Es. $f(x,y) = \operatorname{sen}(xy)$ $\qquad D = \mathbb{R}^2$

$-1 \le f(x,y) \le 1$ $\qquad$ f differenziabile in tutto $\mathbb{R}^2$.

$f_x = y\cos(xy) = 0$

$f_y = x\cos(xy) = 0$

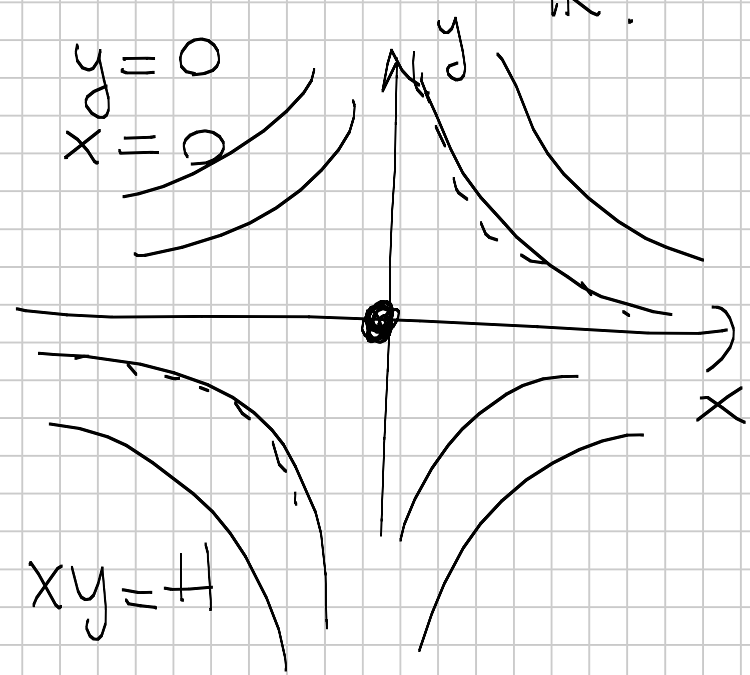
$\cos(xy) = 0$

$xy = \frac{\pi}{2} + k\pi = H$, $k \in \mathbb{Z}$

$k=0$ $\quad xy = \frac{\pi}{2}$ $\quad y = \frac{1}{x}\cdot\frac{\pi}{2}$

$y=0$

$x=0$



$xy = H$

$$\begin{cases} f_{xx} = -y^2 \operatorname{sen}(xy) \\ f_{xy} = f_{yx} = \cos(xy) - yx\operatorname{sen}(xy) \\ f_{yy} = -x^2 \operatorname{sen}(xy) \end{cases}$$

Mi studio il segno del determinante $(f_{xx}f_{yy} - f_{xy}^2)$ nei p.ti critici.

$(0,0)$ $\quad D^2 f(0,0) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad \det D^2 f(0,0) = -1 < 0$

$(0,0)$ p.to di SELLA

$$xy = \frac{\pi}{2} + k\pi \qquad \cos(xy) = 0$$

$\underline{k \text{ pari}}$ $\quad \operatorname{sen}(xy) = 1 \Rightarrow$ p.ti di massimo <u>assoluto</u>.

$$D^2 f\left(\ \right) = \begin{pmatrix} -y^2 & -yx \\ -yx & -x^2 \end{pmatrix} \qquad \det D^2 f = y^2x^2 - y^2x^2 = 0$$

non si può applicare <u>il criterio</u>

$\underline{k \text{ dispari}}$ $\quad \operatorname{sen}(xy) = -1 \Rightarrow$ p.ti di minimo <u>assoluto</u>

$$a_n = \frac{n^2\left(\sqrt{1+\frac{1}{n}} - 1\right) + n(10)^{-\log n} + 2\log n}{n^3 \log n - 3n\cos n + \operatorname{arctg}(n^2)}$$

dire se $\sum a_n$ converge?

N. $(10)^{-\log n} = \dfrac{1}{(10)^{\log n}}$

$$10^{\log n} = e^{\log n \cdot \log 10} = \left(e^{\log n}\right)^{\log 10} = n^{\log 10}$$

$$n\,(10)^{-\log n} = \frac{n}{(10)^{\log n}} = \frac{n}{n^{\log 10}} = \frac{1}{n^{\log 10 - 1}}$$

$$\sqrt{1+\frac{1}{n}} - 1 = \frac{1}{2n} + o\left(\frac{1}{n}\right)$$

<u>N.</u>

$$n^2\left(\frac{1}{2n} + o\left(\frac{1}{n}\right)\right) + \frac{n}{n^{\log 10}} + \log n$$

$$= \frac{n}{2} + o(n) + \frac{n}{n^{\log 10}} + \log n =$$

$$N. = \frac{n}{2}\left(1 + o(1) + \frac{2}{n^{\log 10}} + \frac{2\log n}{n}\right)$$

$$D. = n^3 \log n - 3n\cos n + \operatorname{arctg}(n^2) =$$

$$= n^3 \log n \left(1 - \underbrace{\frac{3\cancel{n}\cos n}{n^{\cancel{3}2}\log n}}_{\to 0} + \underbrace{\frac{\operatorname{arctg} n^2}{n^3 \log n}}_{\to 0}\right)$$

$$a_n \sim \frac{\cancel{n}}{2 n^{\cancel{3/2}\,2} \log n} = \frac{1}{2 n^2 \log n} \longrightarrow 0$$

$\sum a_n$ converge ? $\iff \sum \frac{1}{n^2 \log n}$ converge !

$$\sum \frac{1}{n^2 (\log n)^\beta}$$

$$\sum_{k=1}^{\infty} \left(3k^2 \log \frac{2k^2+2}{2k^2+1}\right)^k$$

$a_k \rightarrow ?$

$$\lim_k \sqrt[k]{a_k} = \lim_k \; 3k^2 \log\left(\frac{2k^2+2}{2k^2+1}\right)$$

$$\Rightarrow \quad \frac{(2k^2+1)+1}{(2k^2+1)} = 1 + \frac{1}{2k^2+1}$$

$$\log\left(\frac{2k^2+2}{2k^2+1}\right) = \log\left(1 + \frac{1}{2k^2+1}\right) = \frac{1}{2k^2+1} + o\left(\frac{1}{k^2}\right)$$

$$3k^2 \log\left(\frac{2k^2+2}{2k^2+1}\right) = 3k^2\left(\frac{1}{2k^2+1} + o\left(\frac{1}{k^2}\right)\right) \xrightarrow{k \to +\infty} \frac{3}{2} > 1$$

la serie <u>diverge</u>

$$\int_0^1 \underbrace{x^5 \log\left(x^3\right)}_{f(x)} \, dx$$

$$\lim_{x \to 0^+} f(x) = 0$$

$f(x)$ non è definita in $x=0$ ma è prolungabile per continuità in $x=0$.

$$\int \underbrace{x^5}_{x^3 \cdot \cancel{x^2}} \log\left(x^3\right) dx =$$

$$x^3 = y$$

$$\underbrace{3x^2 \, dx} = dy$$

$$= \int y \, \frac{1}{3} \log y \; dy = \frac{1}{3} \int y \log y \, dy =$$

= fer forth ....

$$\int_0^1 \frac{x^2 + x^\alpha + e^{-1/x} - \operatorname{sen}(x^2+x^3)}{\sqrt{x}}\,dx$$

$e^{-1/x} = o(x^\beta) \ \forall \beta$

$\operatorname{sen} y = y - \frac{y^3}{3!} + o(y^3)$

Dire per quali $\alpha > 0$ converge.

$x = 0$

$$N_0 = \cancel{x^2} + x^\alpha - (\cancel{x^2} + x^3) + \frac{1}{6}(x^2+x^3)^3 + o(x^6)$$

$$= x^\alpha - x^3 + \frac{1}{6}x^6 + o(x^6)$$

$\alpha > 0$

$D. = \sqrt{x}$

$\alpha < 3$

$$\frac{x^{\alpha}}{\sqrt{x}} = \frac{1}{x^{1/2-\alpha}}$$

$x = 0$

$\int_0^1$ converge $\boxed{\text{I}}$

$\frac{1}{2} - \alpha < 1$

$\alpha > -1/2$

$\forall \alpha < 3$ l'integrale converge

$\alpha > 3$

$$N \quad \frac{x^3}{\sqrt{x}} \xrightarrow[x \to 0]{} 0$$

funzione limitata in intervallo limitato $\Rightarrow$ converge.

$\alpha = 3$

$$\frac{1}{6} \frac{x^6}{\sqrt{x}} \underset{\text{jusqu'à la tota}}{\longrightarrow} 0 \Rightarrow \text{l'intégrale converge.}$$

$$\int_1^{+\infty} \frac{\cos((2-\alpha)x) + 3}{(e^x+1)^\alpha}\,dx$$

per quali $\alpha \in \mathbb{R}$ esiste finito?

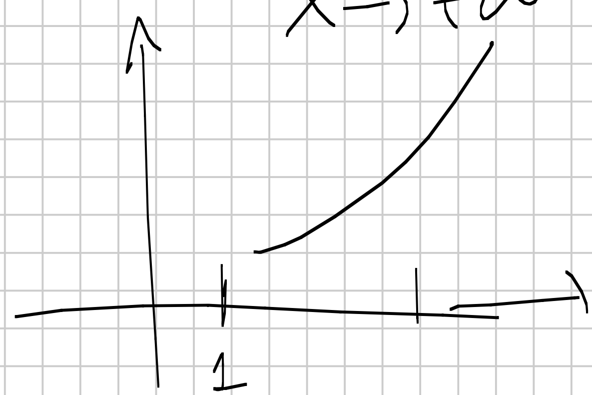
$f(x) > 0$

$x \to +\infty$

$\alpha < 0$

diverge

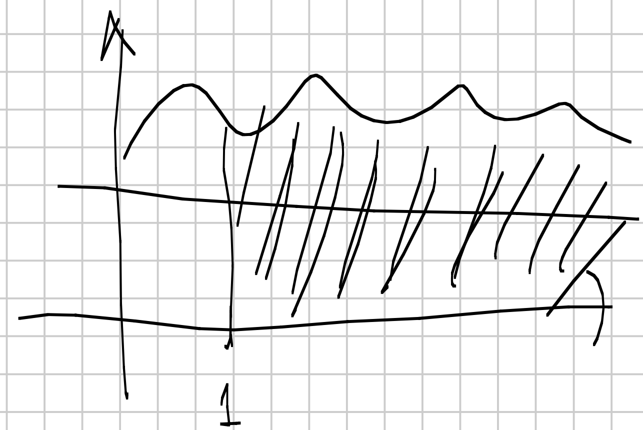
$$f(x) \xrightarrow[x \to +\infty]{} +\infty$$



$\alpha = 0$

$$\int_1^{+\infty} \left(\cos 2x + 3 \right) dx \qquad f(x) \geq 2$$

2



$\Rightarrow$ diverge

$\nexists \lim_{x \to +\infty} \cos(2x) + 3$ me

$\alpha > 0 \qquad f(x) = \dfrac{\cos((2-\alpha)x) + 3}{(e^{x}+1)^{\alpha}}$

$$f(x) = o\left(\frac{1}{x^{2}}\right)_{x \to +\infty}$$

$2 > 1$

$$\int_{1}^{+\infty} \frac{1}{x^{2}}\, dx \rightarrow$$ converge anche l'integrale nostro!

$$\frac{\left[\cos\left((2-\alpha)x\right)\right]+3}{\left(e^{x}+1\right)^{\alpha}} = o\left(\frac{1}{x^2}\right) \qquad x \to +\infty$$

$$\frac{x^2\left(\cos(2-\alpha)x+3\right)}{\left(e^{x}+1\right)^{\alpha}} \xrightarrow[x \to +\infty]{} 0$$

Calcolando per $\underline{\alpha = 2}$

$$\int_1^{+\infty} \frac{\cos 0 + 3}{(e^x+1)^2}\, dx = 4 \int_1^{+\infty} \frac{1}{(e^x+1)^2}\, \cancel{dx} =$$

$$e^x + 1 = y \qquad e^x = y - 1 \qquad x = \log(y-1)$$

$$dx = \frac{1}{y-1}\, dy$$

$$\int_{1+e}^{+\infty} \frac{1}{y^2} \, \frac{1}{(y-1)} \, dy$$

$$\frac{Ay+B}{y^2} + \frac{C}{y-1}$$

$$f(x) = \log |e^x - e^{2x}|$$

Dominio $\quad e^{x} - e^{2x} \neq 0$

$$e^{x}\left(1 - e^{x}\right) = 0 \iff x = 0$$

$$D = \mathbb{R} \setminus \{0\}$$

$$\lim_{x \to 0} f(x) = -\infty$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to +\infty} e^{x} - e^{2x} = \lim_{x \to +\infty} -e^{2x}\left(\frac{-1}{e^{x}} + 1\right) = -\infty$$

$$f(x) = \log\left|e^{x} - e^{2x}\right|$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

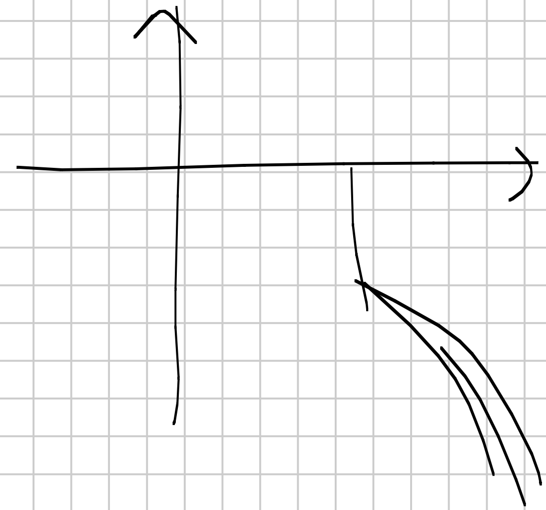
possibili asintoti obliqui

$$\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\log\left(e^{2x} - e^{x}\right)}{x} \overset{H}{=}$$

$$= \lim_{x \to +\infty} \frac{2e^{2x} - e^{x}}{1\left(e^{2x} - e^{x}\right)} = 2$$

$$f(x) = \log|e^x - e^{2x}|$$

$x \to +\infty \qquad \underline{e^x - e^{2x}} \to -\infty$



da un certo x grande in poi

$e^x - e^{2x} < 0$

$e^x - e^{2x} > 0$

Studiare segno di $f(x)$
e anche di $e^x - e^{2x}$

$$\lim_{x \to +\infty} \frac{f(x)}{x} = 2$$

$$\lim_{x \to +\infty} f(x) - 2x = \lim_{x \to +\infty} \log\left(e^{2x} - e^{x}\right) - 2x$$

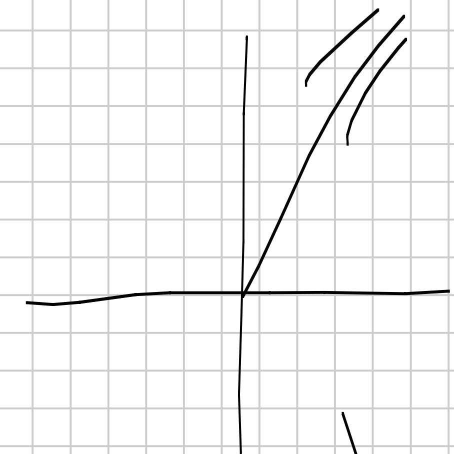
$$= \lim_{x \to +\infty} \log\left(e^{2x} \cdot \left(1 - \frac{1}{e^x}\right)\right) - 2x =$$

$$= \lim_{x \to +\infty} \cancel{\underbrace{\log\left(e^{2x}\right)}_{2x}} + \log\left(1 - \frac{1}{e^x}\right) \cancel{- 2x} = 0$$

$f(x) - ax \to 0$

Ho un asintoto obliquo per $x \to +\infty$

$y = 2x$



<u>PC.</u> $x \to -\infty$

(asintoto obliquo $x \to -\infty$ $y = x$)

$\frac{f(x)}{x} \to 1 \quad x \to -\infty$

$f(x) - x \to 0$

<u>Monotonia</u>

$$f(x) = \log\left|e^{x} - e^{2x}\right|$$

$$e^{x} - e^{2x} = e^{x}\left(1 - e^{x}\right) > 0 \iff e^{x} < 1$$

$$\iff x < 0$$

$$f(x) = \begin{cases} \log\left(e^{2x} - e^{x}\right) & x > 0 \\ \log\left(e^{x} - e^{2x}\right) & x < 0 \end{cases}$$

$(x>0)$

$$f'(x) = \frac{2e^{2x} - e^{x}}{e^{2x} - e^{x}} = \frac{e^{x}\left(2e^{x} - 1\right)}{\underbrace{e^{2x} - e^{x}}_{>0}}$$

$$f' = 0 \qquad 2e^{x} = 1 \implies e^{x} = 1/2 \implies x = \lg 1/2$$

$$2e^{x} - 1 > 0 \implies f'(x) > 0 \quad \forall x > 0$$

$x<0$ $\qquad f(x) = \log\left(e^x - e^{2x}\right)$

$$f'(x) = \frac{e^x - 2e^{2x}}{e^x - e^{2x}} = \frac{e^x\left(1 - 2e^x\right)}{e^x - e^{2x}}$$

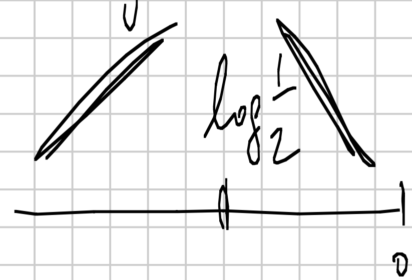
$e^x - e^{2x} > 0$

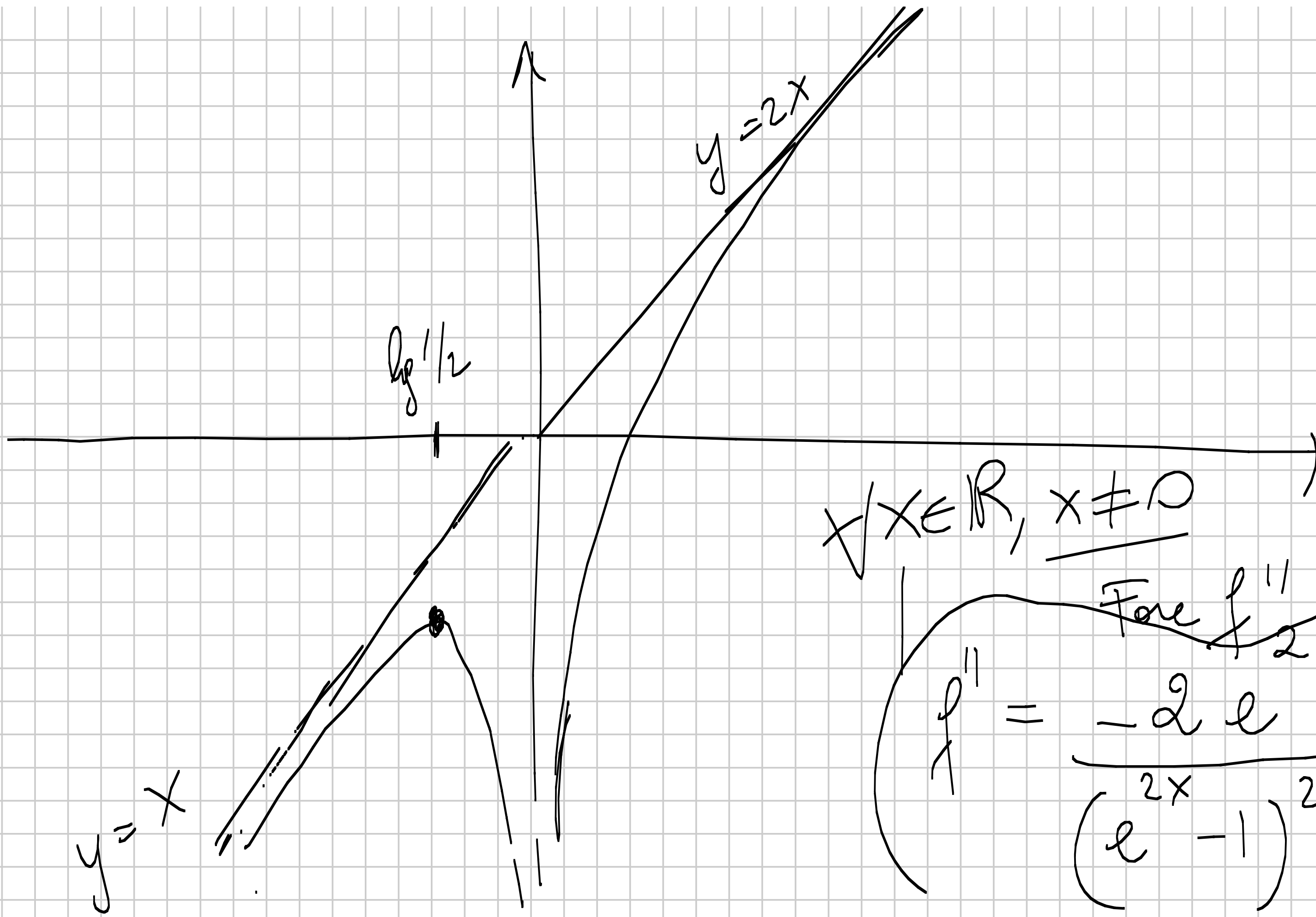
$f' = 0 \qquad e^x = 1/2 \qquad \Rightarrow \quad x = \log 1/2 < 0$

p.to critico

$f' > 0 \qquad e^x < 1/2 \qquad x < \log 1/2$

$x = \log 1/2$ p.to di massimo relativo





$\forall x \in \mathbb{R},\ \underline{x \neq 0}$

Fore f''_{2x}

$$f'' = \frac{-2e^{2x}}{\left(e^{2x}-1\right)^2} < 0$$

$$\sum (n!)^2 \log\left(1 + \frac{1}{n^{\alpha n}}\right) \qquad \alpha \in \mathbb{R}$$

$\alpha \leq 0$ $\qquad a_n \xrightarrow[n \to +\infty]{} +\infty \Rightarrow$ la serie diverge.

$\alpha > 0$ $\qquad a_n \sim (n!)^2 \dfrac{1}{n^{\alpha n}}$

$$\sum \frac{(n!)^2}{(n^\alpha)^n}$$ criterio del rapporto

$$\frac{a_{n+1}}{a_n} = \frac{\left((n+1)!\right)^2}{(n+1)^{\alpha(n+1)}} \cdot \frac{n^{\alpha n}}{(n!)^2} =$$

$$= \frac{\cancel{(n!)^2}(n+1)^2\, n^{\alpha n}}{(n+1)^{\alpha n}(n+1)^{\alpha}\cancel{(n!)^2}} = \frac{1}{(n+1)^{\alpha-2}}\left(\frac{n}{n+1}\right)^{\alpha n}$$

$$= \frac{1}{(n+1)^{\alpha-2}} \left[\frac{1}{\left(1+\frac{1}{n}\right)^{n}} \right]^{\alpha} \Rightarrow \frac{1}{e^{\alpha}}$$

$\alpha - 2 < 0 \qquad \frac{a_{n+1}}{a_n} \to +\infty \Rightarrow$ la serie diverge

$\alpha - 2 > 0 \qquad \frac{a_{n+1}}{a_n} \to 0 < 1$ la serie converge

$\alpha = 2 \qquad \frac{a_{n+1}}{a_n} \longrightarrow \frac{1}{e^2} < 1$ converge

converge $\forall \; \alpha \geq 2$.