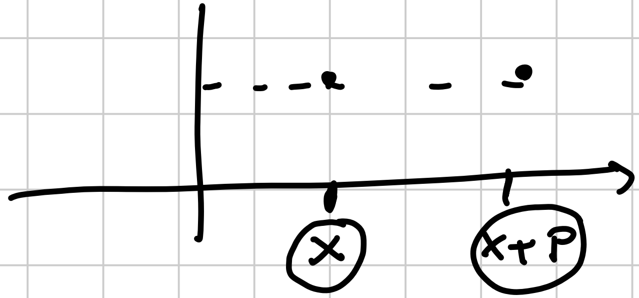
$f : X \longrightarrow \mathbb{R}$, $\quad X \subseteq \mathbb{R}$

$x \in \mathbb{R} \longrightarrow y \in \mathbb{R}$



f è **PERIODICA** in X se $\exists\, P > 0$ t.c.

$$f(x+P) = f(x), \quad \forall x \in X.$$

$\inf \{ P > 0 : f(x+P) = f(x) \}$ si chiama **PERIODO**

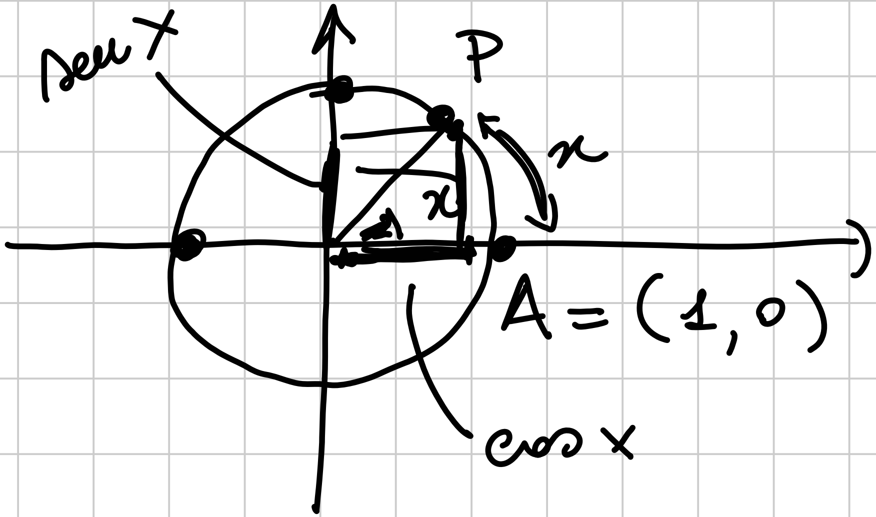
$2P$

$$f(x+2P) = f((x+P)+P) = f(x+P) = f(x)$$

f periodica **non** è iniettiva

se f é periodica di periodo P

FUNZIONI TRIGONOMETRICHE



x = lunghezza dell'arco AP

$A\hat{O}P$

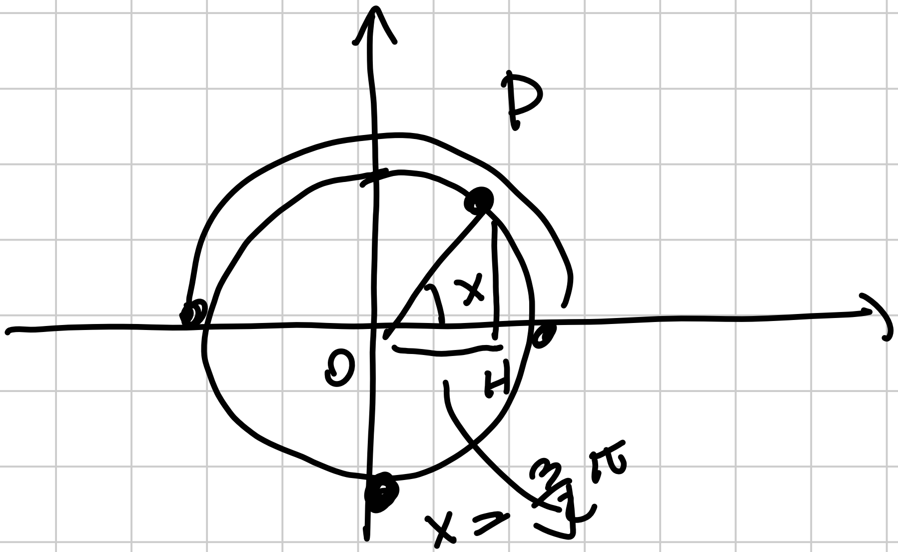
x ampiezza dell'angolo $A\hat{O}P$ se x è la lunghezza dell'arco AP

x = misura in **RADIANTI** dell'angolo $A\hat{O}P$.

$P \equiv A$ $\quad x = 0$ $\quad P = (0, 1)$ $\quad x = \frac{\pi}{2}$

$P = (-1, 0)$ $\quad x = \pi$ $\quad x \in [0, 2\pi)$

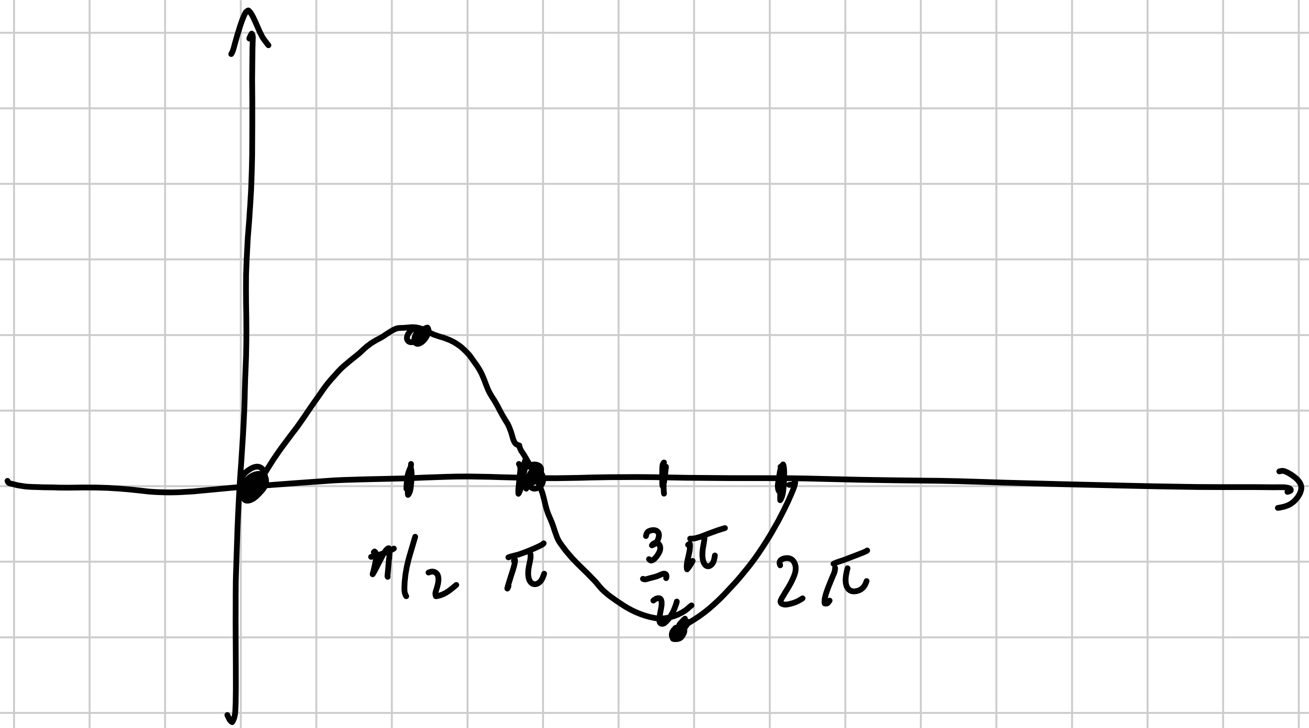
$x = 2\pi$ $\quad P = (\underline{\cos} x, \operatorname{sen} x)$



$\cancel{OH} = \cos x =$ ascissa di P

$\cancel{PH} = \operatorname{sen} x =$ ordinata di P

$\forall x \in [0, 2\pi)$



sen x

$\operatorname{sen} 0 = 0$

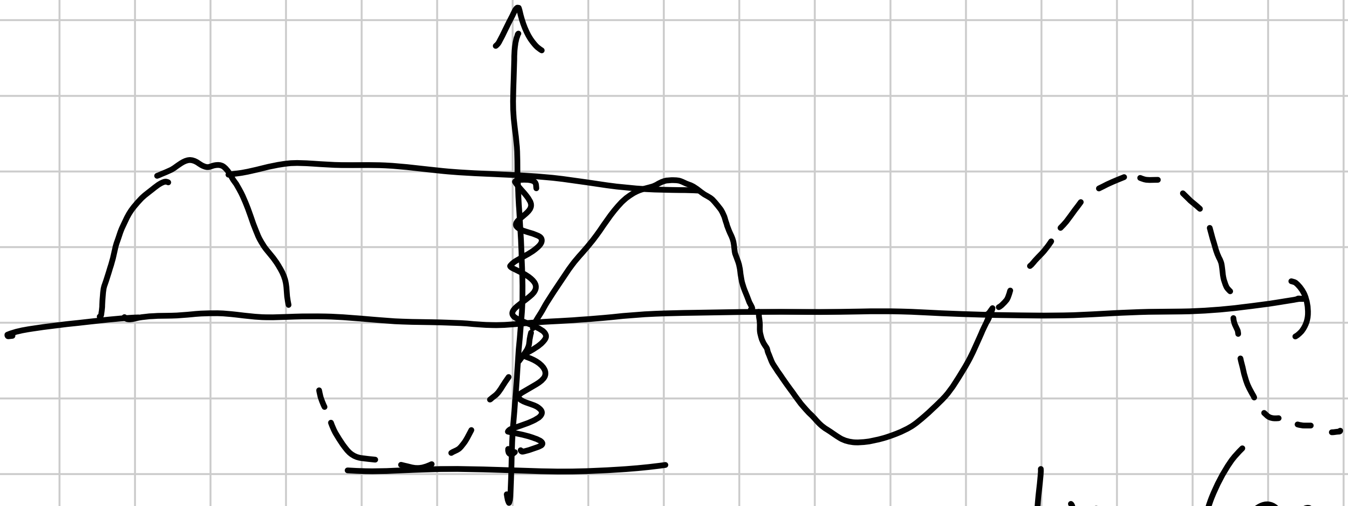
$\operatorname{sen} \frac{\pi}{2} = 1$

$\operatorname{sen} \pi = 0$

$\operatorname{sen} \frac{3\pi}{2} = -1$

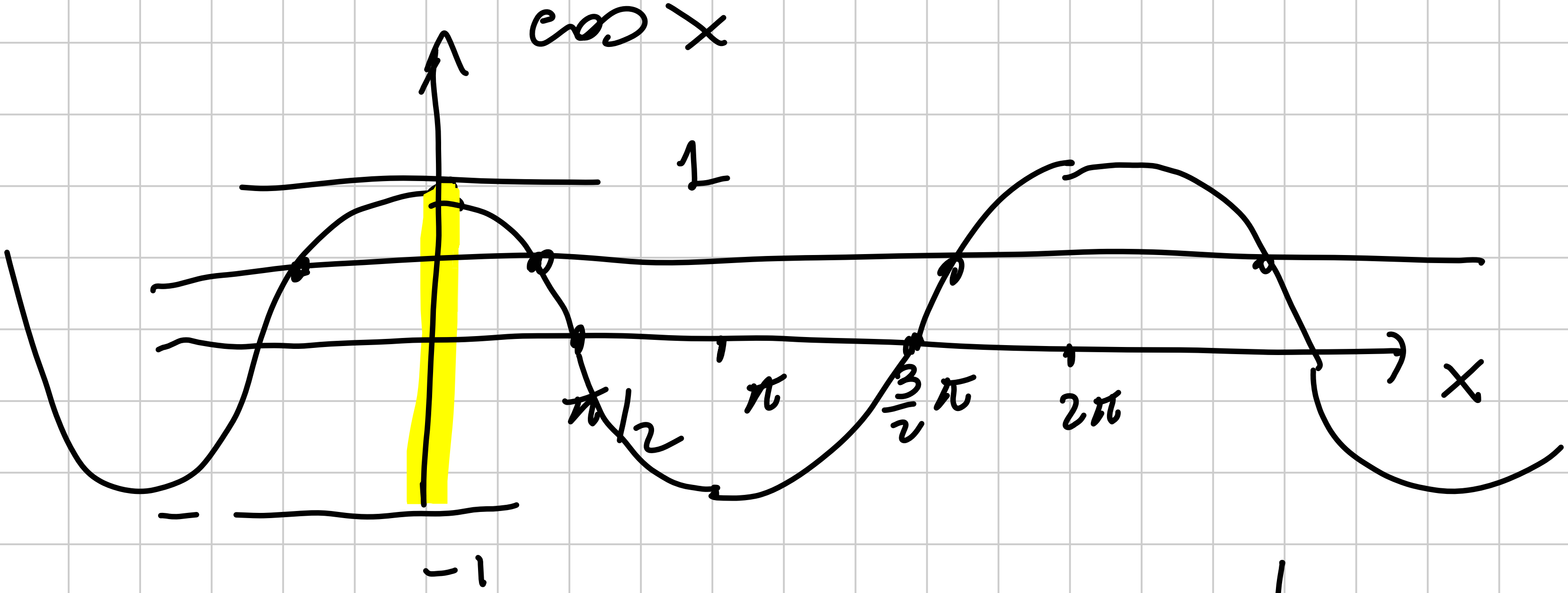
Definisco sen x, $\forall \in \mathbb{R}$ estendendola ad una funzione periodica di periodo 2π

$$\text{sen}(x + 2k\pi) = \text{sen}\, x, \quad \forall x \in [0, 2\pi)$$
$$k \in \mathbb{Z}$$
$$\cos(x + 2k\pi) = \cos x$$



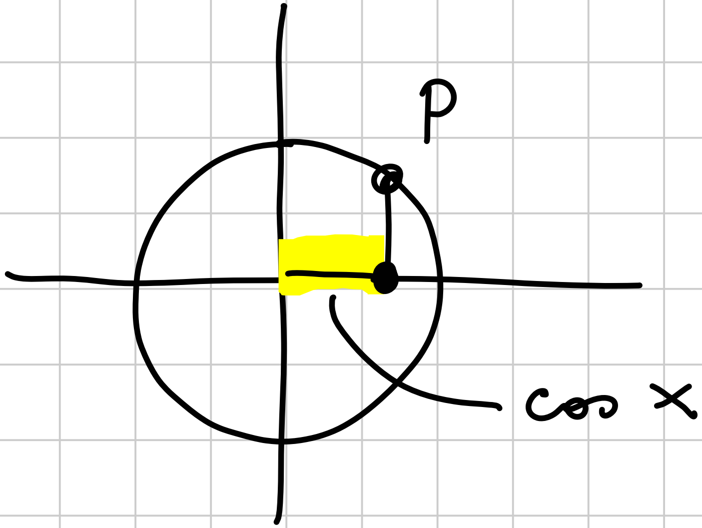
$$\text{Im}(\text{sen}\, x) = [-1, 1]$$

sen x : $\mathbb{R}$
cos x : $\mathbb{R}$



$\cos x : \mathbb{R} \to [-1, 1]$

non sono iniettive



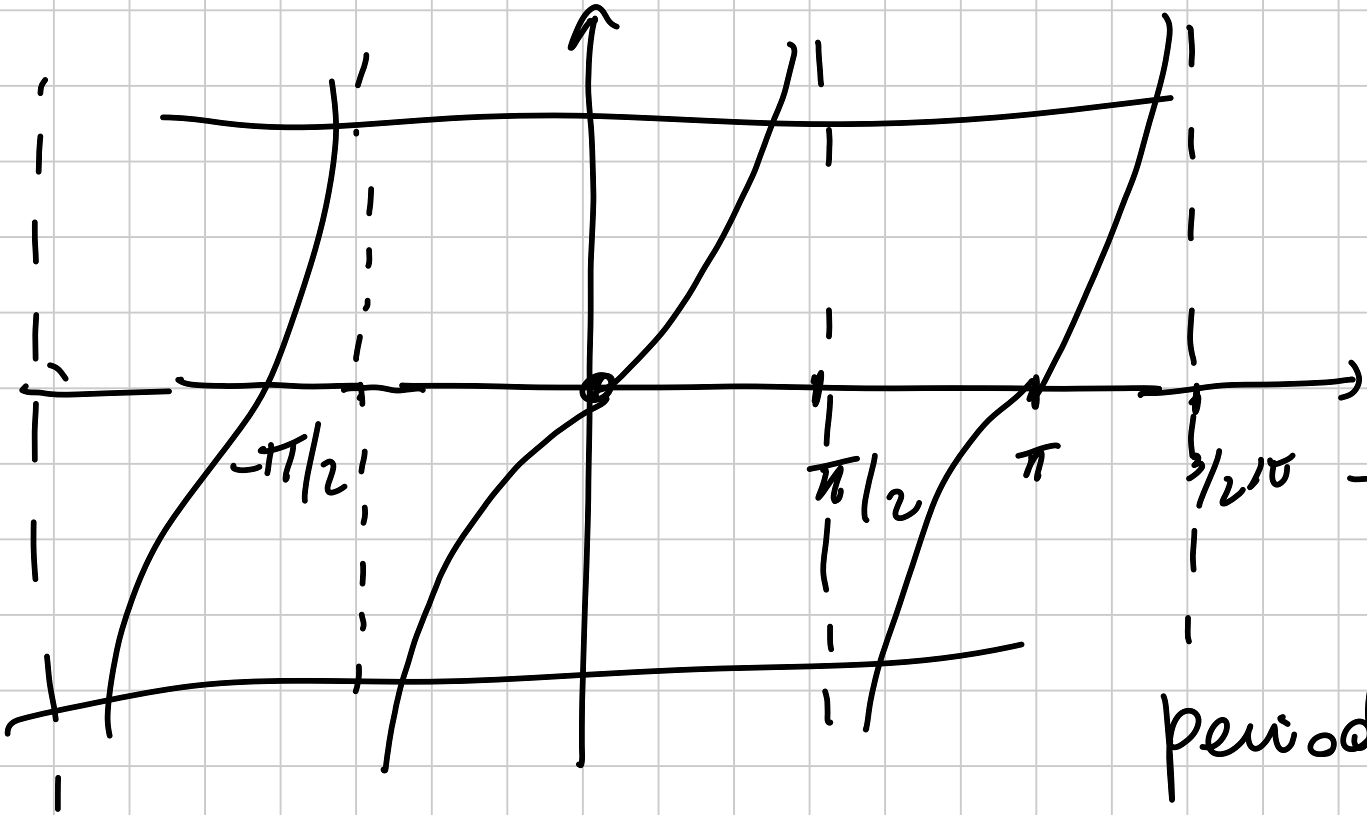
$$\text{tg}\,x := \frac{\text{sen}\,x}{\cos x}$$

$\cos x \neq 0$

$$\boxed{x \neq \frac{\pi}{2} + K\pi}$$

dominio di tg x

$K \in \mathbb{Z}$



$$\text{tg}\,0 = \frac{0}{1} = 0$$

periodica di periodo π

$$\text{tg}\,x : \left\{ x \neq \frac{\pi}{2} + K\pi,\ K \in \mathbb{Z} \right\} \longrightarrow \mathbb{R} = (-\infty, +\infty)$$

im(tg x)

$$\operatorname{im}(\operatorname{sen} x) = [-1, 1]$$

$$\min_{\mathbb{R}} \operatorname{sen} x = -1$$

$$\max_{\mathbb{R}} \operatorname{sen} x = 1$$

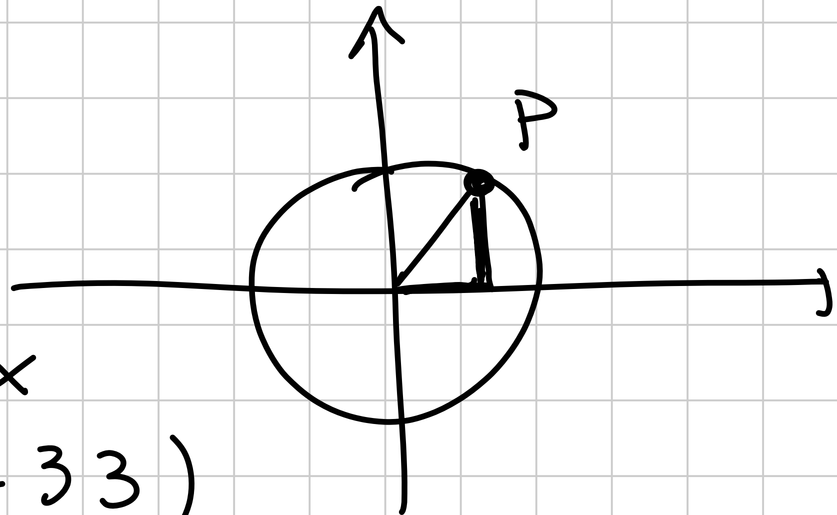
$$\operatorname{sen}^2 x + \cos^2 x = 1$$

$\operatorname{sen} x$, $\cos x$, $\operatorname{tg} x$

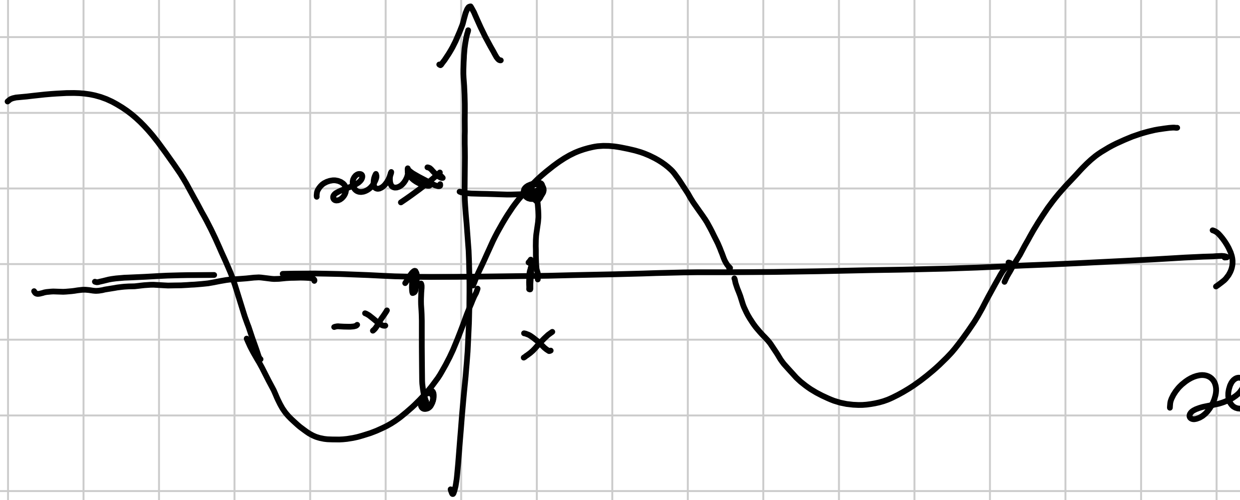
proprietà (p. 32-33)

$$|\operatorname{sen} x| \le 1$$

$$-1 \le \operatorname{sen} x \le 1$$

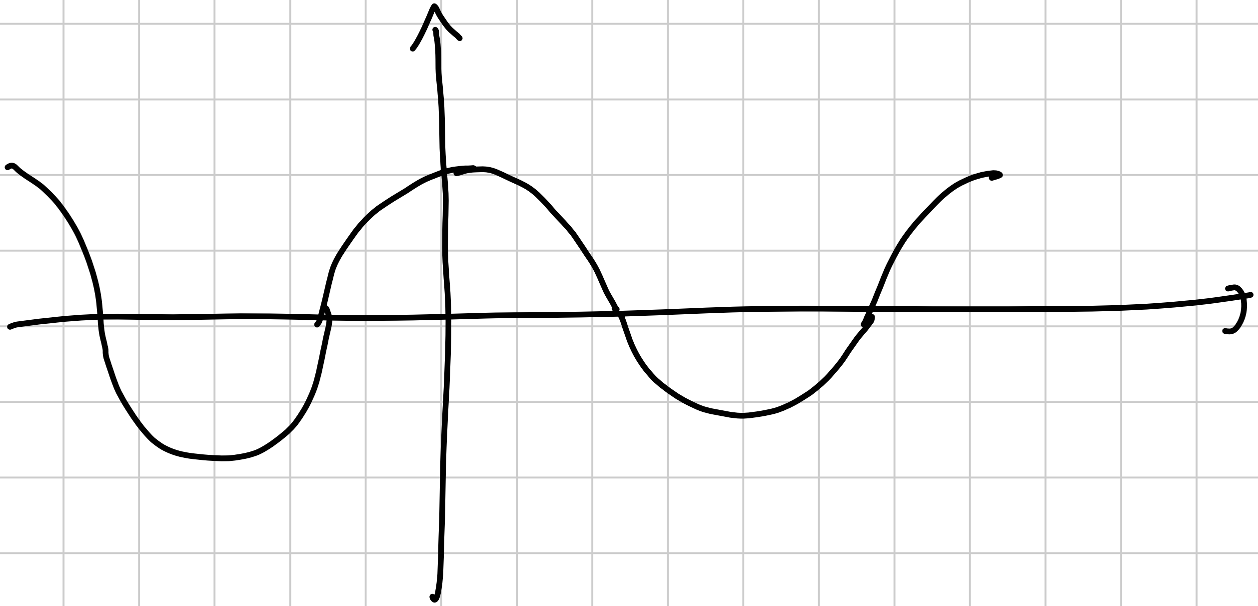


$\text{sen}\, x$



$$\text{sen}(-x) = -\text{sen}\, x$$

sen x è <u>dispari</u>



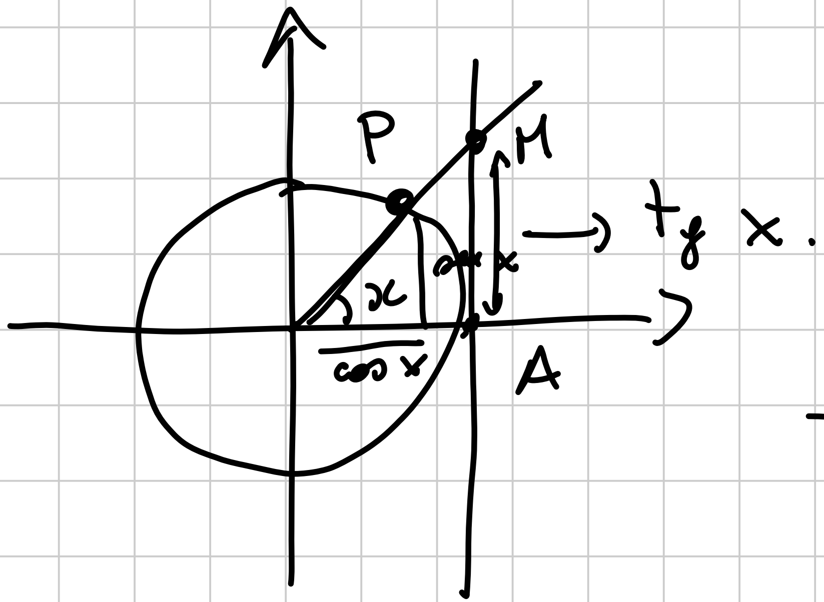
$$\cos(-x) = \cos x$$

cos x è <u>Pari</u>

$$\operatorname{cotg} x := \frac{\cos x}{\operatorname{sen} x} \qquad \forall\, x \neq k\pi \;, \; k \in \mathbb{Z}.$$

$\operatorname{tg} x$

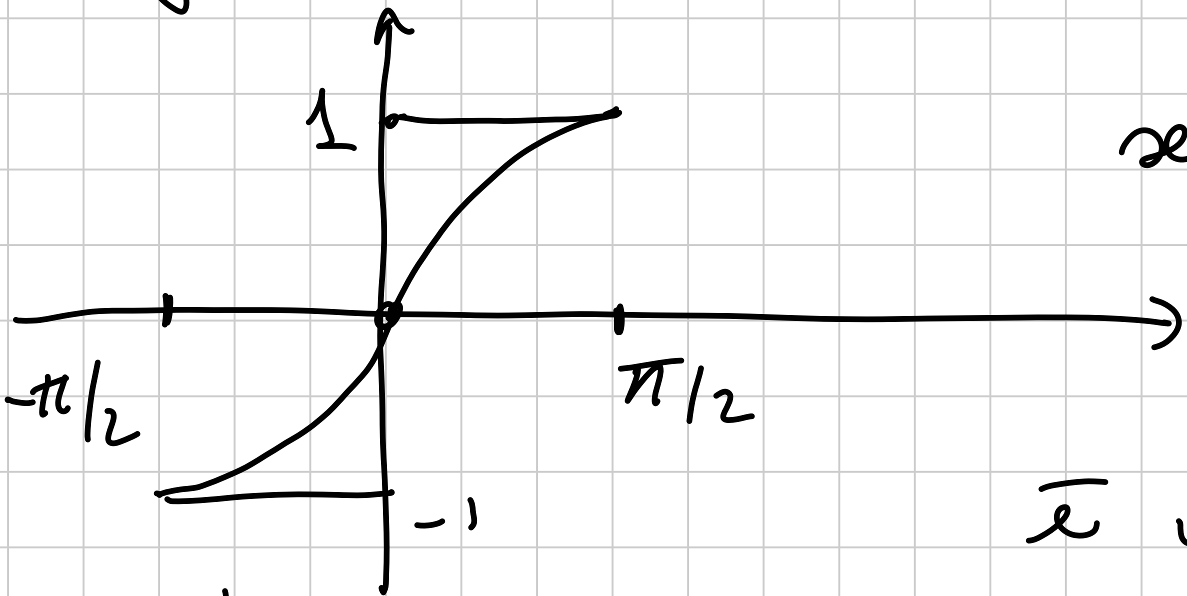
$$MA = \operatorname{tg} x$$



$$\frac{\cos x}{\operatorname{sen} x} = \frac{MA}{1}$$

Funzioni inverse trigonometriche

$f(x) = \operatorname{sen} x \qquad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$\operatorname{sen} x : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$

è biettiva

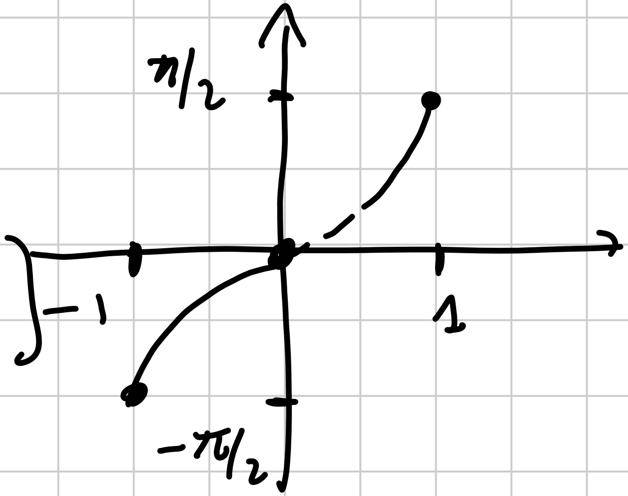
è invertibile

$f^{-1} : [-1, 1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\boxed{x = \operatorname{arcsen} y = f^{-1}(y) \iff y = \operatorname{sen} x \quad \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$$

$\operatorname{sen} x : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$

$\operatorname{arcsen} y : [-1, 1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \qquad x_0 = \frac{\pi}{4}$

$y = \operatorname{sen} x_0 \iff x_0 = \operatorname{arcsen} y$

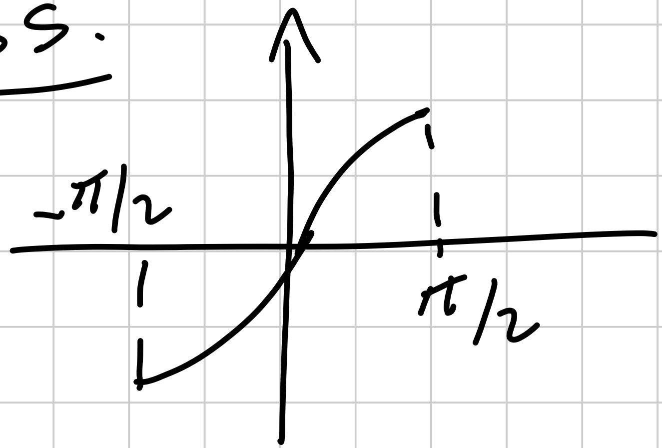
es. $f(x) = \operatorname{arcsen}(x^2 - 5)$

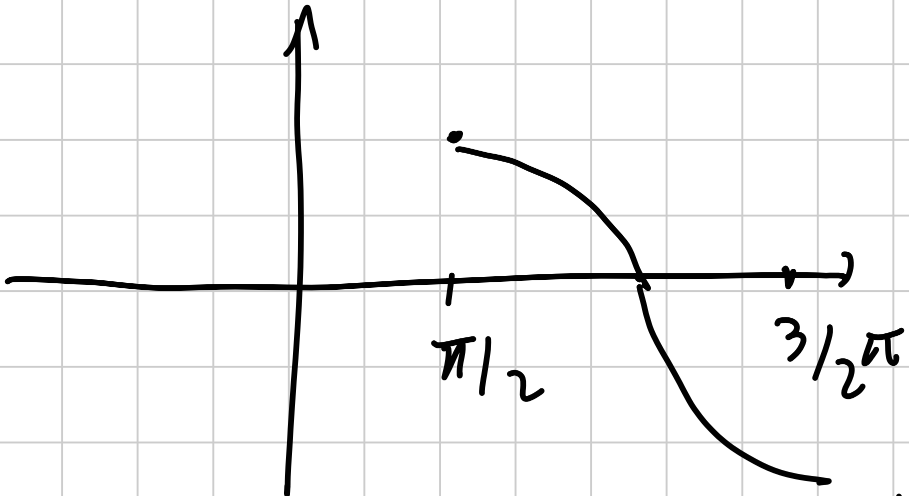
domanda: Trovare il dominio di $f(x)$

$|x^2 - 5| \leq 1$

$-1 \leq x^2 - 5 \leq 1$

OSS.





$\operatorname{sen} x$ è iniettiva anche in $\left[\frac{\pi}{2}, \frac{3}{2}\pi\right]$

$\exists\, f^{-1}$ funzione inversa

$f^{-1} : [-1, 1] \to \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

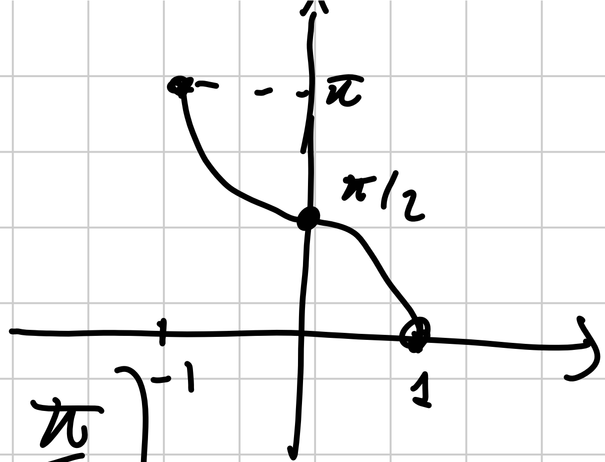
ma f^{-1} non è $\boxed{\operatorname{arcsen} y}$

c'è una relazione tra f^{-1} e $\operatorname{arcsen} y$

Se si parla di

$y = \arcsin x$ intendo

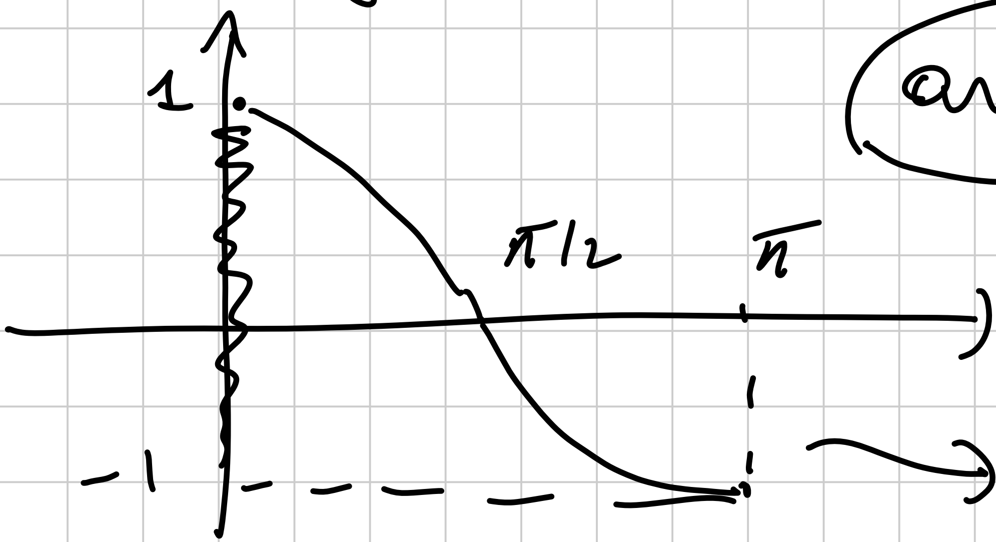
$x \in [-1, 1]$ e $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Analogamente $\cos x : [0, \pi] \to [-1, 1]$

$\arccos y : [-1, 1] \to [0, \pi]$

è la funzione inversa



$\to$ $\cos x$ è decrescente

n° Det. il dominio di

$$f(x) = \arccos(x^3 - 1)$$

$$-1 \leq x^3 - 1 \leq 1$$

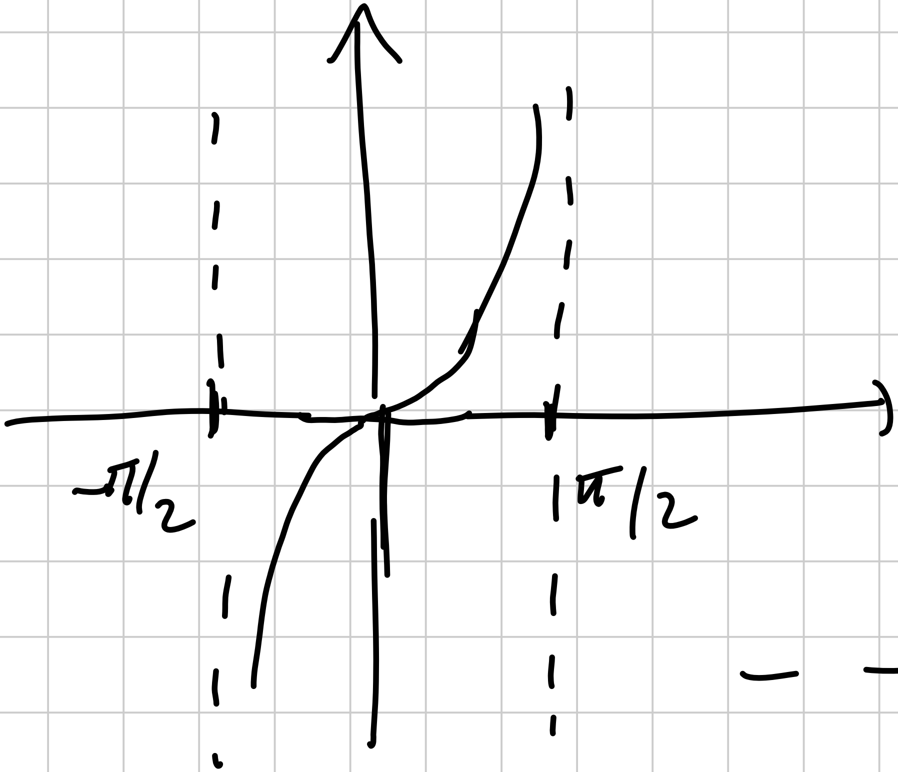
$$0 \leq x^3 \leq 2$$

$$x^3 \geq 0 \Rightarrow x \geq 0$$

$$x^3 \leq 2 \Rightarrow x \leq \sqrt[3]{2}$$

$$D = \{ x \in \mathbb{R} : \quad 0 \leq x \leq \sqrt[3]{2} \}$$

$tg\,x : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \longrightarrow \mathbb{R}$ $\quad \exists f^{-1}$

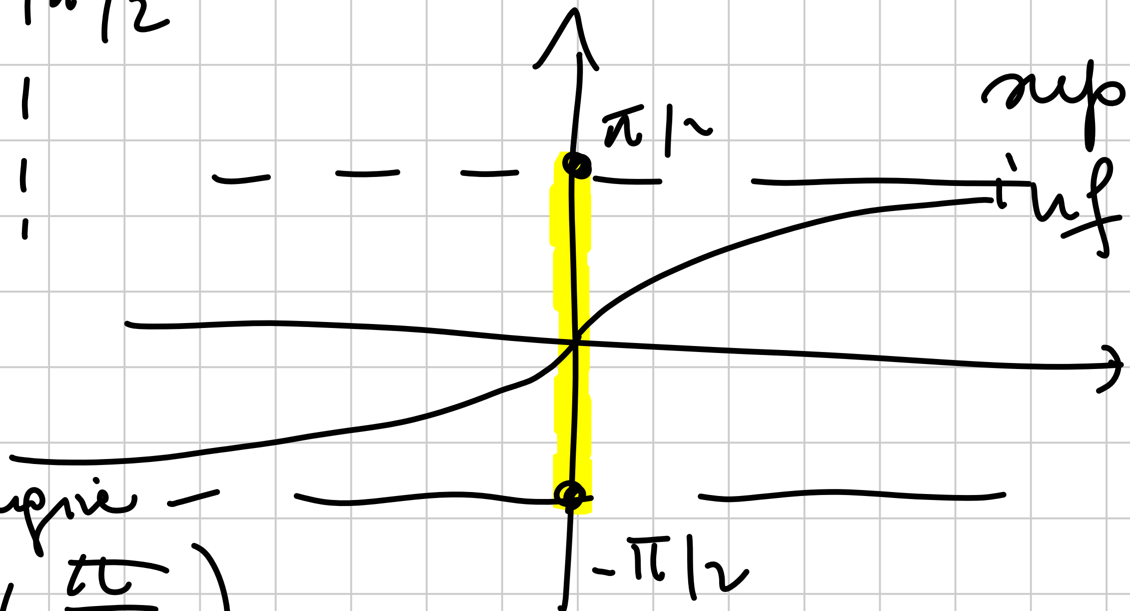


$arctg\, y : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$arctg\, x$, $\underline{\underline{\forall x \in \mathbb{R}}}$

$\sup arctg\, x = \frac{\pi}{2}$

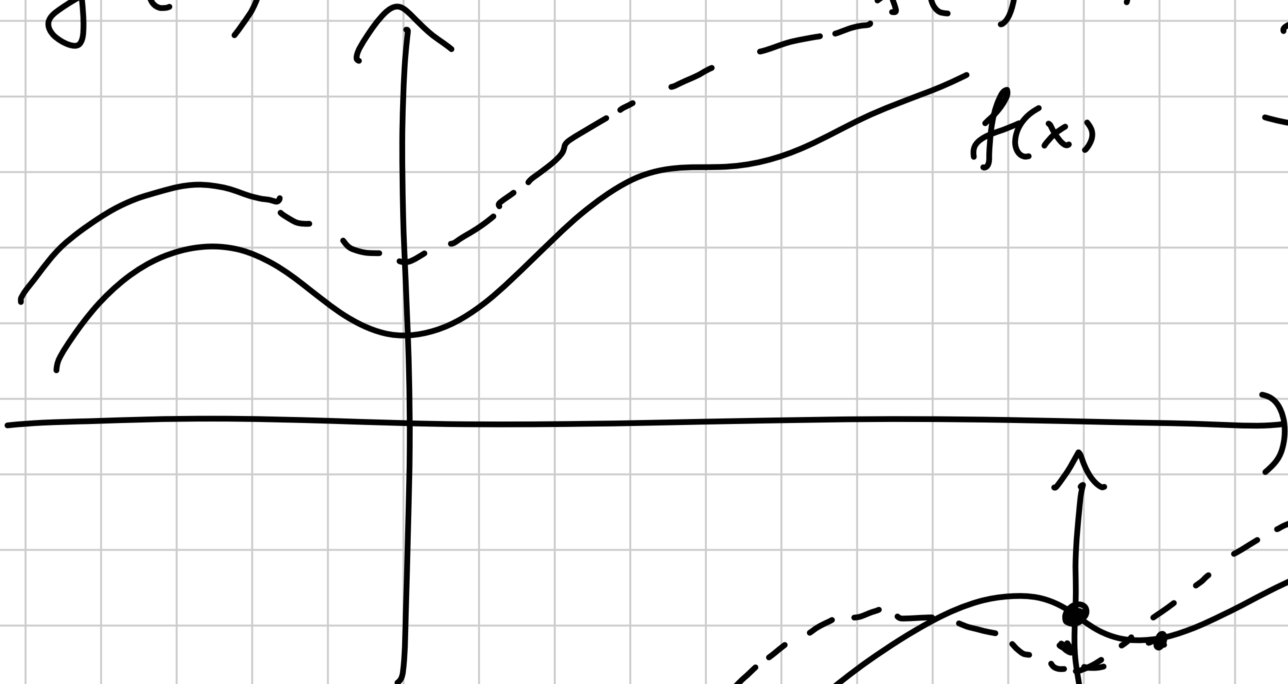
$\inf arctg\, x = -\frac{\pi}{2}$



arctg x è limitata perché l'immagine è $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

p. 66/67 Le operazioni sulle funzioni (sui grafici)

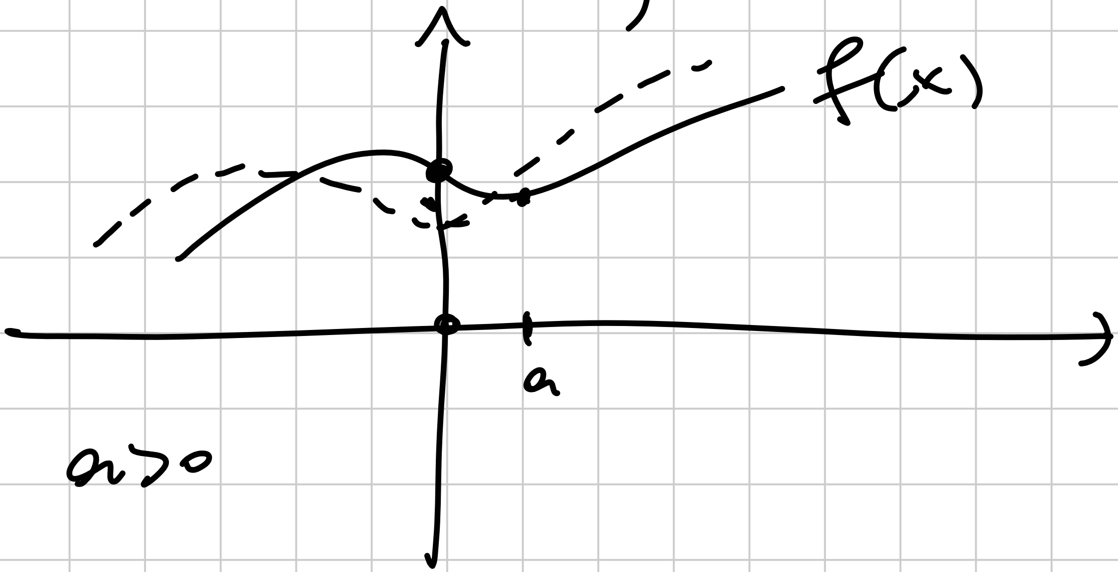
$f(x)$



$f(x) + a$

$f(x+a)$

$f(0+a)$ $a>0$



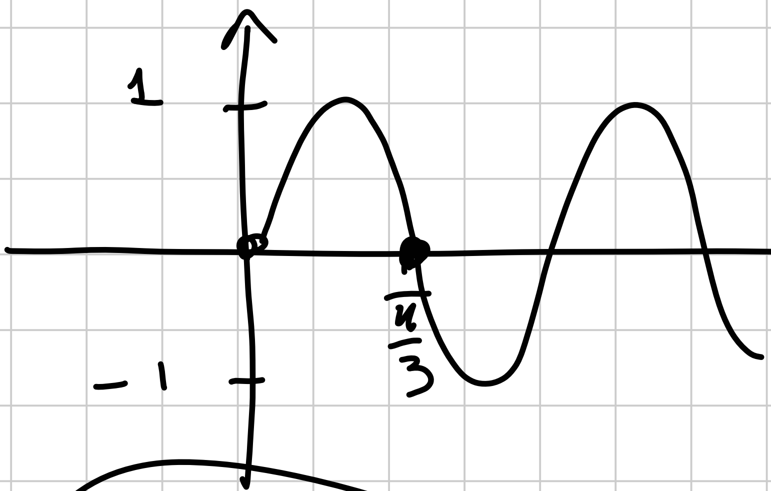
$f(x)$ $\qquad$ $f(kx)$

$k\,f(x)$

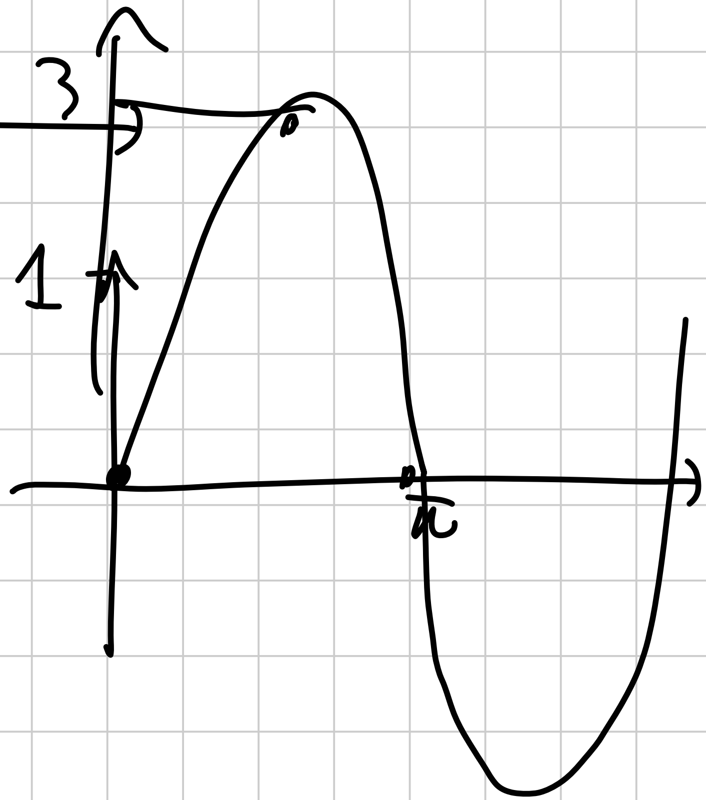
$\text{sen}(3x)$

$3x = \pi$

$x = \dfrac{\pi}{3}$



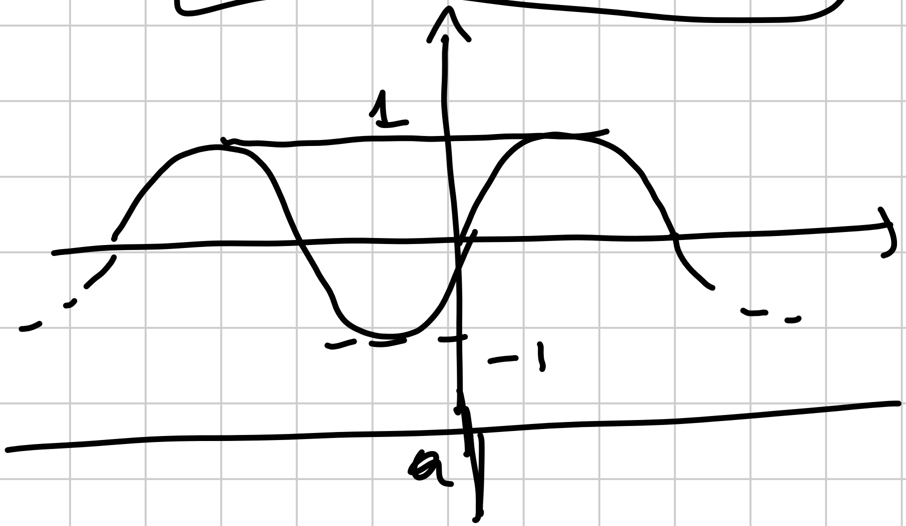
$3\,\text{sen}\,x$



Disequazioni trigonometriche

trovare $x \in \mathbb{R}$ t.c.

$$\boxed{\operatorname{sen} x > a} \quad , \quad a \in \mathbb{R}$$



1) $\boxed{a \geq 1}$ $(a=3)$

$\operatorname{sen} x > 3$

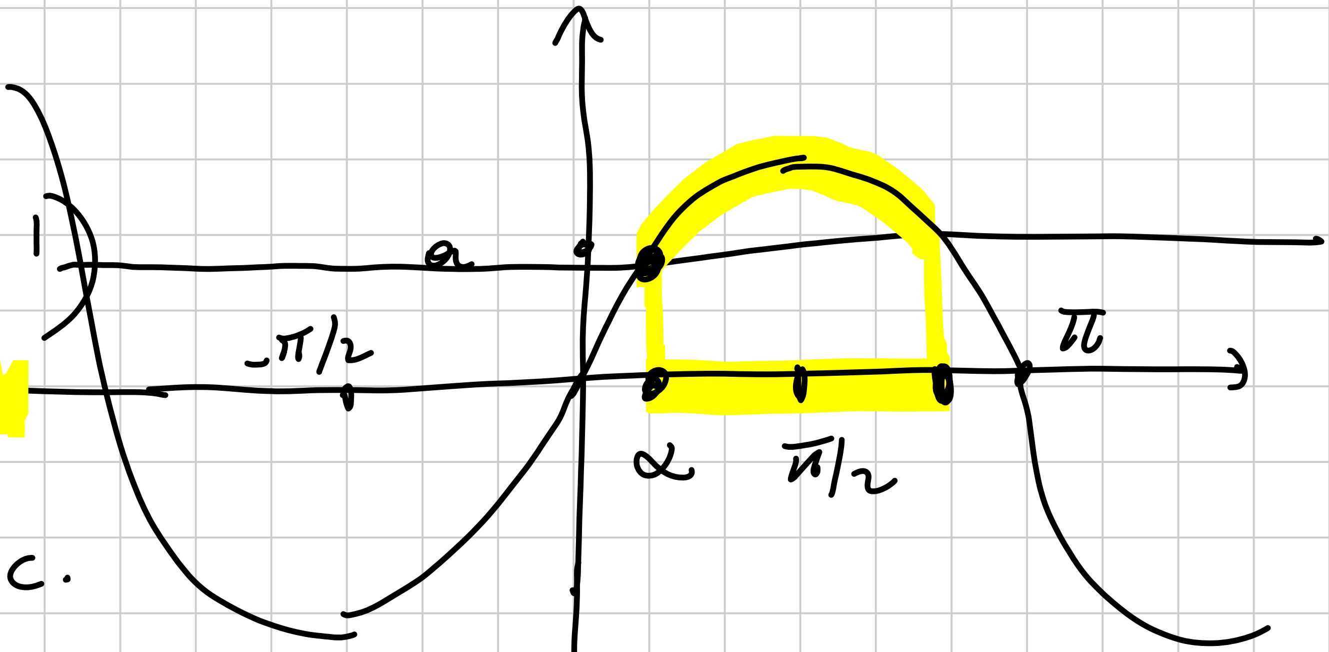
$\nexists\, x$

2) $a < -1 \Rightarrow \forall x \in \mathbb{R}$

3) $a \in [-1, 1)$

$\operatorname{sen} x > |a|$

3) $a \in [-1, 1]$



a

$\exists\, \alpha \in \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ t.c.

$\operatorname{sen} \alpha = a$ $\qquad$ $\alpha = \operatorname{arcsen} a$

$(\alpha, \pi - \alpha)$

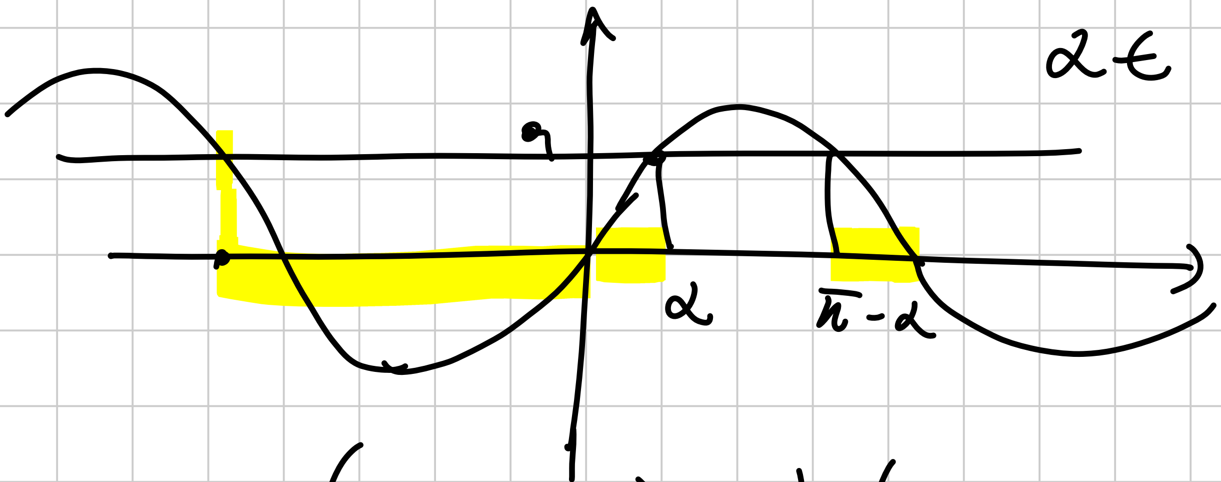
$x \in (\alpha + 2K\pi, \pi - \alpha + 2K\pi)$

$|\operatorname{sen} x| > a$ $\quad$ $\operatorname{sen} x > a$

$\operatorname{sen} x < -a$

$\operatorname{sen} x < a$ $\qquad$ $\alpha = \arcsin a$

$\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



$x \in (-\pi, \alpha) \cup (\pi - \alpha, \alpha)$