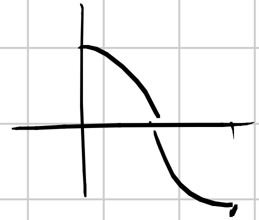
2ª parte lezione del 24 Gennaio

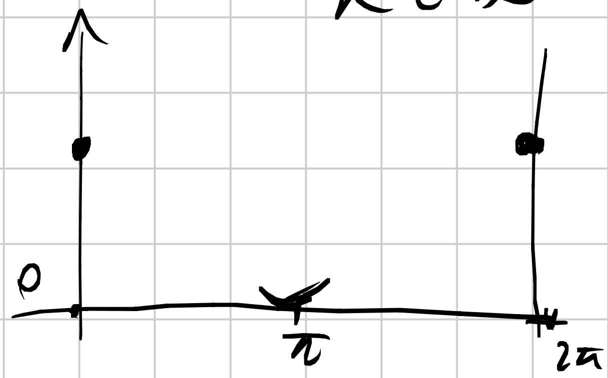
$$f(x) = e^{-\frac{1}{1+\cos x}}$$



$$D = \{1+\cos x \neq 0\} = \{x \neq \pi + 2k\pi,\ k \in \mathbb{Z}\}$$

$$f(x+2\pi) = f(x)$$

Studio f in $[0, 2\pi]$



$f(x) > 0$

$$f(0) = f(2\pi) = \frac{e^{-\frac{1}{2}}}{2} = \frac{1}{2\sqrt{e}}$$

$$\lim_{x \to \pi} f(x) = \lim_{y \to 0^+} \frac{e^{-\frac{1}{y}}}{y} =$$

$1 + \cos x = y$

$x \to \pi$

$y \to 0^+$

$$= \lim_{z \to +\infty} \frac{z}{e^z} = 0$$

$\frac{1}{y} = z$

$$\lim_{x \to \pi} f(x) = 0$$

posso estenderla per continuità

$$f'(x) = \frac{e^{-\frac{1}{1+\cos x}} \cdot \left(\frac{1}{(1+\cos x)^{\cancel{2}}} (-\operatorname{sen} x) \right) \cancel{\cdot (1+\cos x)} - e^{-\frac{1}{1+\cos x}} (-\operatorname{sen} x)}{(1+\cos x)^2} =$$

$$= \frac{e^{-\frac{1}{1+\cos x}} (-\operatorname{sen} x) \left[\frac{1}{1+\cos x} - 1 \right]}{(1+\cos x)^2} =$$

$$= e^{-\frac{1}{1+\cos x}} \frac{(-\operatorname{sen} x)\left(\cancel{1}-\cancel{1}-\cos x\right)}{(1+\cos x)^3} =$$

$$f'(x) = \frac{\operatorname{sen} x \cos x \; e^{-\frac{1}{1+\cos x}}}{(1+\cos x)^3}$$

$\operatorname{sen} x = 0$

$\cos x = 0$

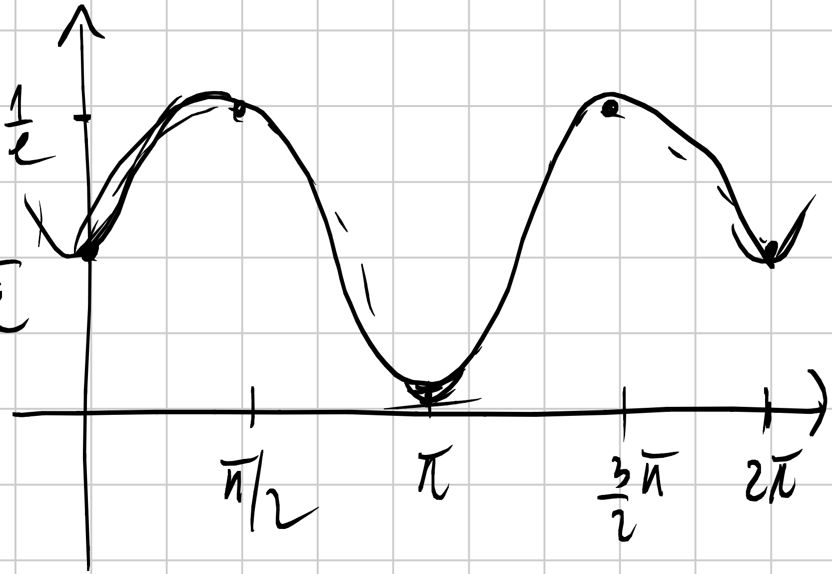
$x = 0 \quad x = \frac{\pi}{2} \quad x = \frac{3}{2}\pi$

. . . .

$x = \frac{\pi}{2} \;,\; \frac{3}{2}\pi$ p.ti di max relativo

$x = 0 \;,\; x = 2\pi$ p.ti di min. relativo

$$f\left(\frac{\pi}{2}\right) = f\left(\frac{3}{2}\pi\right) = \frac{1}{e}$$



$x = \frac{\pi}{2}, \ \frac{3}{2}\pi$

sono p.ti di massimo assoluto

sono p.ti di minimo relativo

$$\lim_{x\to\pi} f'(x) = \lim_{x\to\pi} \frac{\operatorname{sen}x \cos x \, e^{-\frac{1}{1+\cos x}}}{(1+\cos x)^3}$$

-1

$1+\cos x = y \qquad \cos x = y-1$

$-1/\cancel{1}+\cos x$

$$= -\lim_{y\to 0^+} \sqrt{y(2-y)}\,\frac{e^{-\frac{1}{y}}}{y^3}$$

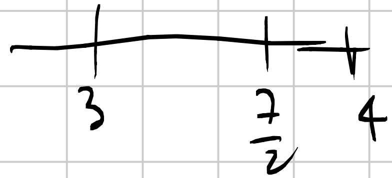
$$-\frac{\operatorname{sen} x\; e}{(1+\cos x)^3}$$

$$\operatorname{sen} x = \sqrt{1-\cos^2 x}$$

$$= \sqrt{1-(y-1)^2} =$$

$$= \sqrt{\cancel{1} - y^2 + 2y - \cancel{1}} = \underbrace{\sqrt{y(2-y)}}_{\operatorname{sen} x}$$

$$= -\lim_{y\to 0^+} \sqrt{y(2-y)}\,\frac{e^{-1/y}}{y^3} = 0$$



es.

$$\int_3^{7/2} \frac{\operatorname{sen}\left((x-3)^{\alpha}\right)(x-1)}{(x-3)^2\, e^{x} \left(\log(x-3)\right)^2}\, dx$$

dire per quali $\alpha > 0$ l'integrale converge.

$x = 3$

$\log(x-3) = 0$

$x - 3 = 1$

$x = 4$

$x - 3 = y$

$x = y + 3$

$$\int_0^{1/2} \frac{\operatorname{sen} y^\alpha \, (y+2)}{y^2 \, e^{y+3} \, (\log y)^2} \, dy$$

$$g(y) \underset{y \to 0}{\sim} \frac{2}{e^3} \operatorname{sen}(y^\alpha) \frac{1}{y^2 (\log y)^2} \sim$$

$$\sim \frac{2}{e^3} \, y^\alpha \frac{1}{y^2 (\log y)^2} =$$

$$= \frac{2}{e^3}\left(\frac{1}{y^{2-\alpha}(\log y)^2}\right)$$

$2-\alpha \leq 1$

$\alpha \geq 1$

$$\int_0^{1/2} \frac{1}{x^\gamma (\lg x)^\beta}$$

$\gamma \leq 1$

$\beta > 1$

es.
$$\int_1^{+\infty} \frac{\operatorname{arctg} x}{(1+x^2)(1+\operatorname{arctg}^2 x)}\,dx$$

- dire se converge
- In caso affermativo, calcolarlo.

$$f(x) \longrightarrow 0 \quad x \to +\infty$$

$$x \to +\infty$$

$$f(x) \sim \frac{\pi}{2} \frac{1}{x^2\left(1+\frac{\pi^2}{4}\right)} = \frac{L}{x^2}$$

$f(x)$ è integrabile in $(1,+\infty)$.

integrabile in $(1,+\infty)$

$$\int \frac{\operatorname{arctg} x}{(1+x^2)(1+\operatorname{arctg}^2 x)}\,dx$$

$\operatorname{arctg} x = y$

$\frac{1}{1+x^2}\,dx = dy$

$$= \int \frac{y}{1+y^2}\,dy = \frac{1}{2}\log(1+y^2)$$

$$\Rightarrow \; = \frac{1}{2}\log(1+\operatorname{arctg}^2 x) + C$$

$$\int_1^{+\infty} f(x)\,dx = \lim_{k\to+\infty} \int_1^{k} f(x)\,dx = \lim_{k\to+\infty}\left[\frac{1}{2}\log\left(1+\operatorname{arctg}^2 k\right) - \right.$$

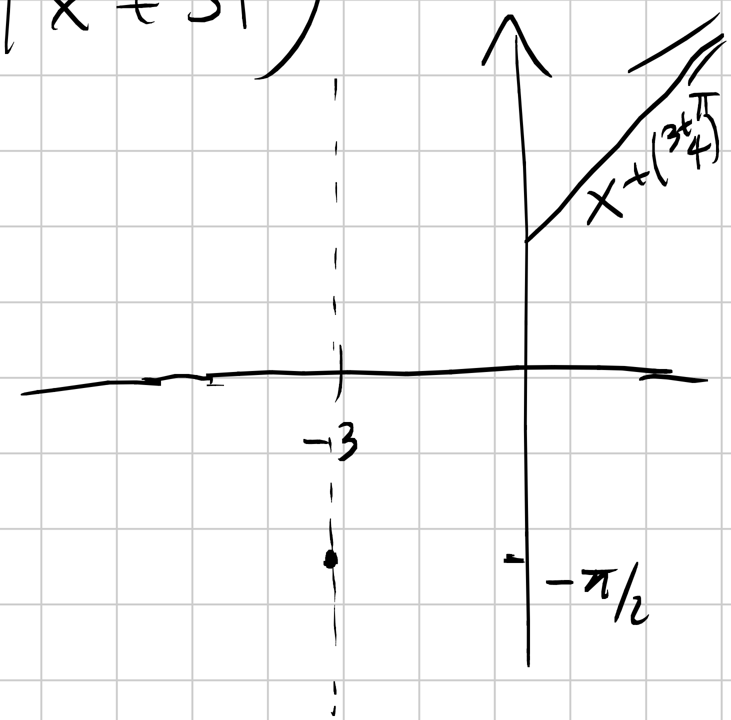
$$\left. - \frac{1}{2}\log\left(1+\operatorname{arctg}^2 1\right)\right) =$$

$$= \frac{1}{2}\log\left(1+\frac{\pi^2}{4}\right) - \frac{1}{2}\log\left(1+\left(\frac{\pi}{4}\right)^2\right).$$

es. $f(x) = |x+3| + \operatorname{arctg}\left(\frac{x+2}{|x+3|}\right)$

$D = \{x \neq -3\}$

$$\lim_{x \to -3} f(x) = -\frac{\pi}{2}$$



$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{x+3}{x} + \frac{1}{x} \operatorname{arctg}\left(\frac{x+2}{x+3}\right)$$

$$= 1$$

$$\lim_{x \to +\infty} f(x) - x =$$

$$= \lim_{x \to +\infty} \cancel{x} + 3 + \arctan\left(\frac{x+2}{x+3}\right) - \cancel{x}$$

$$= 3 + \frac{\pi}{4}$$

$$y = x + \left(3 + \frac{\pi}{4}\right)$$

asintoto obliquo $x \to +\infty$

$$\lim_{x \to -\infty} f(x) = +\infty$$

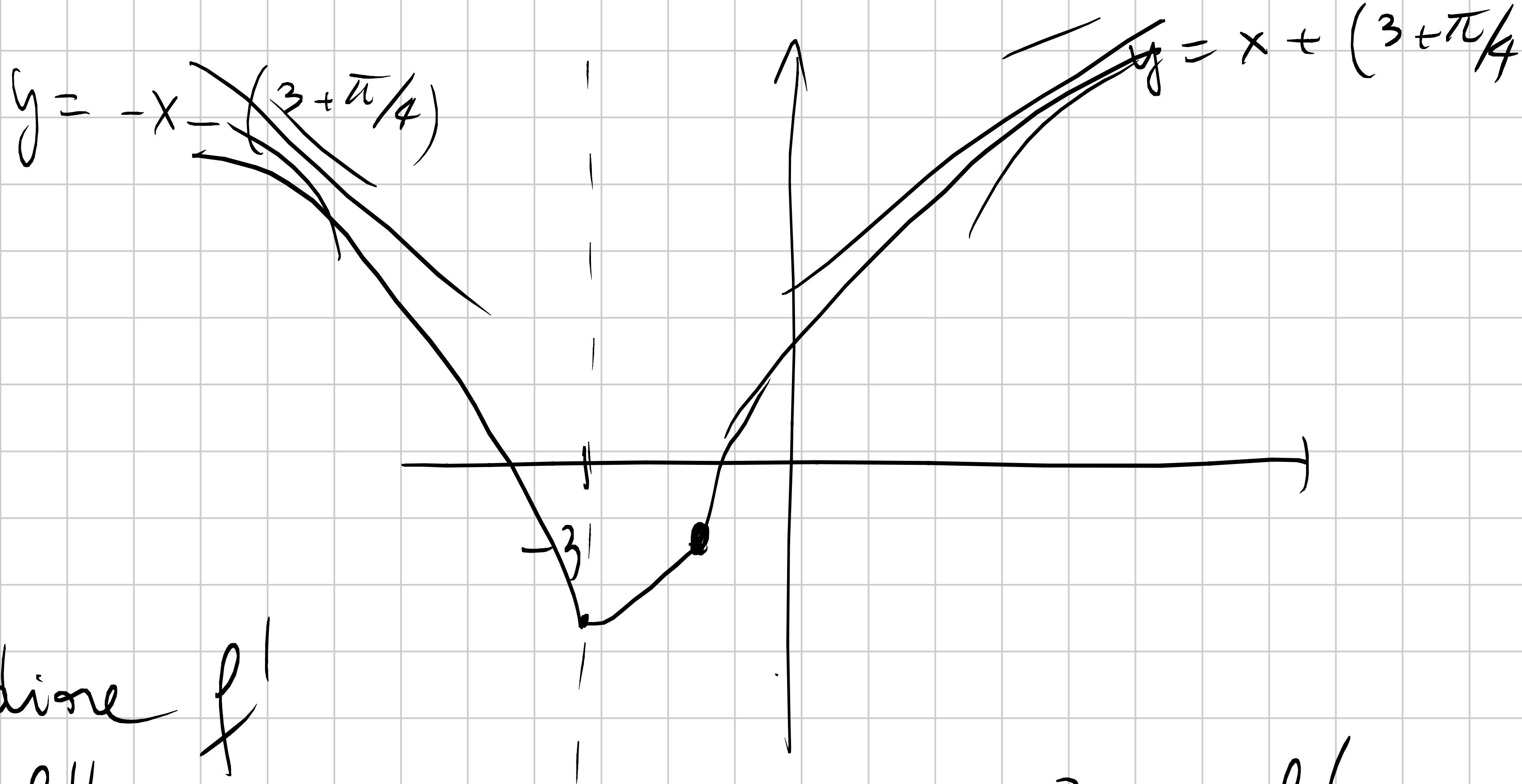
$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{-x-3}{x} + \underbrace{\frac{1}{x}\arctg\left(\frac{x+2}{-x-3}\right)}_{\to 0}$$

$$= -1$$

$$\lim_{x\to-\infty} f(x) + x = \lim_{x\to-\infty} \cancel{-x} - 3 + \arctg\left(\frac{x+2}{-x-3}\right) \cancel{+x}$$

$$= -3 - \frac{\pi}{4}$$

$$y = -x - \left(3 + \frac{\pi}{4}\right)$$ asintoto obliquo per $x \to -\infty$



Studiare f' e f''.

$x + 3 > 0 \quad x > -3 \quad f'(x) > 0$

$x + 3 < 0 \quad x < -3 \quad f''(x) < 0$

$x = -5/2$ flesso . . .

es

$$\int_{-1}^{1} \frac{1}{\left(e^{\sqrt{1-x^2}} - 1\right) \cdot (1+x)^{\alpha}} \, dx =$$

$f(x)$ è illimitata
sia per $x \to 1$
che per $x \to -1$

$$= \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$$

1^o $\downarrow$ 2^o

1^o

$$\int_0^1 \frac{1}{\left(e^{\sqrt{1-x^2}} - 1\right)(1+x)^2}\,dx = \left| \begin{array}{l} 1-x=y \\ \Downarrow \\ x = 1-y \\ 1+x = 2-y \\ -dx = dy \end{array} \right.$$

$x=1 \quad y=0$

$x = 0 \quad y = 1$

$$= \int_1^0 \frac{1}{\left(e^{\sqrt{y(2-y)}} - 1\right)(2-y)^\alpha} \, (-dy)$$

$1 - x^2 = (1-x)(1+x)$

$= y(2-y)$

$$= \int_0^1 \frac{1}{\left(e^{\sqrt{y(2-y)}} - 1\right)(2-y)^\alpha} \, dy$$

$y \to 0$

$$g(y) \sim \frac{1}{2^\alpha} \cdot \frac{1}{\sqrt{y(2-y)}} = \frac{C}{\sqrt{y}}$$

$$e^{\sqrt{y(2-y)}} - 1 \sim \sqrt{y(2-y)}$$

$$e^{z} - 1 \sim z \qquad z \to 0$$

$$g(y) \sim \frac{c}{\sqrt{y}}$$ è integrabile in $(0,1)$

$\Rightarrow$ $g(y)$ è integrabile in $(0,1)$

In maniera analoga si fa il 1°

$$\int_{-1}^{0} \frac{1}{\left(e^{\sqrt{1-x^2}}-1\right)(1+x)^{\alpha}}\,dx \qquad 1+x=y$$

$$g(y) \sim \frac{c}{\sqrt{y}\; y^{\alpha}} = \frac{c}{y^{\alpha+\frac{1}{2}}}$$

è integrabile se

$$\alpha+\frac{1}{2}<1$$

se $\alpha < 1/2$

$$\int_{-1}^{1} f(x)\,dx \quad \text{converge ssi} \quad \alpha < 1/2 .$$