Ricerca di primitive

21/12/11

Integrazione di f. trigonometriche

$$\int \operatorname{sen}^2 x \, dx \,, \qquad \int \cos^2 x \, dx \quad \ldots$$

$$\int \operatorname{sen}^3 x \, dx = \int \operatorname{sen}^2 x \cdot \operatorname{sen} x \, dx = \int (1 - \cos^2 x) \operatorname{sen} x \, dx$$

$$= \int (1 - t^2)(-dt) =$$

$$\cos x = t$$

$$- \operatorname{sen} x \, dx = dt$$

$$= \int (t^2 - 1)\, dt = \dots$$

es. $$\int \operatorname{sen}^3 x \cos^2 x \, dx =$$

$$= \int \operatorname{sen} x \underbrace{\operatorname{sen}^2 x}_{1-\cos^2 x} \cos^2 x \, dx = \int (1-\cos^2 x) \cos^2 x \underbrace{\operatorname{sen} x \, dx}$$

$\cos x = t$

$$= \int (1 - t^2)\, t^2 \, (-dt)$$

es. $$\int \operatorname{sen}(3x) \cdot \cos(2x)\, dx$$

$$\operatorname{sen} u + \operatorname{sen} v =$$

$$= 2 \operatorname{sen}\left(\frac{u+v}{2}\right) \cdot \cos\left(\frac{u-v}{2}\right)$$

$$\begin{cases} \dfrac{u+v}{2} = 3x \\ \dfrac{u-v}{2} = 2x \end{cases}$$

$$\begin{cases} u+v = 6x \\ u-v = 4x \end{cases}$$

sommando $2u = 6x + 4x = 10x$

$u = 5x$

$v = 6x - u = 6x - 5x = x$

$u = 5x$ $\qquad$ $\operatorname{sen} 5x + \operatorname{sen} x = 2 \operatorname{sen}(3x) \cos(2x)$

$v = x$

$$\int \operatorname{sen}(3x) \cos 2x \, dx = \int \frac{1}{2} \left(\operatorname{sen} 5x + \operatorname{sen} x \right) dx$$

$= \ldots\ldots$

$$\int \operatorname{sen}(3x) \cos(3x) \, dx =$$

$\operatorname{sen}(3x) = t$

$\underbrace{3 \cos(3x) \, dx}$ $= dt$

$$= \int t \frac{1}{3} dt = \ldots$$

Funzioni razionali in $\operatorname{sen} x$ e $\cos x$

Es.
$$\frac{1}{\operatorname{sen} x} \qquad \frac{\cos x}{\operatorname{sen} x + 1} \qquad \frac{\operatorname{sen} x + \cos x}{\operatorname{sen} x - \cos x + 1}$$

$$\boxed{t = \operatorname{tg}\left(\frac{x}{2}\right)} \qquad \frac{x}{2} = \operatorname{arctg} t$$

$$x = 2 \operatorname{arctg} t \qquad \boxed{dx = 2 \frac{1}{1+t^2}\, dt}$$

$$\boxed{\operatorname{sen} x = \frac{2t}{1+t^2}} \qquad \boxed{\cos x = \frac{1-t^2}{1+t^2}}$$

es. 1

$$\int \frac{1}{\operatorname{sen} x}\,dx = \int \frac{1}{\frac{\cancel{2}t}{\cancel{1+t^2}}} \cdot \frac{\cancel{2}\cdot 1}{\cancel{1+t^2}}\,dt$$

$$= \int \frac{1}{t}\,dt = \log|t| = \log\left|tg\left(\frac{x}{2}\right)\right| + K$$

es. 2

$$\int \frac{1}{1+\operatorname{sen} x - \cos x}\,dx = \int \frac{1}{\left(1+\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2}{1+t^2}\,dt$$

$$= \int \frac{1}{\frac{\cancel{1}+t^2+2t-\cancel{1}+t^2}{\cancel{1+t^2}}} \cdot \frac{2}{\cancel{1+t^2}}\,dt$$

$$= \int \frac{1}{2t^2+2t} \, 2\,dt = \int \frac{1}{t(t+1)} \, dt$$

$$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)} \Rightarrow A, B.$$

fratti semplici

e si integra ...

Funzioni irrazionali del tipo

$$\sqrt{a^2 - x^2}, \quad \sqrt{a^2 + x^2}, \quad \sqrt{x^2 - a^2}$$

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es. 1 $\int \sqrt{3 - x^2}\, dx = \int \sqrt{3\left(1 - \left(\frac{x}{\sqrt{3}}\right)^2\right)}\, dx$

$= \sqrt{3} \int \sqrt{1 - \left(\frac{x}{\sqrt{3}}\right)^2}\, dx =$ $\frac{x}{\sqrt{3}} = \operatorname{sen} t$

$\rightarrow \left|\frac{x}{\sqrt{3}}\right| \leq 1$ / funzione corretta $\sqrt{}$

$= \sqrt{3} \int \sqrt{1 - \operatorname{sen}^2 t}\; \sqrt{3} \cos t\, dt$

$x = \sqrt{3} \operatorname{sen} t$

$dx = \sqrt{3} \cos t\, dt$

$= 3 \int \sqrt{\cos^2 t}\; \cos t\, dt$

$$= 3 \int \underbrace{|\cos t|} \cos t \, dt$$

$\int \cos^2 t \, dt$ se $\cos t \geq 0$

$\int - \cos^2 t \, dt$ $\cos t < 0$

$$\int \cos^2 t \, dt$$

$\Rightarrow$ e poi scrivere tutto in funzione di x

$$x = \sqrt{3} \, \text{sen}\, t$$

$$\cos^2 t = 1 - \text{sen}^2 t$$

$$\cos t = \pm \sqrt{1 - \text{sen}^2 t} =$$

$$= \pm \sqrt{1 - \left(\frac{x}{\sqrt{3}}\right)^2}$$

$$\int \sqrt{a^2 - x^2}\, dx = \int \sqrt{a^2\left(1 - \left(\frac{x}{a}\right)^2\right)}\, dx =$$

$$= |a| \int \sqrt{1 - \left(\frac{x}{a}\right)^2}\, dx \qquad \frac{x}{a} = \sin t$$

$$dx = a \cos t\, dt$$

$$= |a| \int \sqrt{1 - \sin^2 t}\; a \cos t\, dt$$

$$= \dots$$

$$\sim \circ \sim$$

$$\int \sqrt{a^2 + x^2}\, dx \qquad \cosh^2 t - \sinh^2 t = \underline{1}$$

$$\cosh^2 t = 1 + \underbrace{\sinh^2 t}$$

$$\int \sqrt{5+x^2}\,dx = \int \sqrt{5\left(1+\left(\frac{x}{\sqrt{5}}\right)^2\right)}\,dx =$$

$$= \sqrt{5}\int \sqrt{1+\left(\frac{x}{\sqrt{5}}\right)^2}\,dx =$$

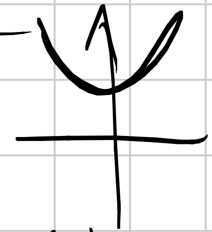
$$\frac{x}{\sqrt{5}} = \operatorname{senh} t$$

$$x = \sqrt{5}\operatorname{senh} t$$

$$dx = \sqrt{5}\cosh t\,dt$$

$$= \sqrt{5}\int \sqrt{1+\operatorname{senh}^2 t}\;\sqrt{5}\cosh t\,dt =$$

$$= 5\int \sqrt{\cosh^2 t}\;\cosh t\,dt =$$



$\cosh t > 0$

$$= 5\int \cosh t \cdot \cosh t\,dt =$$

$$= 5 \int \cosh^2 t \, dt$$ $\longrightarrow$ per parti $\int \cos^2 t \, dt$

? sostituzione di x e integrazione?

$\sim \cdot \sim$

3° tipo

$$\int \sqrt{x^2 - a^2} \, dx$$

es.

$$\int \sqrt{x^2 - 7} \, dx = \sqrt{7} \int \sqrt{\left(\frac{x}{\sqrt{7}}\right)^2 - 1} \, dx$$

$$\cosh^2 t - \sinh^2 t = 1$$

$$\frac{x}{\sqrt{7}} = \cosh t \qquad\qquad \cosh^2 t - 1 = \operatorname{senh}^2 t$$

$$x = \sqrt{7}\cosh t \qquad\qquad dx = \sqrt{7}\operatorname{senh} t\, dt$$

$$= \sqrt{7}\int \sqrt{\cosh^2 t - 1}\;\sqrt{7}\operatorname{senh} t\, dt$$

$$= 7\int \sqrt{\operatorname{senh}^2 t}\;\operatorname{senh} t\, dt =$$

$$= 7\int |\operatorname{senh} t|\operatorname{senh} t\, dt =$$

$$= \begin{cases} 7\int \operatorname{senh}^2 t\, dt & \operatorname{senh} t > 0 \end{cases}$$

$$\left(-7\int \operatorname{senh}^2 t \, dt \right. \qquad \operatorname{senh} t < 0$$

$$\int \operatorname{senh}^2 t \, dt = \int \underbrace{\operatorname{senh} t}\cdot \underbrace{\operatorname{senh} t} \, dt =$$

$$= \cosh t \cdot \operatorname{senh} t - \int \cosh t \cosh t \, dt$$

$$= \cosh t \operatorname{senh} t - \int (1 + \operatorname{senh}^2 t)\, dt \qquad \boxed{\cosh^2 t = 1 + \operatorname{senh}^2 t}$$

$$= \cosh t \operatorname{senh} t - t - \int \operatorname{senh}^2 t \, dt$$

$$2\int \operatorname{senh}^2 t \, dt = \cosh t \operatorname{senh} t - t$$

$$\int \operatorname{senh}^2 t \, dt = \frac{\cosh t \operatorname{senh} t - t}{2} + K$$

$\sim \cdot \sim$

es. $$\int \sqrt{x^2 + 6x} \, dx$$

completiamo il quadrato

x^2+6x

$$= x^2 + 6x + 9 - 9 = (x+3)^2 - 9$$

$$= \int \sqrt{(x+3)^2 - 9} \, dx =$$

$$= 3 \int \sqrt{\left(\frac{x+3}{3}\right)^2 - 1} \, dx =$$

$$\frac{x+3}{3} = \cosh t$$

$$x + 3 = 3 \cosh t$$

$$x = 3 \cosh t - 3$$

$$dx = 3 \operatorname{senh} t \, dt$$

$$= 3 \int \sqrt{\operatorname{senh}^2 t} \; 3 \operatorname{senh} t \, dt$$

$$= \cdots\cdots$$

$\sim \cdot \sim$

es.

$$\int \frac{\sqrt{x}}{x\left(2\sqrt[3]{x} + 3\right)} \, dx =$$

$\sqrt{x} = t \qquad x = t^2$

$\sqrt[3]{x} = x^{1/3} = t^{2/3}$

$x = t^6 \qquad \sqrt{x} = t^3 \qquad \sqrt[3]{x} = t^2$

$dx = 6t^5\, dt$

Integrale $= \displaystyle\int \frac{\cancel{t^3}^{\,2}}{\cancel{t^6}\,(2t^2+3)}\, 6\,\cancel{t^5}\, dt$

$$= \quad = 6\int \frac{t^2}{2t^2+3}\,dt$$

$$= \frac{6}{2}\int \frac{2t^2+3-3}{2t^2+3}\,dt =$$

$$= 3\left(\int\left(1-\frac{3}{2t^2+3}\right)dt\right) =$$

$$= 3\left(t - \underbrace{3\int\frac{1}{2t^2+3}\,dt}\right)$$

↳ tipo arctg (. . .)

e poi $t = x^{1/6}$

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es. $\displaystyle\int \frac{1}{x\sqrt{x-1}}\,dx =$

$\sqrt{x-1} = t$

$x-1 = t^2$

$x = t^2+1$

$dx = \underbrace{2t\,dt}$

$= \displaystyle\int \frac{1}{(t^2+1)\,\cancel{t}}\,\cancel{2t}\,dt$

$$= 2\,\operatorname{arctg}\left(\sqrt{x-1}\right) + K$$

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es.

$$\int \frac{1}{\operatorname{tg} x \,\log(\operatorname{sen} x)}\,dx = \int \frac{\cos x}{\operatorname{sen} x \,\log(\operatorname{sen} x)}\,dx$$

$$\operatorname{sen} x = t \qquad \cos x\,dx = dt$$

$$= \int \frac{1}{t \,\log(t)}\,dt =$$

$$\log t = z$$

$$\frac{1}{t}\,dt = dz$$

$$= \int \frac{1}{z}\,dz = \log|z| + K =$$

$$= \log|\log(\operatorname{sen} x)| + K$$

es.

$$\int \frac{\operatorname{arctg} x}{(1+x^2)(1+\operatorname{arctg}^2 x)}\,dx =$$

$$\operatorname{arctg} x = t$$

$$\frac{1}{1+x^2}\,dx = dt$$

$$= \int \frac{t}{1+t^2}\,dt$$

$$= \int \frac{1}{2} \frac{dz}{z} =$$

$$1 + t^2 = z$$

$$2t\,dt = dz$$

$$= \frac{1}{2} \log \left| 1 + \operatorname{arctg}^2 x \right| + K$$