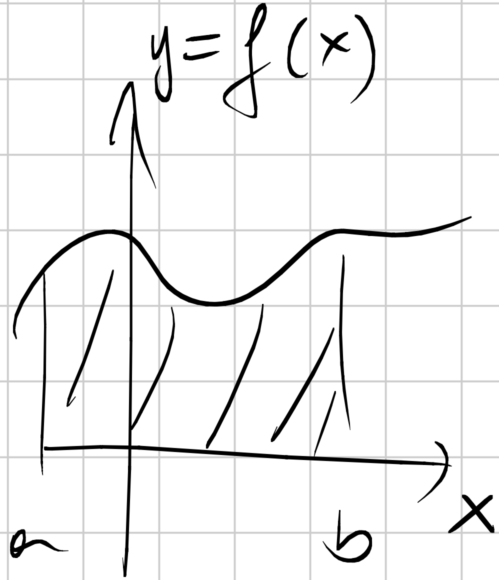
Canale 2   <u>1 Ottobre</u>

<u>Programma del corso</u>

A1 $f : \mathbb{R} \to \mathbb{R}$

$x \to y = f(x)$



A2 $f : \mathbb{R}^n \to \mathbb{R}^m$

$$\underline{x} = (x_1, \dots x_n) \longrightarrow (y_1, \dots y_m) = \underline{y}$$

$$f(\underline{x}) = \underline{y}$$

caso particolare $m = 1$

$$f : \mathbb{R}^n \to \mathbb{R}$$

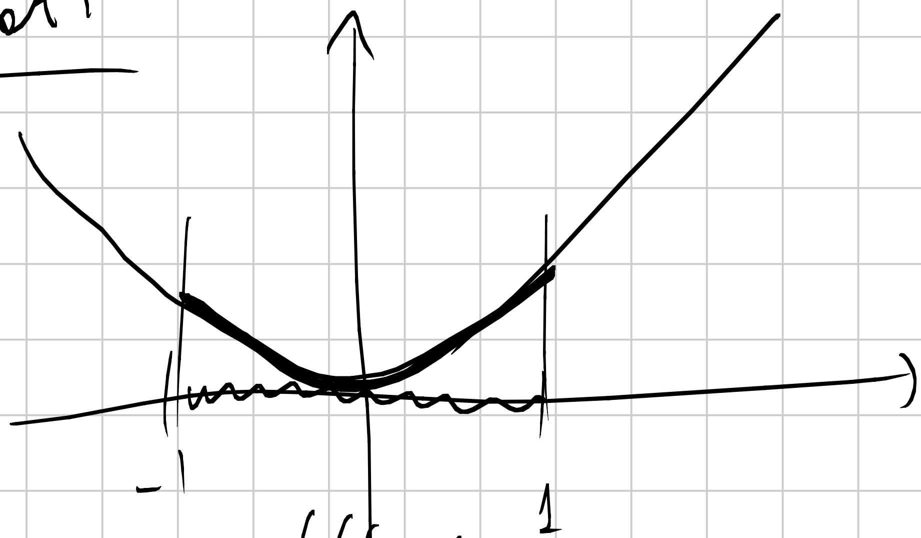
funzioni di più variabili a valori reali

$n=2$ $f(x,y) \in \mathbb{R}$

$n=3$ $f(x,y,z) \in \mathbb{R}$

1) ricerca di max e min "liberi"
o "vincolati"

$f(x) = x^2$



2) $\iint f(x,y)$, $\iiint f(x,y,z)$

$m = 1$

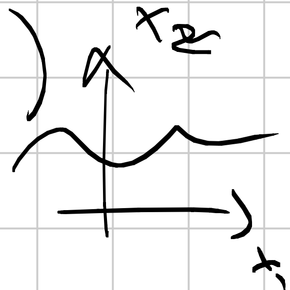
$f : \overline{\mathbb{R}^n \to \mathbb{R}^m}$

$n = 1$

curve

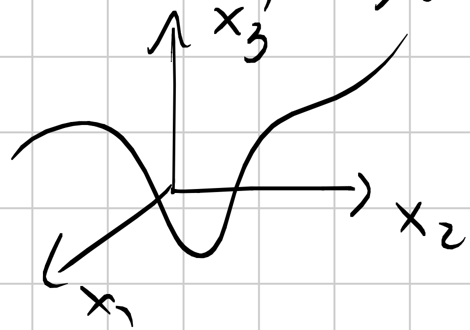
$$f : \mathbb{R} \to \mathbb{R}^m$$

$$t \longrightarrow (x_1, x_2, \ldots\ldots, x_m)$$



$m = 2 \qquad t \longrightarrow \left(x_1(t), \; x_2(t) \right)$

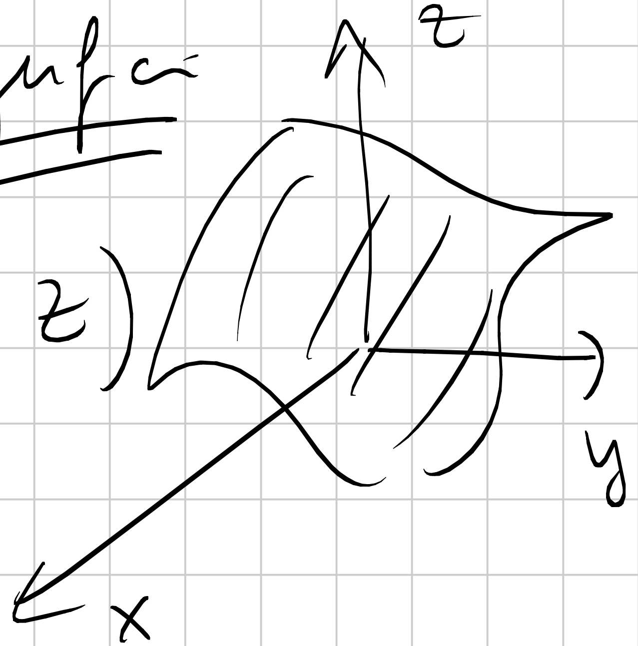
$m = 3 \qquad t \longrightarrow \left(x_1(t), \; x_2(t), \; x_3(t) \right)$



Scurve

• $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ <u>superficie</u>

$(u,v) \longrightarrow (x, y, z)$



• Eq. differenziali $F(y, y', y'', \dots) = 0$

$y(x)$

Ripasso su funzioni di più variabili

$$f : \mathbb{R}^n \to \mathbb{R}$$ o meglio

A = dominio

$$f : A \subseteq \mathbb{R}^n \to \mathbb{R}$$

$n = 2$ $\quad f : A \subseteq \mathbb{R}^2 \to \mathbb{R}$

$$(x, y) \longrightarrow f(x, y) \in \mathbb{R}$$

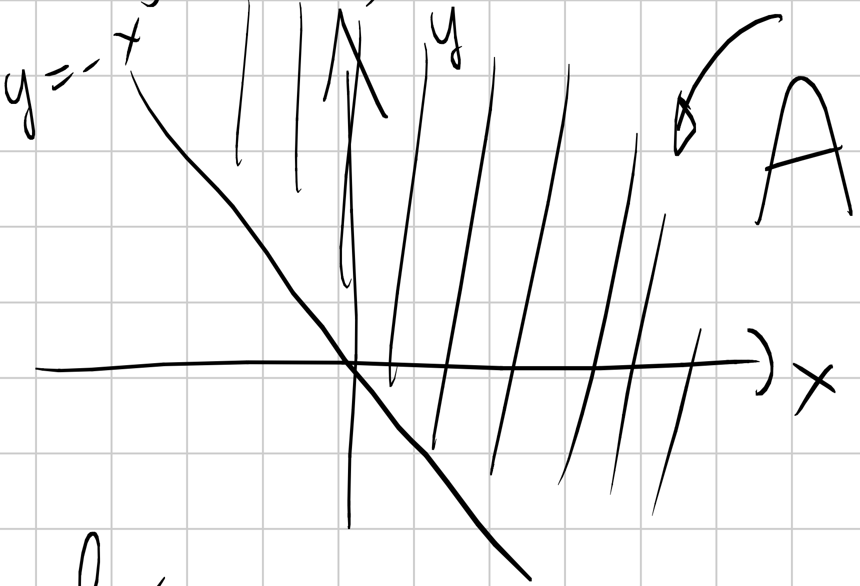
$n = 3$ $\quad f : A \subseteq \mathbb{R}^3 \to \mathbb{R}$

$$(x, y, z) \longrightarrow f(x, y, z) \in \mathbb{R}$$

es. $f(x,y)=\sqrt{x+y}$

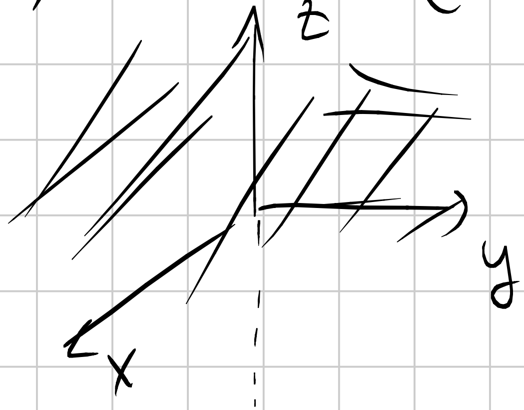
$x+y\geq 0$

$y\geq -x$



es. $f(x,y,z)=\sin(x+y)+\log z$

$z>0$

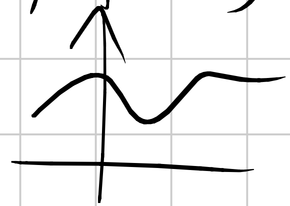


$$f : A \subseteq \mathbb{R}^2 \to \mathbb{R}$$

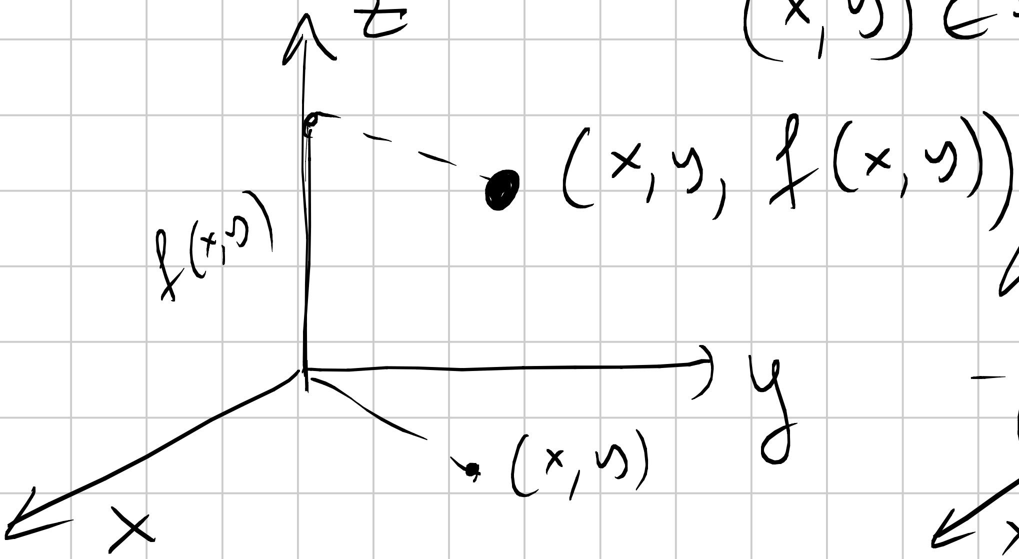
$$(x, y) \to f(x, y)$$

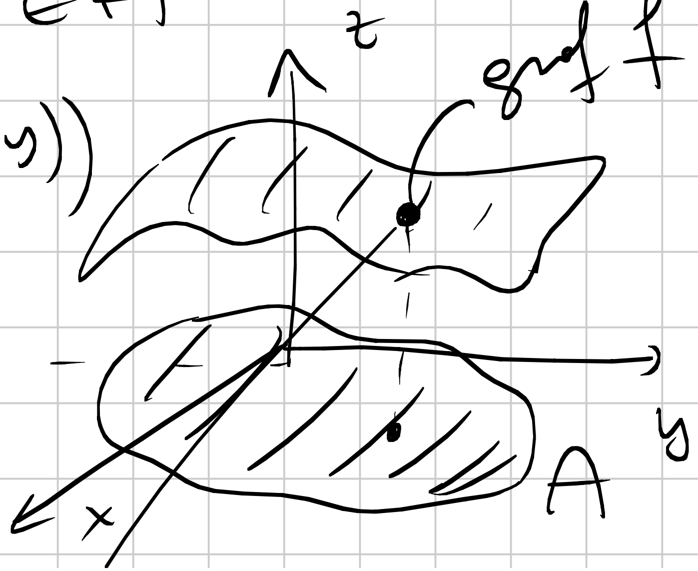
$y = f(x)$

$(x, f(x))$



$$\text{graf } f = \{ (x, y, z) : z = f(x, y),\ (x, y) \in A \}$$





$\checkmark\ (x, y, f(x,y))$.

Grafico di funzioni

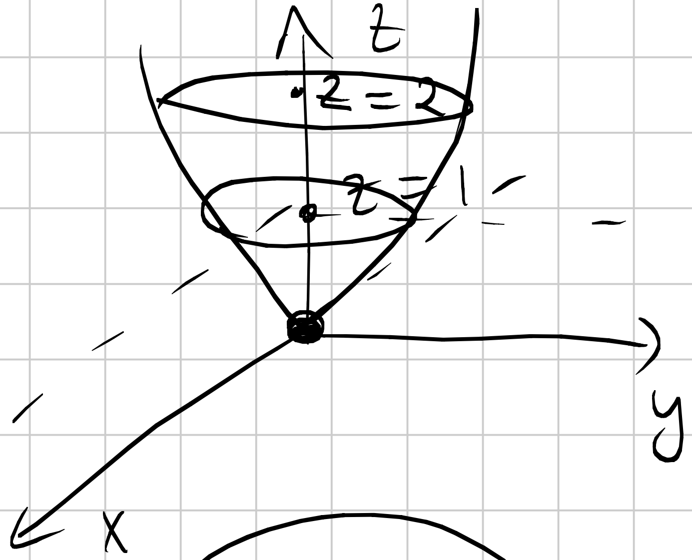
$$f(x,y) = x^2 + y^2$$

$$f(x,y) \geq 0$$

$$z = x^2 + y^2 \geq 0$$

$(0,0,0) \in \text{graf}\, f$

$A = \mathbb{R}^2$



$y = 0$
$z = x^2$

$z = 1 \qquad x^2 + y^2 = 1 \qquad x = 0$

$z = 2 \qquad x^2 + y^2 = 2 \qquad z = y^2$

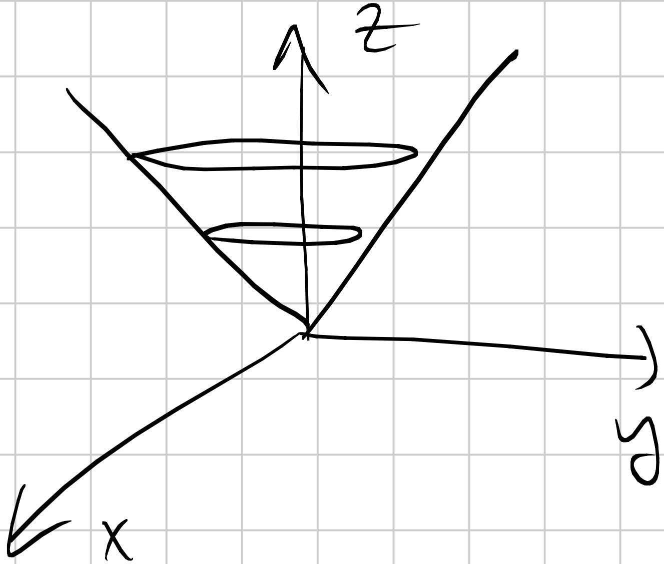
$y = 0$

es. per cono $f(x,y) = 1 - (x^2 + y^2)$

es. $f(x,y) = \sqrt{x^2 + y^2}$

$z = \sqrt{x^2 + y^2} \geq 0$

$x^2 + y^2 = z^2$



$z = c \Rightarrow x^2 + y^2 = c^2$ circonferenza di raggio c

$y = 0 \qquad z = \sqrt{x^2} = |x|$

$x = 0 \qquad z = |y|$

---

limiti di funzioni

$$f : \mathbb{R}^n \to \mathbb{R}$$

$$\underline{x} = (x_1, \dots x_n)$$

$$\lim_{\underline{x} \to \underline{x}_0} \underbrace{f(\underline{x})}_{\in \mathbb{R}} = \underbrace{l}_{\in \mathbb{R}}$$

bisogna
definire un intorno di $\underline{x}_0$ e
quindi dare una nozione di
distanza

modulo di un vettore $\underline{x} = (x_1, \dots x_n)$

$$|\underline{x}| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2} \quad (= \|x\|)$$

(o norma)

es. $\underline{x} = (1, 2, 3)$

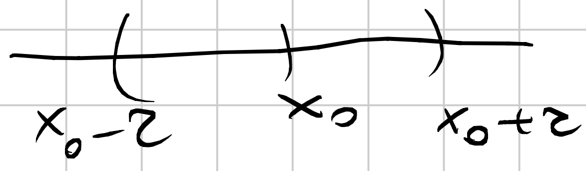
$$|\underline{x}| = \sqrt{1+4+9} = \sqrt{14}$$

Intorno **sferico** di $\underline{x}_0 \in \mathbb{R}^n$

$$U_r(\underline{x}_0) = \{ \underline{x} \in \mathbb{R}^n : \overbrace{|\underline{x} - \underline{x}_0|}^{d(\underline{x},\, \underline{x}_0)} < r \}$$

tutti i punti di $\mathbb{R}^n$ che distano da x_0 meno di r

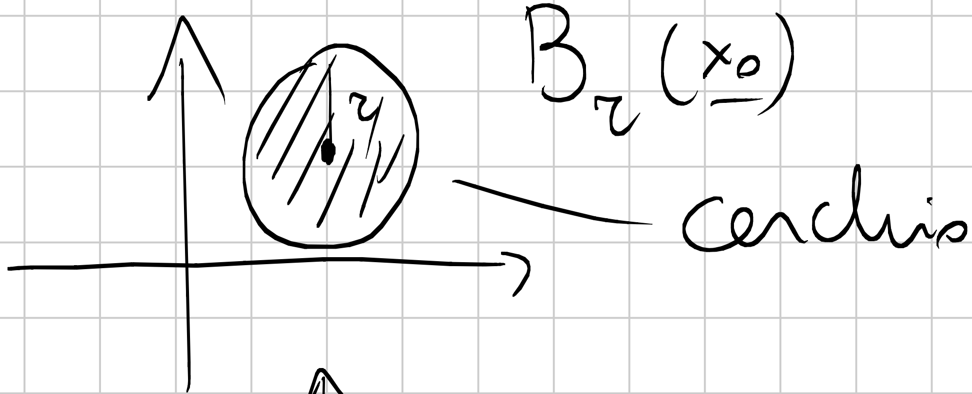
es. in $\mathbb{R}$ $(n=1)$



$U_r(x_0) = (x_0 - r,\ x_0 + r)$

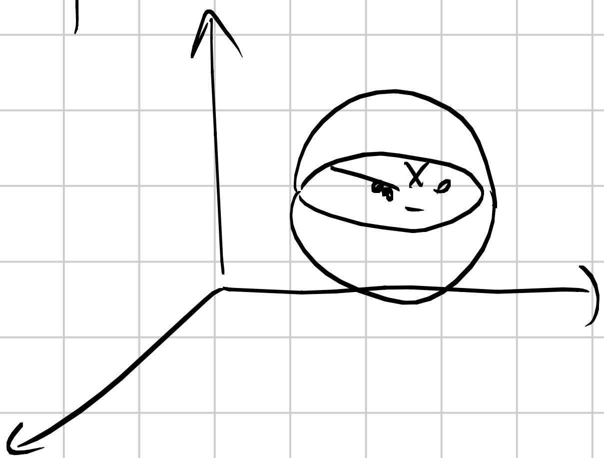
in $\mathbb{R}^2$

$U_r(\underline{x}_0)$



in $\mathbb{R}^3$

$U_r(\underline{x}_0)$



$$\lim_{\underline{x} \to \underline{x}_0} f(x) = l \iff \forall \text{ intorno } V \text{ di } l$$
$$\exists \text{ intorno } U \text{ di } \underline{x}_0$$
$$\text{t.c. } f(x) \in V, \ \forall x \in U$$

f <u>continua</u> in $\underline{x}_0$

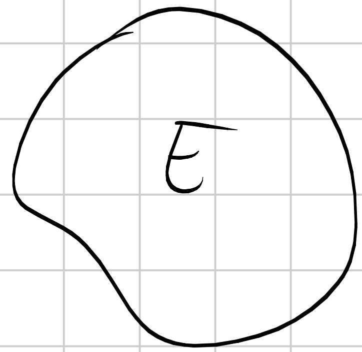
$f : \mathbb{R}^n \to \mathbb{R}$

se $\lim_{\underline{x} \to \underline{x}_0} f(x) = f(\underline{x}_0)$

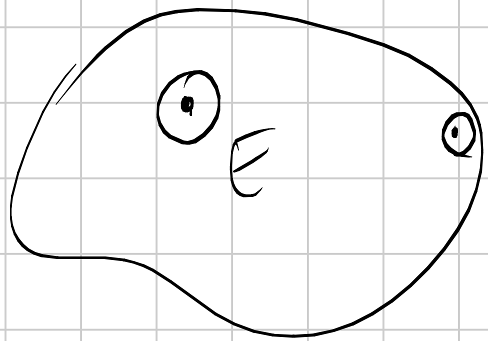
$$f(x, y) = \sqrt{x + y}$$

Topologia in $\mathbb{R}^2$ e $\mathbb{R}^3$

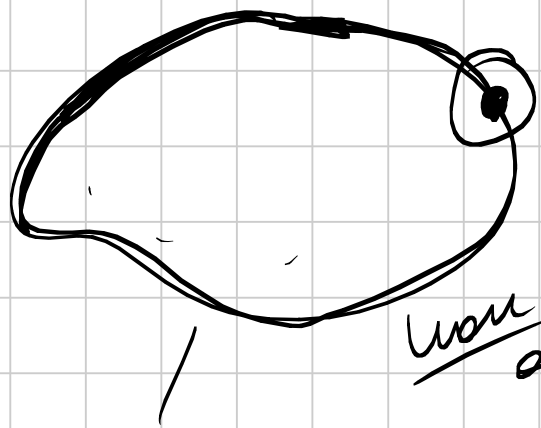
$E \subseteq \mathbb{R}^n$ è **aperto** se ogni suo p.to è **interno**



(cioè se esiste un intorno sferico del p.to tutto contenuto in E)



aperto



non è aperto

∴ E è chiuso se il complementare di E è aperto

↓ è chiuso!

esistono insiemi che non sono né aperti, né chiusi.

in $\mathbb{R}$ (a,b), $(a,+\infty)$, $(-\infty,a)$ aperti

$[a,b]$, $[a,+\infty)$, $(-\infty,a]$ chiusi

$[a, b)$, $(a, b]$ né aperti

né chiusi