2) $f(x) = \log\left(\frac{x+4}{(x+1)^2}\right)$ (13/12/12)

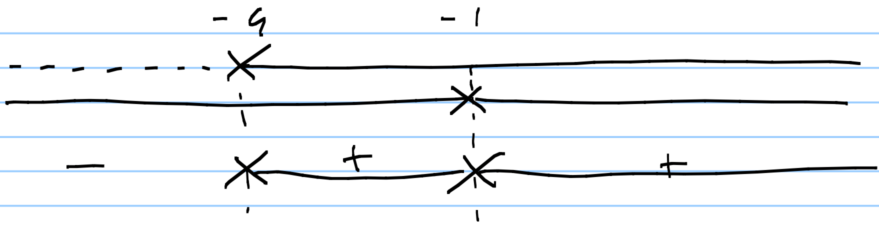
1) dom f

$\rightarrow \frac{x+4}{(x+1)^2} > 0$

$\rightarrow x+1 \neq 0$

$N \geq 0 \qquad x+4 > 0 \qquad x > -4$

$D > 0 \qquad (x+1)^2 > 0 \qquad x \neq -1$



$\text{dom} f = \left\{ x \in \mathbb{R} : \frac{x+4}{(x+1)^2} > 0 \text{ e } x \neq -1 \right\} =$

$= \{ x \in \mathbb{R} : x > -4, x \neq -1 \} =$

$= (-4, -1) \cup (-1, +\infty)$

2) $A = \{$ pti di accumulazione del dom $f \}$

$\forall x_0 \in A, \quad \lim_{x \to x_0} f(x)$

$A = [-4, -1] \cup [-1, +\infty) \cup \{+\infty\}$

$= [-4, +\infty) \cup \{+\infty)$

$\lim_{x \to x_0} f(x) = ? \qquad \forall x_0 \in A$

$x_0 \in A \cap (\mathrm{dom}\, f) = (-4, -1) \cup (-1, +\infty)$

$x_0 \in A \setminus (\mathrm{dom}\, f) = \{-4\} \cup \{-1\} \cup \{+\infty\}$

$$\lim_{x \to x_0} f(x) = f(x_0) \qquad \forall\, x_0 \in \mathrm{dom}\, f\, (\cap A)$$

$$\lim_{x \to -4^+} \log\left(\frac{x+4}{(x+1)^2}\right) = -\infty$$

$$\lim_{x \to -1} \log\underbrace{\left(\frac{x+4}{(x+1)^2}\right)}_{+\infty} = +\infty$$

$$\lim_{x \to +\infty} \log\left(\underbrace{\frac{x+4}{(x+1)^2}}_{\to 0}\right) = -\infty$$

3) determinare eventuali asintoti

verticali : $x = -4$, $x = -1$ sono asintoti verticali

orizzontali : non ce ne sono

obliqui : $$\lim_{x \to +\infty} \frac{\log\left(\frac{x+4}{(x+1)^2}\right)}{x} =$$

$$= \lim_{x \to +\infty} \underbrace{\frac{1}{x}}_{\to 0} \underbrace{\log\left(\underbrace{\frac{x+4}{(x+1)^2}}_{\to 0}\right)}_{\to -\infty} =$$

$$= \lim_{x \to 0^+} \boxed{\frac{(x+1)^2}{x(x+4)}}\ \boxed{\frac{x+4}{(x+1)^2} \log\left(\frac{x+4}{(x+1)^2}\right)} = 0$$

$\frac{x+4}{(x+1)^2} = t$; $t \to 0$ se $x \to +\infty$

$$\lim_{t \to 0} t \log t = 0$$

non ci sono asintoti obliqui!

4) Determinare l'insieme B dei punti nei quali f è derivabile e calcolare $f'(x)$, $\forall x \in B$.

Per il teorema sull'algebra delle derivate e il teorema sulla composizione di funzioni derivabili

f è derivabile in x, $\forall x \in \operatorname{dom} f$

$\operatorname{dom} f = B = (-4,-1) \cup (-1,+\infty)$

$$f'(x) = \frac{\cancel{(x+1)}^2}{x+4} \cdot \frac{(x+1)^{\cancel{2}} - 2(x+4)\cancel{(x+1)}}{(x+1)^{\cancel{4}\cancel{3}}} =$$

$$= \frac{(x+1) - 2x - 8}{(x+4)(x+1)}$$

$$= \frac{-x-7}{(x+4)(x+1)}$$

Calcolare $\lim_{x \to -1^{\pm}} f'(x) = \lim_{x \to -1^{\pm}} \frac{\overset{<0}{(-x-7)}}{\underbrace{(x+4)}_{>0}\underbrace{(x+1)}_{>0,\ <0}} = \mp \infty$

5) Determinare gli intervalli di monotonia.

$$f'(x) = -\frac{(x+7)}{(x+4)(x+1)} > 0$$

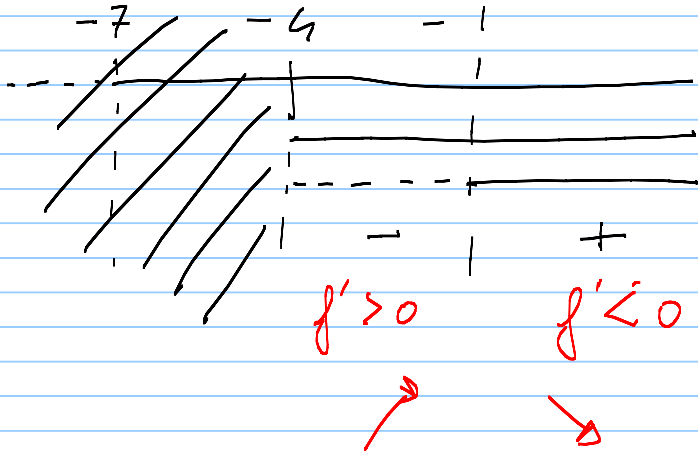
$\Updownarrow$

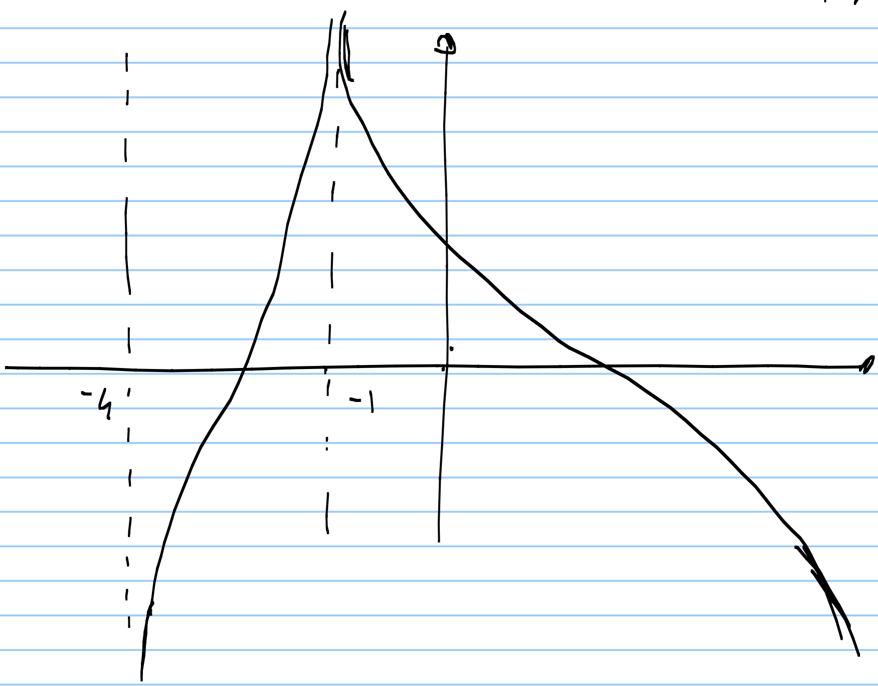
$$\frac{(x+7)}{(x+4)(x+1)} < 0$$

$(x+7) > 0 \iff x > -7$

$(x+4) > 0 \iff x > -4$

$(x+1) > 0 \iff x > -1$





f è strettamente crescente $(-4, -1)$

f è strettamente decrescente $(-1, +\infty)$

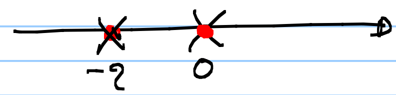
6) Determinare eventuali punti di max/min rel e/o assoluto.

non ci sono max/min rel né assoluti

3) $f(x) = \dfrac{(x+2)}{x}\, e^{-\frac{1}{(x+2)}}$ (19/12/11)

1) $\mathrm{dom} f = \{x \in \mathbb{R},\ x \neq 0,\ x+2 \neq 0\}$

$= \{x \in \mathbb{R} :\ x \neq 0,\ x \neq -2\}$



$= (-\infty, -2) \cup (-2, 0) \cup (0, +\infty)$

2) $A = \{\text{pti di acc. di } \mathrm{dom} f\} =$

$= \{-\infty\} \cup (-\infty, -2] \cup [-2, 0] \cup [0, +\infty) \cup \{+\infty\}$

$= \mathbb{R} \cup \{-\infty\} \cup \{+\infty\}$

$\lim\limits_{x \to x_0} f(x) \qquad \forall x_0 \in A$

•) $x_0 \in A \cap (\mathrm{dom} f) = \mathrm{dom} f = (-\infty, -2) \cup (-2, 0) \cup (0, +\infty)$

f è continua in $x_0 \Rightarrow \lim\limits_{x \to x_0} f(x) = f(x_0)$

•) $x_0 \in A \setminus (\mathrm{dom} f) = \{+\infty\} \cup \{-\infty\} \cup \{-2\} \cup \{0\}$

$$\lim_{\substack{x \to +\infty \\ -\infty}} \underbrace{\frac{(x+2)}{x}}_{\to 1} \underbrace{e^{\overbrace{-\frac{1}{(x+2)}}^{\to 0}}}_{\to 1} = 1$$

$$\lim_{x \to -2^+} \underbrace{\frac{(x+2)}{x}}_{\to 0} \underbrace{e^{-\frac{1}{(x+2)}}}_{\to 0} = 0$$

$$\lim_{x \to -2^-} \underbrace{\frac{(x+2)}{x}}_{\to 0} \cdot \underbrace{e^{\overbrace{-\frac{1}{(x+2)}}^{t}}}_{\to +\infty} =$$

$t \to +\infty$

$$\lim_{t \to +\infty} \frac{e^t}{t} = +\infty$$

$$\frac{\cancel{x+2}}{x}\left(-\frac{1}{\cancel{(x+2)}}\right)\frac{e^{-\frac{1}{(x+2)}}}{-\frac{1}{(x+2)}} = -\underbrace{\boxed{\frac{1}{x}}}_{\to +\frac{1}{2}}\ \underbrace{\boxed{\frac{e^{-\frac{1}{(x+2)}}}{-\frac{1}{(x+2)}}}}_{\to +\infty}$$

$$= +\infty$$

$$\lim_{x\to 0^{\pm}} \boxed{\frac{x+2}{x}}\ \boxed{e^{-\frac{1}{x+2}}} = \begin{matrix} +\infty \\ -\infty \end{matrix}$$

3) Asintoti

orizzontali: $y=1$ sia a $+\infty$ che a $-\infty$

verticali: $x=0$, $x=-2$

obliqui non ce ne sono (ci sono gli asintoti orizzontali)

4) Determinare $B = \{$punti dove f è derivabile$\}$

e calcolare $f'(x)$, $\forall x \in B$.

$$f(x) = \frac{x+2}{x}\, e^{-\frac{1}{x+2}}$$

per il teo sull'algebra delle derivate e sulla composizione di funzioni derivabili $B = \mathrm{dom}\, f$ e

$$f'(x) = \left[\frac{\cancel{x} - (\cancel{x}+2)}{x^2}\right] \underline{e^{-\frac{1}{x+2}}}$$

$$+ \left[\frac{\cancel{x+2}}{x}\right] \underline{e^{-\frac{1}{x+2}}} \left(\frac{1}{(x+2)^{\cancel{2}}}\right)$$

$$= e^{-\frac{1}{x+2}} \left[-\frac{2}{x^2} + \frac{1}{x(x+2)} \right]$$

$$= e^{-\frac{1}{x+2}} \, \frac{-2x-4+x}{x^2(x+2)} =$$

$$= -e^{-\frac{1}{x+2}} \, \frac{4+x}{x^2(x+2)}$$

5) Intervalli di monotonia

$$f'(x) = -e^{-\frac{1}{x+2}} \, \frac{4+x}{x^2(x+2)} > 0$$

$$\Updownarrow$$

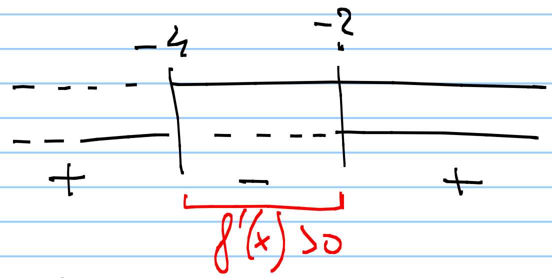
$$\underbrace{\boxed{e^{-\frac{1}{x+2}}}}_{>0} \, \frac{4+x}{\underbrace{x^2}_{>0}(x+2)} < 0$$

$$\Updownarrow$$

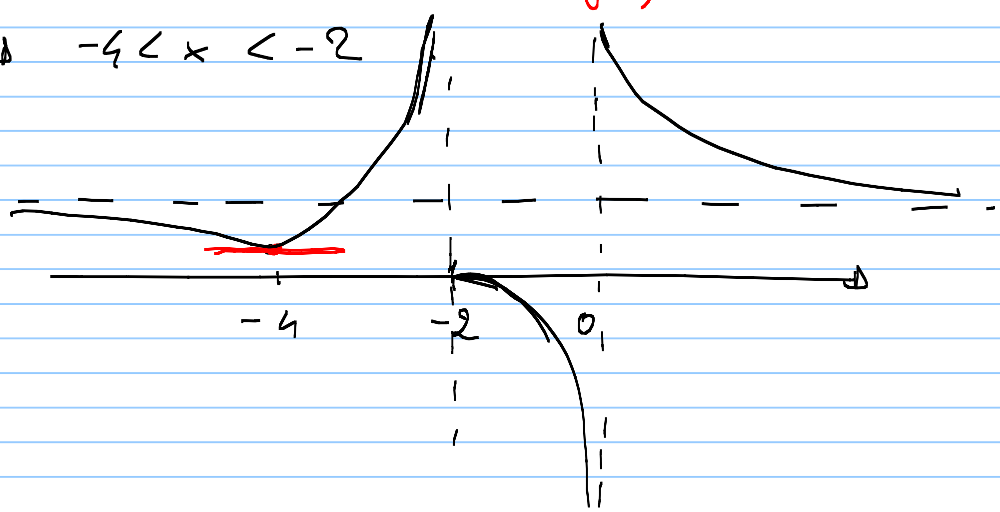
$$\frac{4+x}{(x+2)} < 0$$

$(4+x) > 0 \iff x > -4$

$(x+2) > 0 \iff x > -2$



$f'(x) > 0 \iff -4 < x < -2$



f è decrescente in $(-\infty, -4)$
in $(-2, 0)$
in $(0, +\infty)$

f è crescente in $(-4, -2)$

6) max / min.

$f'(x) = 0 \iff x = -4$

$x = -4$ è di min relativo
non assoluto $\left(\lim_{x \to 0^-} f(x) = -\infty \right.$

$$\lim_{x \to -2^+} f'(x) = \lim_{x \to -2^+} -\underbrace{e^{-\frac{1}{x+2}}}_{\downarrow \atop 0} \underbrace{\boxed{\frac{4+x}{x^2(x+2)}}}_{\downarrow \atop +\infty} =$$

$$= \lim_{x \to -2^+} - \frac{\frac{4+x}{x^2(x+2)}}{e^{\frac{1}{x+2}}} =$$

$$= \lim_{x \to -2^+} - \underbrace{\boxed{\frac{4+x}{x^2}}}_{\downarrow \atop \frac{1}{2}} \boxed{\frac{\frac{1}{(x+2)}}{e^{\frac{1}{(x+2)}}}} = 0$$

$t = \frac{1}{x+2}$ $\qquad x \to -2^-$
$\qquad t \to +\infty$

$$\lim_{t \to +\infty} \frac{t}{e^t} = 0$$