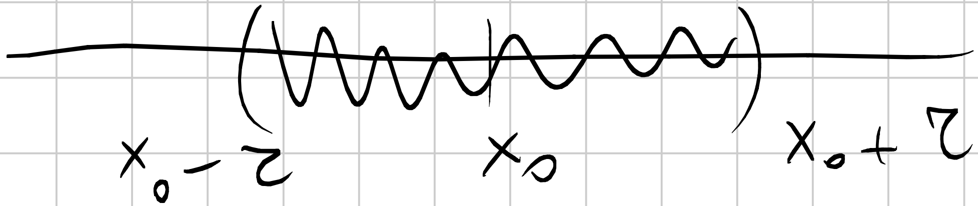
$$U_r(\underline{x}_0) = \{ \underline{x} \in \mathbb{R}^n : \underbrace{|\underline{x} - \underline{x}_0|}_{\text{distanza di } \underline{x} \text{ da } \underline{x}_0} < r \}$$

sono i punti che distano da $\underline{x}_0$ meno di r

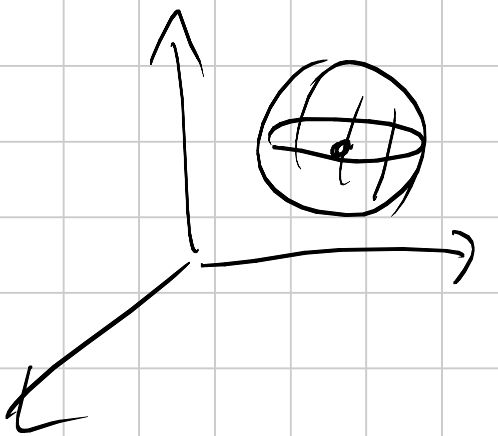
in $\mathbb{R}^2$



in $\mathbb{R}$



in $\mathbb{R}^3$



Def. di limite $f: \mathbb{R}^n \to \mathbb{R}$

$\lim_{\underline{x} \to \underline{x}_0} f(x) = l \iff \forall$ intorno V di l esiste intorno U di $\underline{x}_0$ t.c. $f(x) \in V, \forall x \in U$

Def. di funzione continua in $\underline{x}_0$

$f: \mathbb{R}^n \to \mathbb{R}$

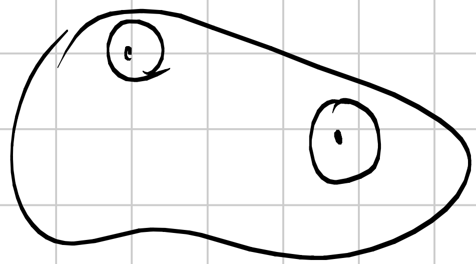
$$\lim_{\underline{x} \to \underline{x}_0} f(\underline{x}) = f(\underline{x}_0)$$

es. $f(x,y,z) = \operatorname{sen}(x+y) + \lg z$

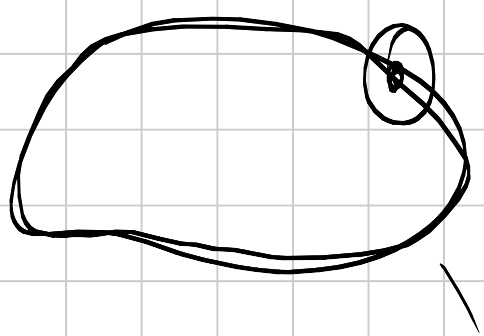
è continua dove è definita.

Topologia in $\mathbb{R}^2$ e $\mathbb{R}^3$

E è *aperto* se ogni suo punto è interno (cioè $\exists$ un intorno sferico tutto contenuto in E)



aperto



non *aperto*

E è chiuso se il suo complementare è aperto
↓ chiuso

$\exists$ insiemi né aperti, né chiusi.

in $\mathbb{R}$ (a, b) $(a, +\infty)$, $(-\infty, a)$
aperti

$[a, b]$, $[a, +\infty)$, $(-\infty, a]$
chiusi

$[a, b)$, $(a, b]$ né aperti né chiusi