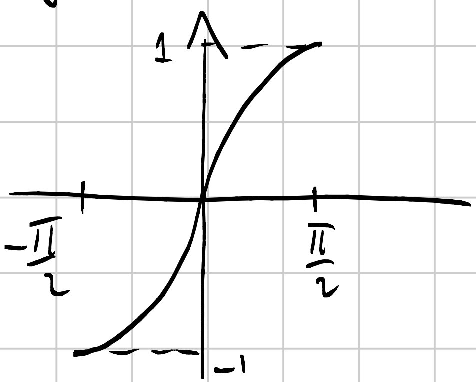
Lezione del 21 Ottobre

Funzioni trigonometriche *inverse*

$y = \operatorname{sen} x$ non è invertibile se $x \in \mathbb{R}$



se $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

sen x è strett. crescente

$$\operatorname{sen} x : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1,1]$$

$$f^{-1} : [-1,1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

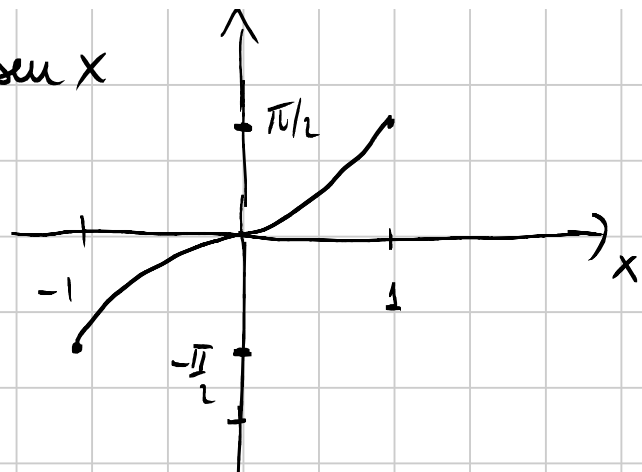
$$f^{-1}(y) =: \operatorname{arcsen} y$$

$$x = \operatorname{arcsen} y \iff y = \operatorname{sen} x$$

$y \in [-1,1]$ $\qquad$ $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$y = \arcsin x$

$x \in [-1, 1]$



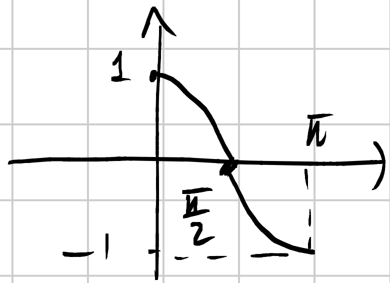
1) $\arcsin x$ è def. in $[-1, 1]$

2) $\arcsin(-x) = -\arcsin x$

Analogamente

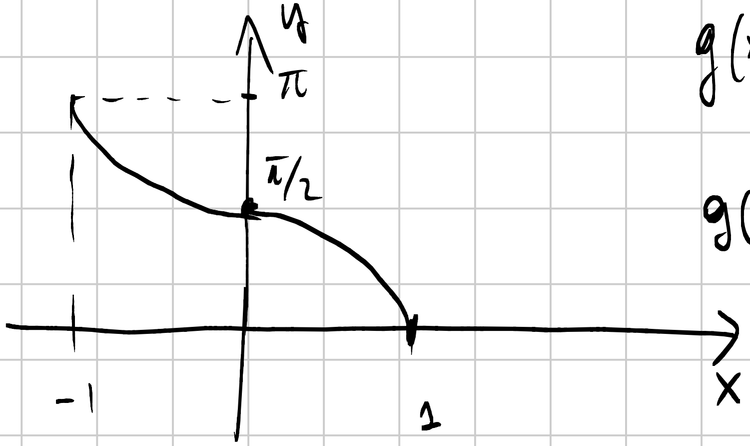
$y = \cos x$

$x \in [0, \pi]$



$\cos x : [0, \pi] \longrightarrow [-1, 1]$

$f^{-1}(y) =: \arccos y : [-1, 1] \to [0, \pi]$



$g(x) = \arccos x$
definita $x \in [-1, 1]$

$g(-1) = \pi$
$g(0) = \pi/2$
$g(1) = 0$

es. Trovare il dominio di

$$f(x) = \arccos |x^3 - 1|$$

$$|x^3 - 1| \leq 1 \iff -1 \leq x^3 - 1 \leq 1$$

$$0 \leq x^3 \leq 2 \qquad 0 \leq x \leq \sqrt[3]{2}$$

oss. sen x , $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ la funzione inversa è arcsen y

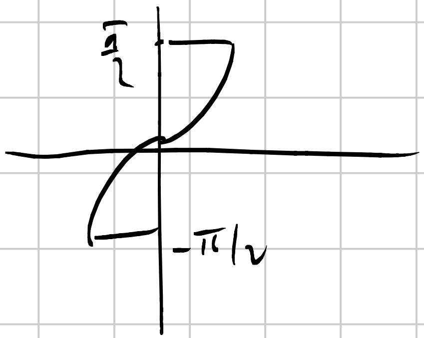
$x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ sen x è strett. decrescente

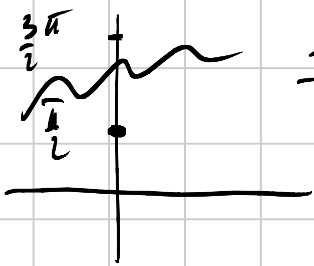
$\text{sen}\, x : \left[\frac{\pi}{2}, \frac{3}{2}\pi\right] \to [-1, 1]$

$f^{-1} : [-1, 1] \to \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

non è la funzione arcsen y

$\text{arcsen}\, y : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



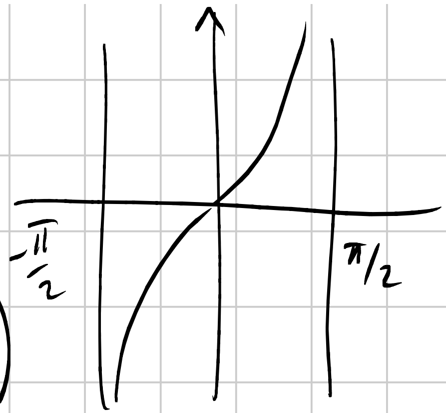


$f^{-1}(y) = \pi - \text{arcsen}\, y$

Analogamente

$\operatorname{tg} x : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$

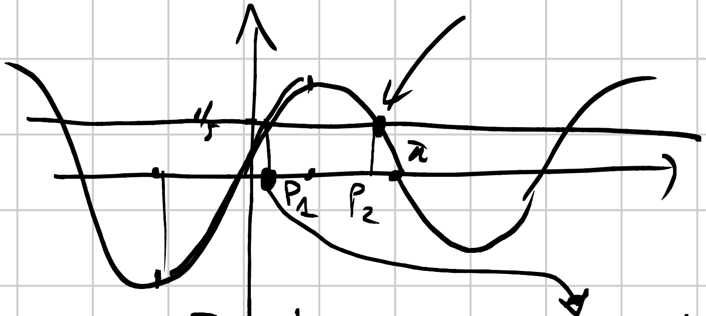
$f^{-1}(y) =: \operatorname{arctg} y : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



arctg x è **dispari**

es. Trovare le soluzioni di

$\operatorname{sen} x = \frac{1}{5}$



$\bar{x} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ t.c. $\operatorname{sen} \bar{x} = \frac{1}{5}$ $\quad \bar{x} = \arcsin \frac{1}{5}$

$P_1 = \arcsin \frac{1}{5}$ $\qquad$ $P_2 = \pi - \arcsin \frac{1}{5}$

tutte le soluzioni sono

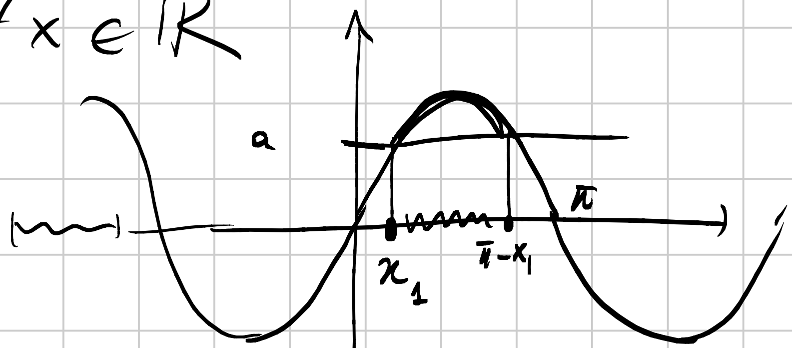
$$\begin{cases} x = \arcsin \frac{1}{5} + 2k\pi, \quad k \in \mathbb{Z} \\ \quad\text{e} \\ x = \pi - \arcsin \frac{1}{5} + 2k\pi, \quad k \in \mathbb{Z} \end{cases}$$

<u>es.</u> $\sin x > a, \quad a \in \mathbb{R}$

1) $a \geq 1 \qquad \nexists\, x$

2) $a < -1 \qquad \forall x \in \mathbb{R}$

3) $a \in [-1, 1)$



$x_1 = \arcsin a$

$x \in (x_1, \pi - x_1) = (\arcsin a, \pi - \arcsin a)$

$x \in (\arcsin a + 2k\pi, \pi - \arcsin a + 2k\pi)$.

- $a = -1$ fare esplicitamente

Fare $\operatorname{sen} x < a$, $a \in \mathbb{R}$.

Funzioni iperboliche

$$Sh x = \operatorname{senh} x := \frac{e^x - e^{-x}}{2}$$ seno iperbolico

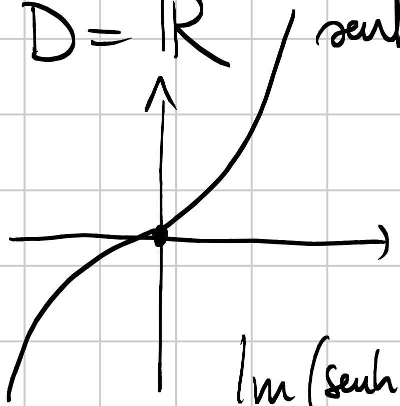
$$Ch x = \cosh x := \frac{e^x + e^{-x}}{2}$$ coseno iperbolico

$$Th x = \operatorname{tgh} x := \frac{\operatorname{senh} x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{senh} x = \frac{e^{x} - e^{-x}}{2}$$

$D = \mathbb{R}$

senh x



dispari

$(\operatorname{senh}(-x) = -\operatorname{senh} x)$

Fare

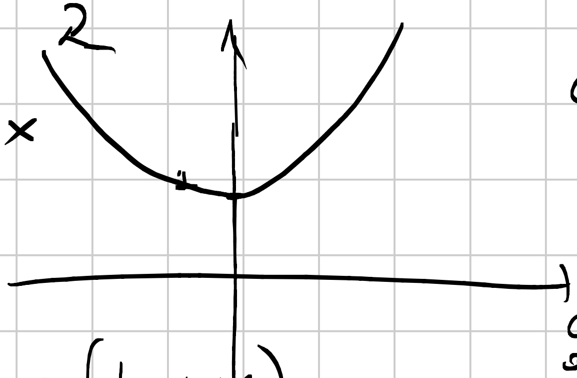
$\operatorname{Im}(\operatorname{senh} x) = \mathbb{R}$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

$D = \mathbb{R}$

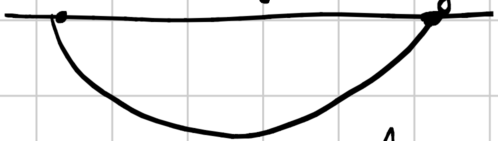
catenaria

$\cosh(-x) = \cosh x$



$\operatorname{Im}(\cosh x) = [1, +\infty)$

catena appesa in due punti

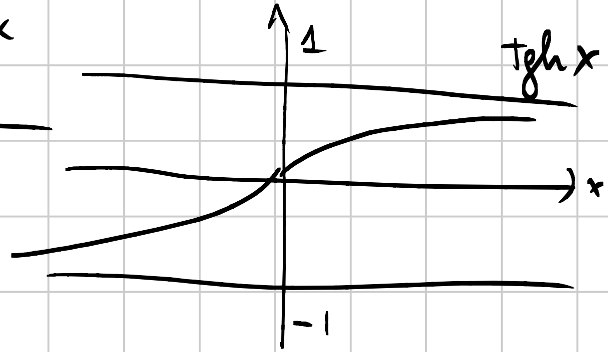


$y = \cosh x$

$$\boxed{\underbrace{\cosh^2 x}_{y^2} - \underbrace{\operatorname{senh}^2 x}_{z^2} = 1} \longrightarrow z = \operatorname{senh} x$$

$$y^2 - z^2 = 1$$

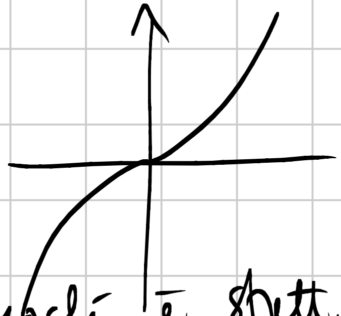
$$\operatorname{tgh} x := \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$



$$\operatorname{Im}(\operatorname{tgh} x) = (-1, 1).$$

Funzioni iperboliche inverse

$y = \operatorname{senh} x$



$\operatorname{senh} x : \mathbb{R} \to \mathbb{R}$

è invertibile su tutto $\mathbb{R}$ perché è strett. crescente

$$f^{-1}(y) = \operatorname{settsenh} y : \mathbb{R} \to \mathbb{R}$$

settore seno iperbolico

si può dimostrare :

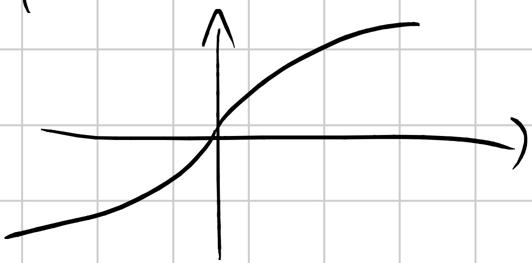
$$y = \operatorname{senh} x = \frac{e^{x} - e^{-x}}{2} = y$$

$\downarrow$ se esplicitate x in funzione di y

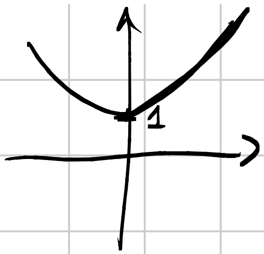
$$x = \log\left(y + \sqrt{1+y^2}\right) = \operatorname{settsenh} y$$

$$\operatorname{settsenh} x = \log\Big(\underbrace{x + \sqrt{1+x^2}}_{>0\ \forall x \in \mathbb{R}}\Big)$$

(PC. verificare)



Analogamente



$\cosh x : [0, +\infty) \to [1, +\infty)$

$f^{-1}(y) : [1, +\infty) \to [0, +\infty)$

$f^{-1}(y) = \text{settcosh}\, y \; : [1, +\infty)$

$$\text{settcosh}\, x = \log\left(x + \sqrt{x^2 - 1}\right)$$

es: per casa Trovare dominio di log( )

$(x \geq 1)$

esercizio

$$f(x) = \frac{1}{x^2 - \operatorname{sen}\left(\frac{\pi}{2}x\right)}$$

oss. $\operatorname{sen} x$ è periodica di periodo 2π

$\rightarrow f(x) = \operatorname{sen}(\omega x)$ è periodica ?

sì! di periodo $\frac{2\pi}{\omega} = T$

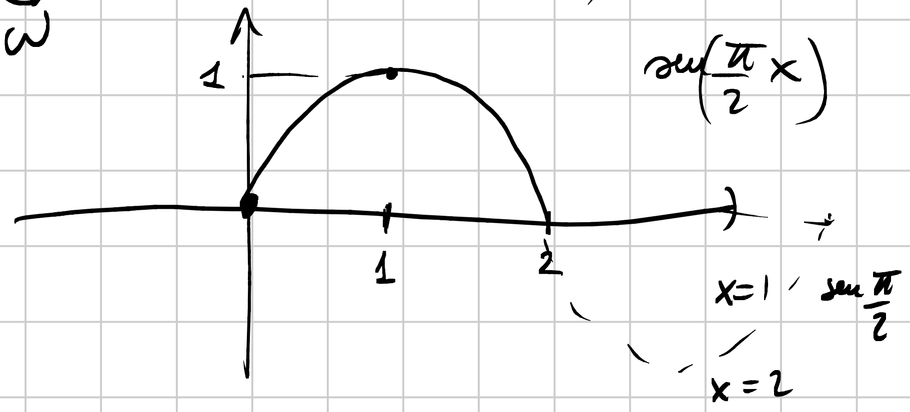
$$f(x+T) = f(x), \quad \forall x$$

$$f(x+T) = \operatorname{sen}(\omega(x+T)) = \operatorname{sen}\left(\omega x + \cancel{\omega}\frac{2\pi}{\cancel{\omega}}\right)$$

$$= \operatorname{sen}(\omega x + 2\pi) = \operatorname{sen}(\omega x) = f(x)$$

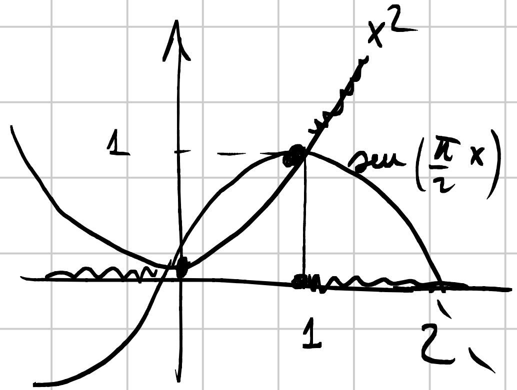
$$f(x) = \operatorname{sen}\left(\underset{\omega}{\frac{\pi}{2}}\, x\right) \qquad T = \frac{2\cancel{\pi}}{\cancel{\pi}} \cdot 2 = 4$$

$\operatorname{sen} x$



$$f(x) = \frac{1}{x^2 - \operatorname{sen}\left(\frac{\pi}{2} x\right)}$$

$$\underbrace{\operatorname{sen}\left(\frac{\pi}{2} x\right)}_{g(x)} = \underbrace{x^2}_{h(x)}$$



$g(1) = 1 \qquad h(1) = 1$

$x = 0$

$x = 1$

$D = \{ x \in \mathbb{R},\ x \neq 0 \text{ e } x \neq 1 \}$

$$\boxed{f(x) = \frac{1}{x^2 - \operatorname{sen}\left(\frac{\pi}{2} x\right)}}$$

- $f(x) > 0$

se $\underbrace{x^2}_{h(x)} > \underbrace{\operatorname{sen}\left(\frac{\pi}{2} x\right)}_{g(x)}$

se e solo se

$x > 1$

e

$x < 0$

- f periodica? no!

$f(x+4) = f(x) \quad \forall x$

- f simmetrica no!

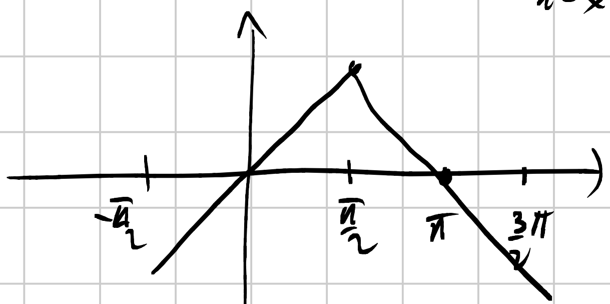
né pari, né dispari

$$f(-x) = \frac{1}{x^2 - \operatorname{sen}\left(\frac{\pi}{2}(-x)\right)} = \frac{1}{x^2 + \operatorname{sen}\left(\frac{\pi}{2} x\right)}$$

no! $\cancel{f(x)}$

Es. disegnare il grafico di $\arcsin(\sin x)$

$\pi - x$



se $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\arcsin(\sin x) = x$

$\arcsin(\sin x) = h(x)$?

$x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Cambio variabili

$y = \pi - x$ $\quad y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

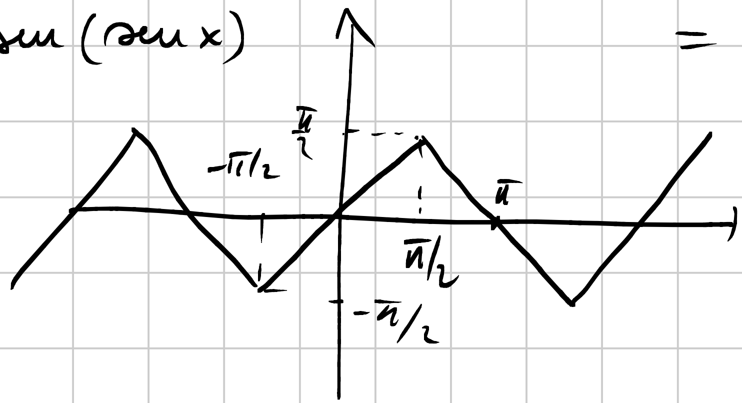
$(f^{-1}(f(x)) = x)$

$\boxed{\arcsin(\sin y) = y}$ $\longrightarrow$

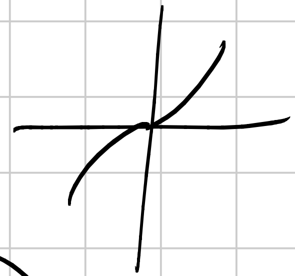
$\sin y = \sin(\pi - x) = \sin x$

$\boxed{\arcsin(\sin x) = \pi - x}$ $\quad x \in \left(\frac{\pi}{2}, \frac{3}{2}\pi\right)$

$\arcsin(\sin x)$



$$= \begin{cases} x & x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \pi - x & x \in \left(\frac{\pi}{2}, \frac{3}{2}\pi\right) \\ \vdots \end{cases}$$



Operazioni sui grafici

oss. $f(x) = \arcsin(\sin x)$

Dominio ?

$x \in \mathbb{R}$

$|\sin x| \le 1$

$\arcsin x \qquad |x| \le 1$

$f(x+T) = f(x) \quad \forall x$

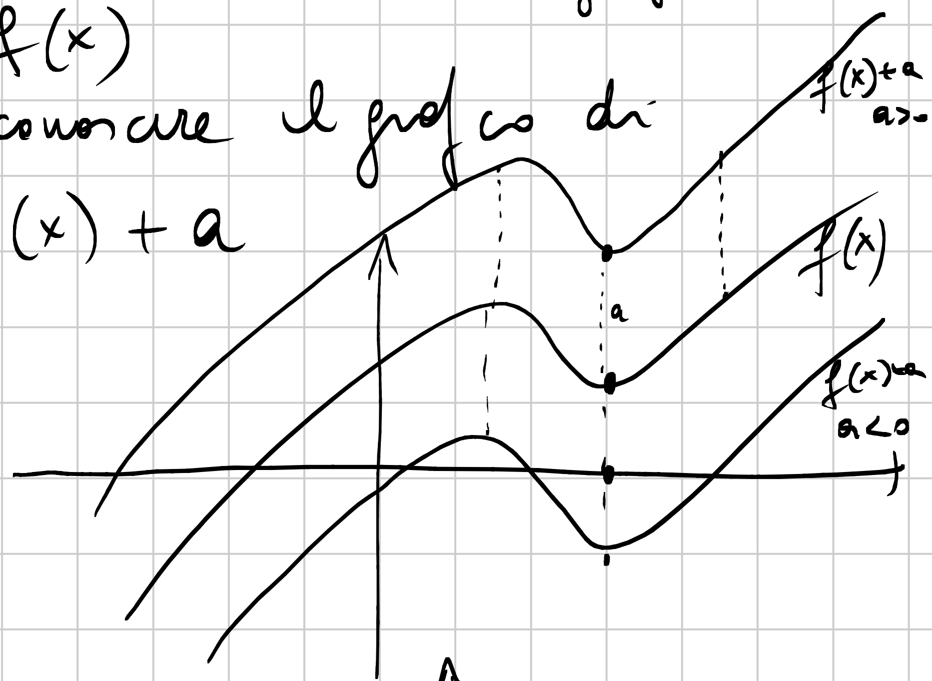
$f(x+2\pi) = \arcsin(\sin(x+2\pi)) = \arcsin(\sin x)$,
$= f(x)$

Operazioni sui grafici

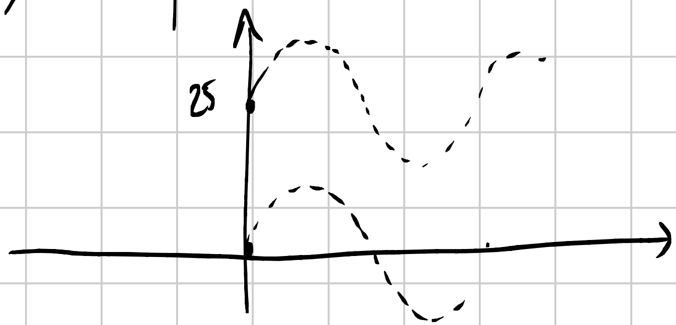
Supponiamo di conoscere il grafico di $y = f(x)$ e vogliamo conoscere il grafico di

$$y_1 = f(x) + a$$

$a > 0$



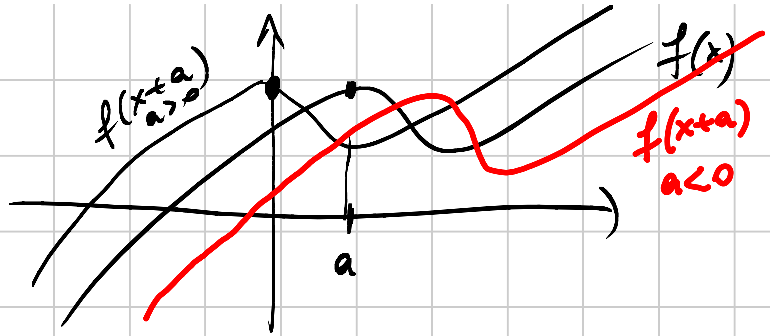
$y = \operatorname{sen} x + 25$



Traslazione in alto o in basso

$y_2 = f(x+a)$

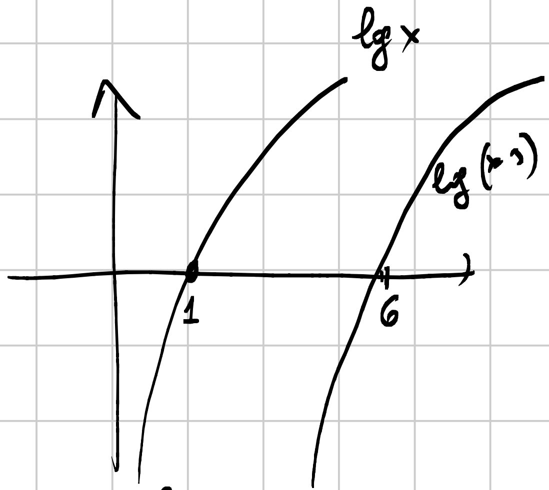
$x=0 \qquad y_2 = f(a)$



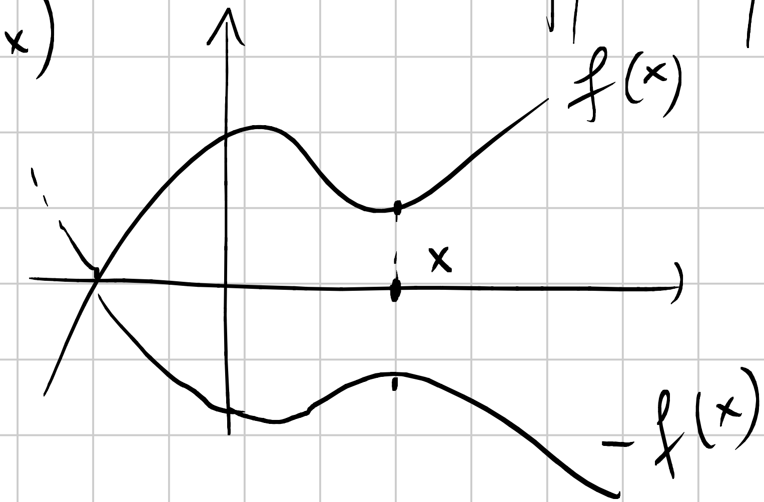
se $a>0$ traslazione a sx

se $a<0$ traslazione a dx

<u>es.</u> $\log(x-5) \qquad x>5$



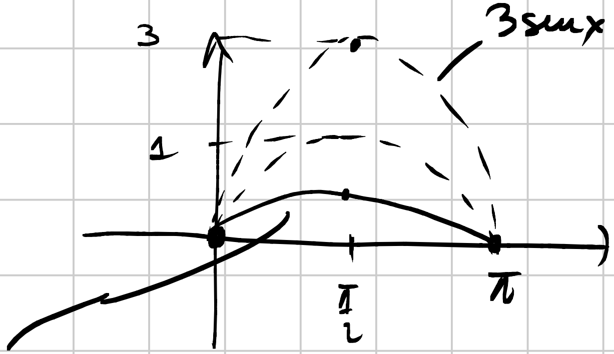
$y_3 = -f(x)$



$y_4 = k\, f(x)$

es. $y_4 = 3 \operatorname{sen} x$

$= \frac{1}{2} \operatorname{sen} x$



$k > 1$ stiramento verso l'alto

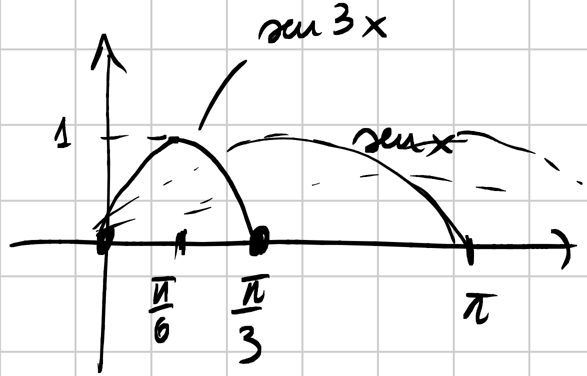
$k < 1$ contrazione " "

$y_5 = f(kx)$

$y_5 = \operatorname{sen} 3x$

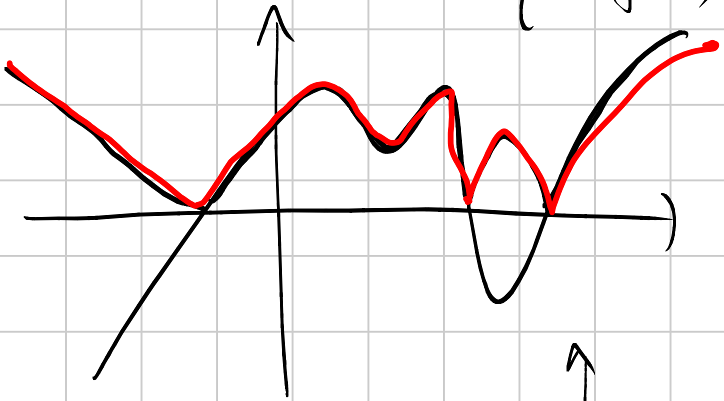
$3x = \pi \qquad x = \frac{\pi}{3}$

$3x = \frac{\pi}{2} \qquad x = \frac{\pi}{6}$

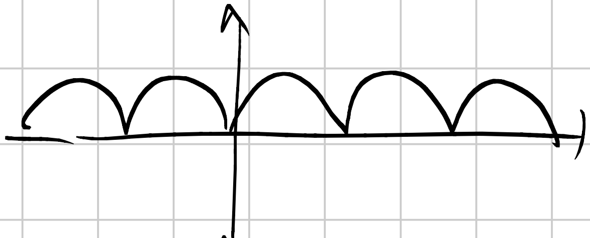


$\operatorname{sen} \frac{1}{3} x$

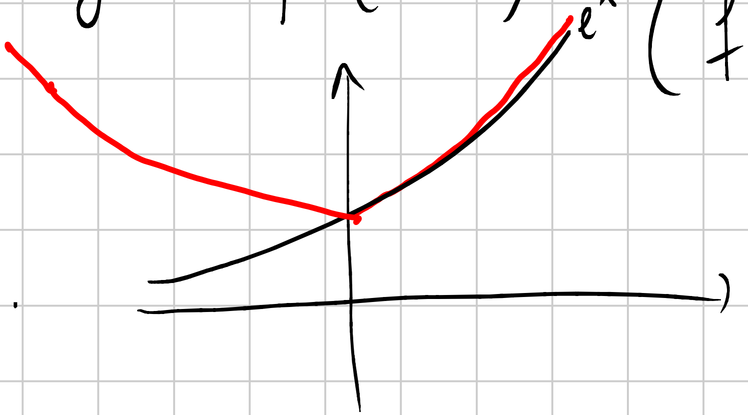
$$y_5 = |f(x)| = \begin{cases} f(x) & f(x) \geq 0 \\ -f(x) & f(x) < 0 \end{cases}$$



es. $|\text{sen}\, x|$



$$y_6 = f(|x|) = \begin{cases} f(x) & x \geq 0 \\ f(-x) & x < 0 \end{cases}$$



$f(x) = e^x$ sempre pari

$e^{|x|}$

oss. su diseq. con il valore assoluto.

$$|f(x)| \leq g(x)$$

es. $|x^2-1| \leq 3x$

1) $g(x) < 0$ mai verificata

2) $g(x) \geq 0$ $\boxed{-g(x) \leq f(x) \leq g(x)}$

es. Dominio

$$f(x) = \sqrt{\frac{1}{3-x-|6x^2-13x-15|}}$$

$$|\underbrace{6x^2-13x-15}_{f(x)}| < \underbrace{3-x}_{g(x)} \quad -g(x) < f(x) < g(x)$$

1) se $3 - x < 0$ mai! $x > 3$

2) se $3 - x \geq 0$ $x \leq 3$

a) $6x^2 - 13x - 15 < 3 - x$

b) $6x^2 - 13x - 15 > x - 3$

R. $\left(-1, -2/3\right)$