Lezione del 19 Ottobre

$$f : A \longrightarrow B$$
$$x \longrightarrow y = f(x)$$

A D dominio della funzione

$$f(A) = \operatorname{im} f = \{ y \in B : \exists\, x \in A : f(x) = y \}$$
$$f(A) \subseteq B$$

$$\operatorname{graf} f = \{ (x, y) \in \mathbb{R}^2 : y = f(x) \}$$

Funzioni simmetriche

f PARI : $f(-x) = f(x)$

Dominio = simmetrico rispetto all'origine

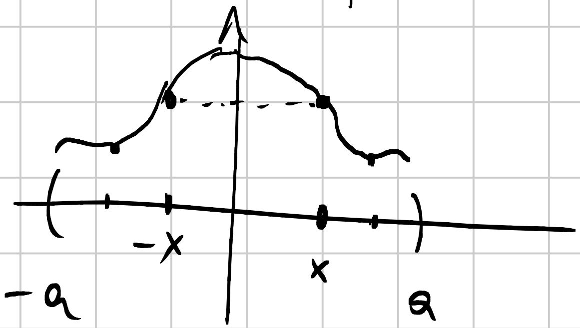
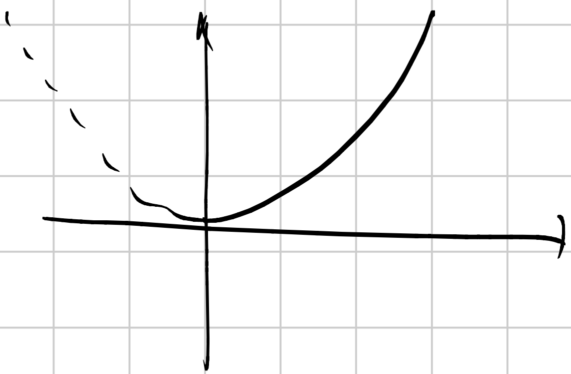


grafico simmetrico rispetto all'asse delle y.

es. $f(x) = x^2$ $\quad f(-x) = (-x)^2 = x^2 = f(x)$



f dispari

$$f(-x) = -f(x)$$

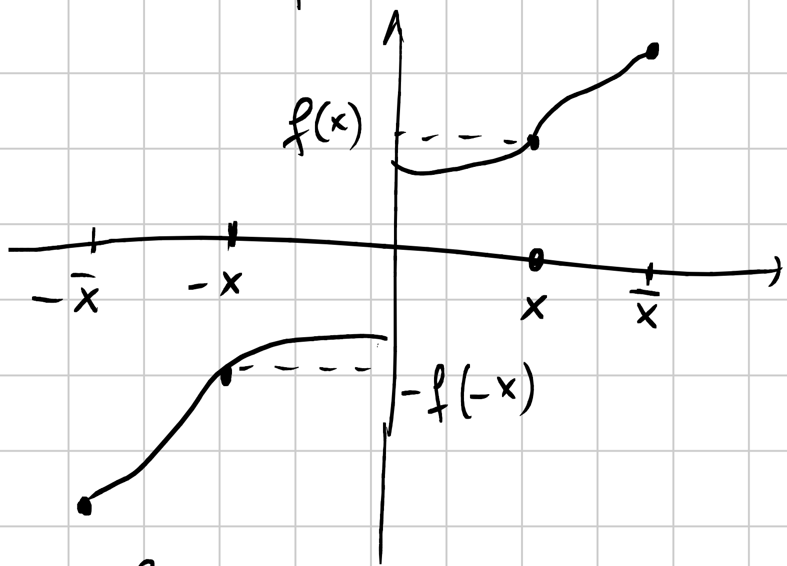
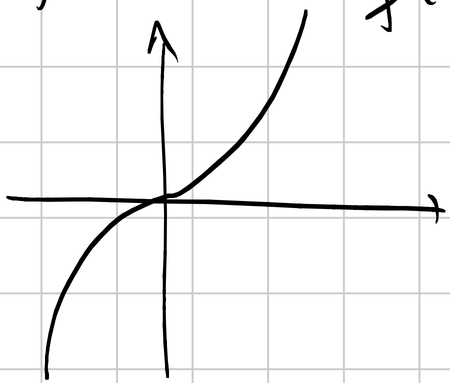


grafico simmetrico rispetto all'origine degli assi

$$f(x) = x^3 \qquad f(-x) = (-x)^3 = -x^3 = -f(x)$$



$$f(x) = x^n$$

n pari f pari

n dispari f dispari

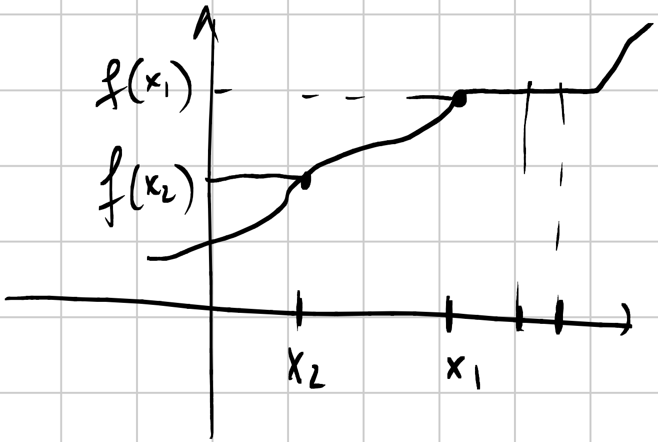
Funzione <u>monotòna</u>

$f : D \to \mathbb{R}$

f è monotona crescente (non decrescente) se $\forall x_1, x_2 \in D$

$x_1 > x_2$

$\Rightarrow f(x_1) \geq f(x_2)$

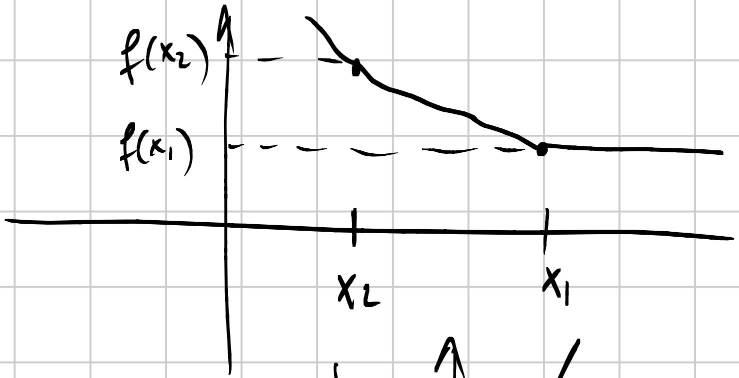


se $f(x_1) \underset{\neq}{\geq} f(x_2)$

f si dice strettamente crescente

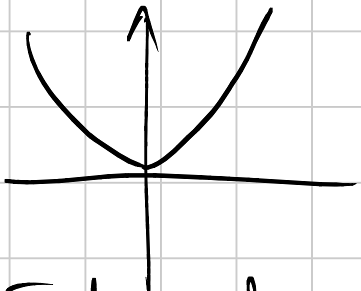
f è monotona decrescente (non crescente) se $\forall x_1 > x_2 \Rightarrow$

$f(x_1) \leq f(x_2)$



es.

$f(x) = x^2$



non è monotone

oppure si può dire che $f(x) = x^2$ è
strett. crescente in $[0, +\infty)$ e
è strett. decrescente in $(-\infty, 0]$

oss. $\quad x_1 > x_2 \quad \overset{?}{\not\Rightarrow} \quad \underbrace{x_1^2}{} > x_2^2$

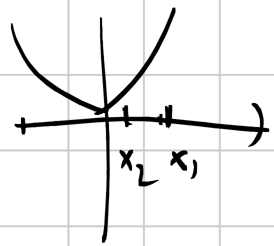
$f(x_1) > f(x_2)$

è vero se f è strett. crescente

$f(x) = x^2$

non è strett. crescente su $\mathbb{R}$

non vale l'implicazione.



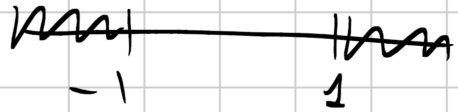
$x_1 > x_2$ e $x_1, x_2 > 0 \Longrightarrow x_1^2 > x_2^2$

es. $\underbrace{\sqrt{x^2-1}}_{x_1} \overset{?}{>} \underbrace{\frac{x}{2}}_{x_2}$

$x^2-1 \geq 0$

$|x| \geq 1$

$(x \geq 1, \ x \leq -1)$



1) se $\frac{x}{2} < 0$ $\quad x < 0 \quad$ sempre vero

$x < -1$ sempre vero

2) se $\frac{x}{2} \geq 0$ $\quad x \geq 0$

$$\left(\sqrt{x^2-1}\right)^2 > \left(\frac{x}{2}\right)^2$$

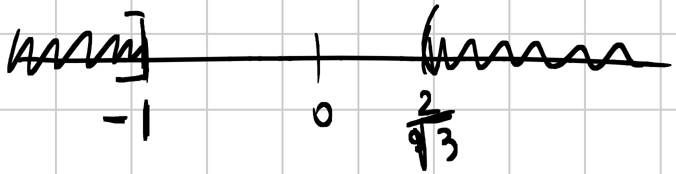
!

$x^2 - 1 > \frac{x^2}{4}$ $\qquad \frac{3}{4}x^2 > 1$

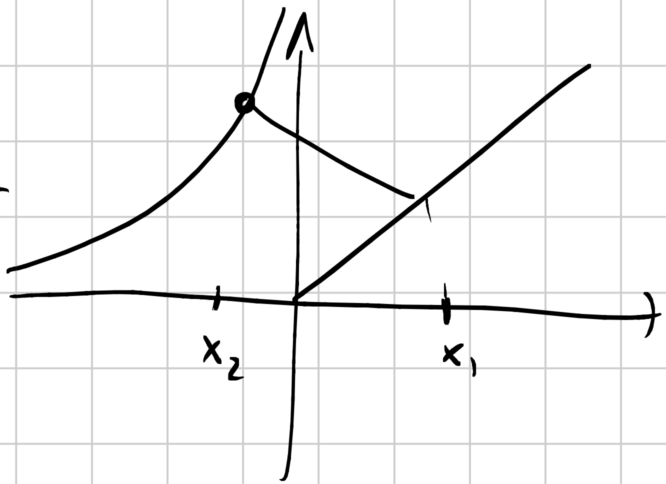
$x^2 > \frac{4}{3}$ $\qquad \left(x > \frac{2}{\sqrt{3}}, \quad \cancel{x < -\frac{2}{\sqrt{3}}} \right)$

2) $\quad x > \frac{2}{\sqrt{3}}$

$(-\infty, -1] \cup \left(\frac{2}{\sqrt{3}}, +\infty\right)$



<u>es.</u>



f <u>non</u> è monotone nel suo dominio di definizione.

Funzioni periodiche

$f : D \to \mathbb{R}$ è <u>periodica di periodo T</u>, $T > 0$

se T è il più piccolo numero reale positivo t.c. $f(x+T) = f(x)$, $\forall x \in D$

Ogni intervallo di lunghezza T si chiama intervallo di periodicità $(0,T)$

$f(x) = \operatorname{sen} x$ $\quad T = 2\pi$

$= \cos x$

$f(x) = \operatorname{tg} x$ $\quad T = \pi$

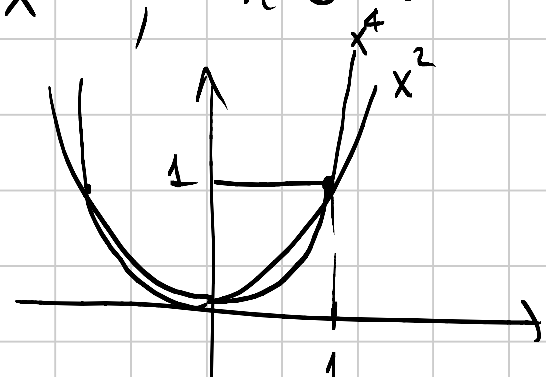
Funzioni elementari

Funzioni potenza

$f(x) = x^n \quad , \quad n \in \mathbb{N}$

$D = \mathbb{R}$

f

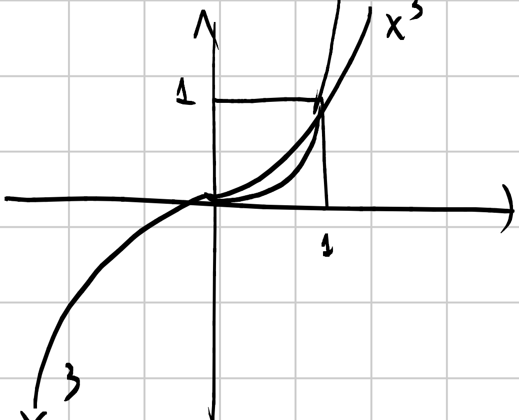


n pari

f pari

$f(1) = 1 \quad , \forall n$

$f(\mathbb{R}) = [0, +\infty)$



n dispari

f dispari

strett. crescente

$x_1 > x_2 \Rightarrow x_1^3 > x_2^3$

es. $$\underbrace{\sqrt[3]{7+8x^3}}_{x_1} < \underbrace{2x+1}_{x_2}$$

$$(\quad)^3 < (\quad)^3$$

$$7+8x^3 < (2x+1)^3$$

$$\left(x > \frac{1}{2} \text{ e } x < -1\right)$$

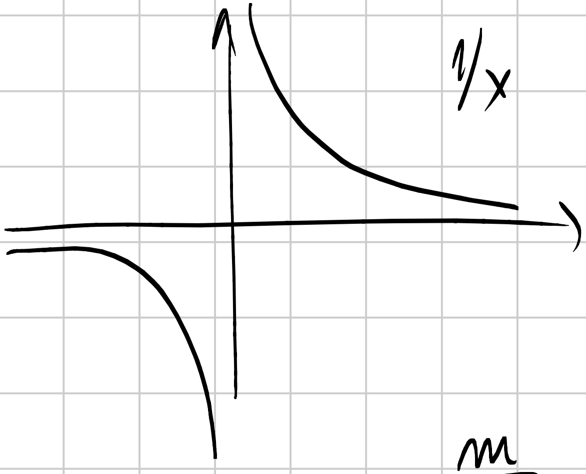
Finire!

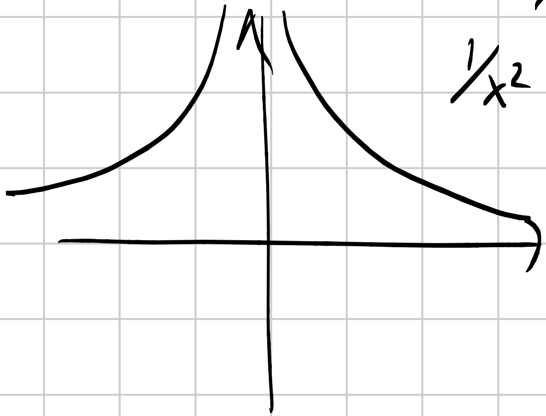
- $f(x) = x^r$, $r \in \mathbb{Z}$, $r < 0$

f $\quad D = \mathbb{R} \setminus \{0\}$

x $-r$ è pari $\quad$ f è pari $\quad$ es. $f(x) = \dfrac{1}{x^2}$

se -2 è dispari f è dispari es. $f(x) = \frac{1}{x}$





• $f(x) = x^{\frac{m}{n}}$

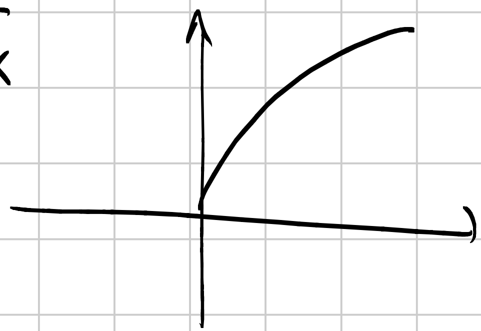
in generale def. per $x > 0$

se n è dispari anche se $x < 0$

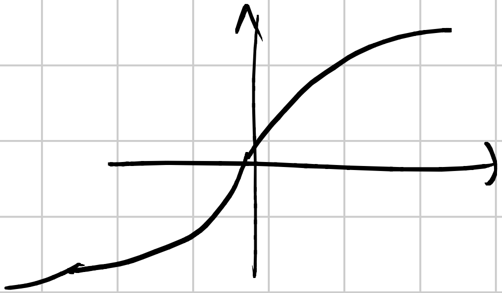
$f(x) = x^{1/2} = \sqrt{x}$

$D = \{x \geq 0\}$

$\text{Im} f = \{x \geq 0\}$



$f(x) = x^{1/3}$



$D = \mathbb{R}$

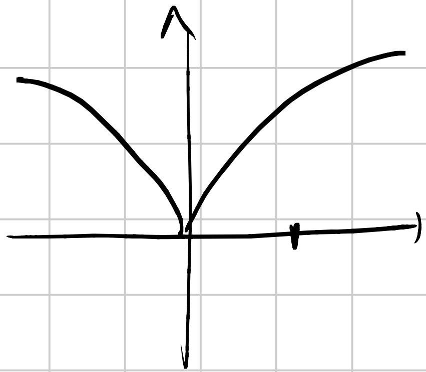
$\operatorname{im} f = \mathbb{R}$

$\left(\sqrt[n]{y} = -\sqrt[n]{-y}\right)$

$y < 0$

$f(x) = x^{2/3} = \left(x^2\right)^{1/3}$

f pari



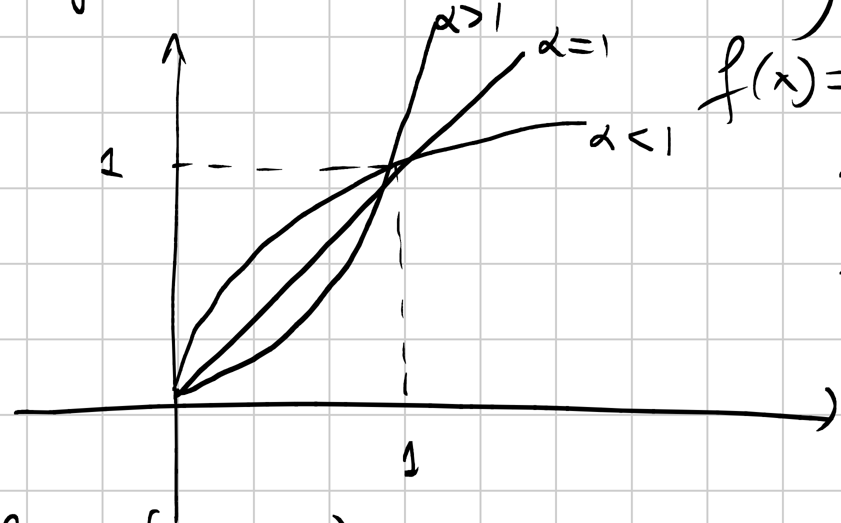
$f(x) = x^{-\frac{1}{2}} = \dfrac{1}{\sqrt{x}}$ $\qquad D = \{x > 0\}$

- $f(x) = x^{\alpha}$, $\alpha \in \mathbb{R}$

definita solo per $x > 0$

(def per $x \geq 0$ se $\alpha \geq 0$)

$\alpha > 0$



$f(x) = x^{\alpha}$

$x \geq 0$

$f(1) = 1$

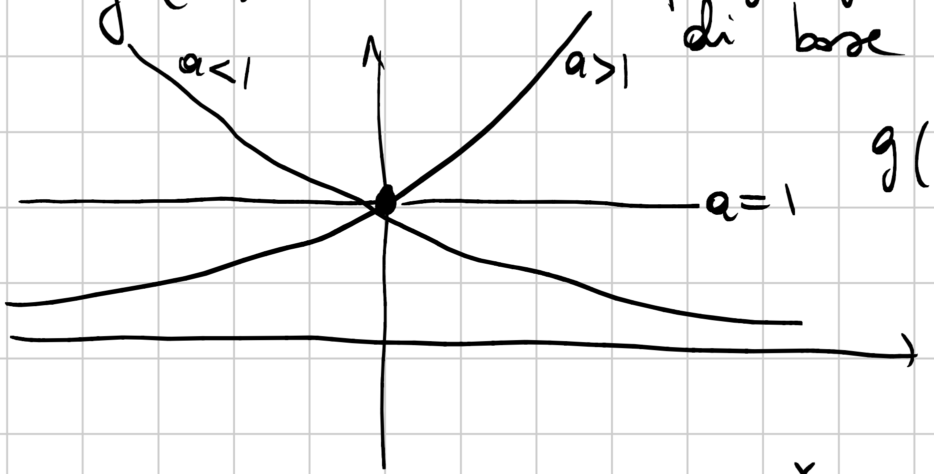
$\text{dom} f = [0, +\infty)$

$\text{Im} f = [0, +\infty)$

Funzione <u>esponenziale</u> e <u>logaritmica</u>

$a \in \mathbb{R}, \quad a > 0$

$g(x) = a^x$ funz. esponenziale di base a



$g(0) = 1 \quad \forall a$

$a = 1$

$1^x = 1$

$a > 1$ <u>es.</u> $3^x \quad (\sqrt{2})^x \quad (28)^x$

$\text{dom}\, g = \mathbb{R} \qquad \text{Img} = (0, +\infty)$

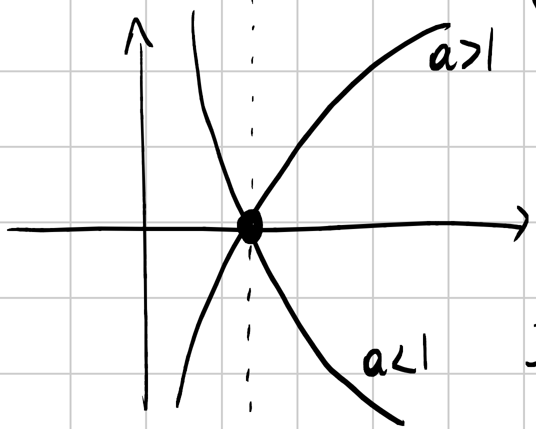
$0 < a < 1 \qquad g(x) = a^x \qquad$ es. $\left(\frac{1}{3}\right)^x$

funzione logaritmica

$f(x) = \log_a x$, $\boxed{x > 0}$

$y =: \log_a x \iff x = a^y$

$a^y = a^{\log_a x} = x$ $\quad (x > 0,\ a > 0,\ a \neq 1)$



$a > 1$

$f(x) = \log_a x$

$f(1) = \log_a x = 0$

$\log_2 x$ è strett. crescente

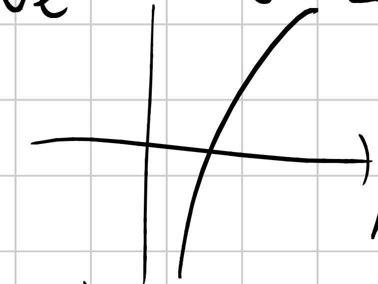
$\log_{\frac{1}{2}} x$ è strett decrescente

Spesso si prende come $a = e$

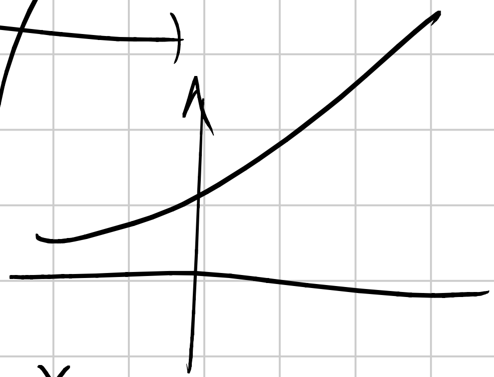
numero di Nepero

$e \sim 2,71$ $\quad e > 1$

$$f(x) = \log_e x = \log x = \ln x$$



$$f(x) = e^x$$



$$a^x = \left(e^{\log a}\right)^x = e^{\log a \, x}$$

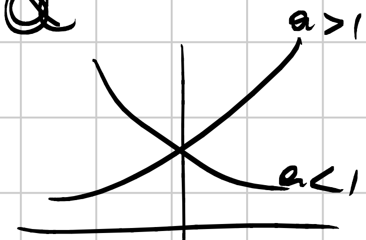
$$a = e^{\log a}$$

esercizio

$$\log\left(e^{2x} - 4e^{x} + 4\right) > 0$$

$$x_1 > x_2 \not\Rightarrow a^{x_1} > a^{x_2}$$

$\Rightarrow$ solo se $a > 1$



se $a < 1$ $\quad x_1 > x_2 \Rightarrow a^{x_1} < a^{x_2}$

$\underbrace{\log\left(e^{2x}-4e^{x}+4\right)}_{x_1} > \underbrace{0}_{x_2}$

prima

$e^{2x}-4e^{x}+4>0$

$e>1$

$e^{x_1} > e^{x_2}$

$$\boxed{e^{2x}-4e^{x}+4>1}$$

$e^{x}=y$

$y^2-4y+3>0$

$y = 2 \pm \sqrt{4-3} = 2 \pm 1 \begin{cases} 3 \\ 1 \end{cases}$

$y>3 \qquad y<1$

$e^{x}>3 \qquad e^{x}<1$

$x>\log 3 \qquad x<0$

se fosse stato

$$\log_{1/2}\left(e^{2x}-4e^{x}+4\right)>0$$

$$e^{2x}-4e^{x}+4>0$$

$$\left(\frac{1}{2}\right)^{\log_{1/2}(\qquad)} < \left(\frac{1}{2}\right)^{0}$$

$x_1 > x_2 \Rightarrow a^{x_1} < a^{x_2}$ se $a<1$

$$\boxed{\left(e^{2x}-4e^{x}+4\right)<1}$$