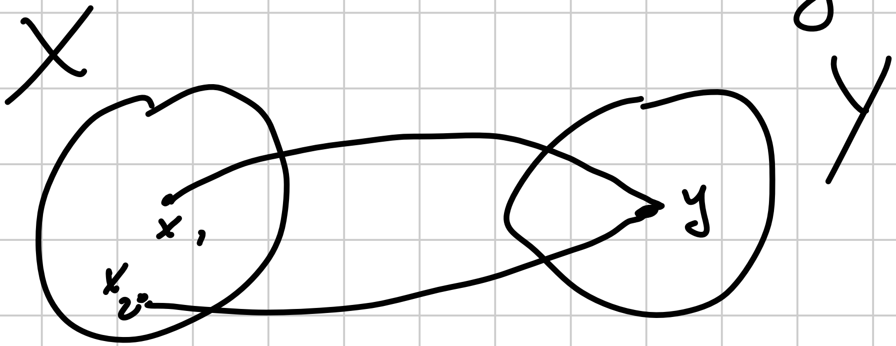
$f : X \longrightarrow Y$

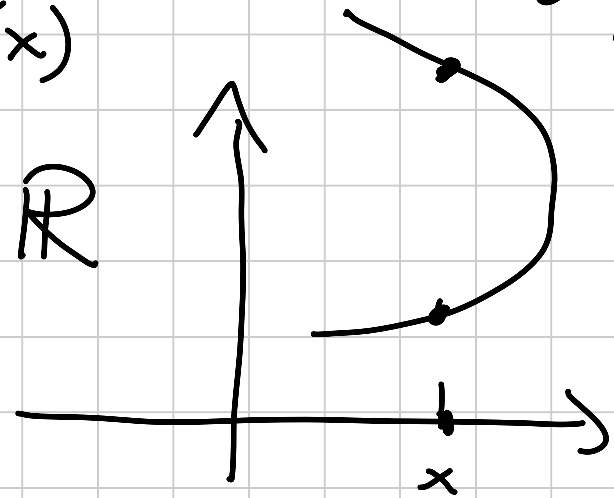
$x \in X \longrightarrow y \in Y$ $\qquad y = f(x)$

no funzione!



$X, Y \subseteq \mathbb{R}$

$(x, f(x))_{x \in X}$



$f(x) = x^2$ $\qquad x_1 = 1$ $\quad f(x_1) = f(x_2) = 1$

$x_2 = -1$

Def.

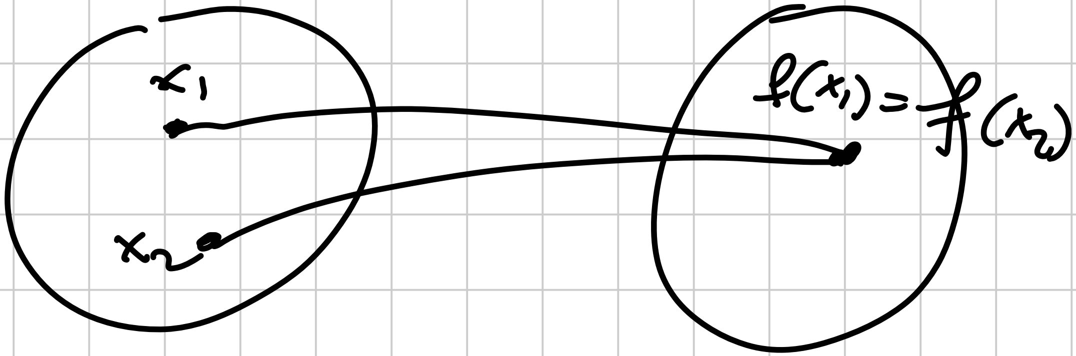
Def. $f : X \to Y$ si dice **INIETTIVA** se

$$\forall\, x_1, x_2 \in X : \; f(x_1) = f(x_2) \Longrightarrow x_1 = x_2$$

$$\left(\forall\, x_1, x_2 \in X \quad x_1 \neq x_2 \Longrightarrow f(x_1) \neq f(x_2) \right)$$

NO!

Es.



$$f(x) = x^3 + 3$$

$$f(x_1) = x_1^3 + 3 = f(x_2) = x_2^3 + 3$$

$$x_1^3 + \cancel{3} = x_2^3 + \cancel{3} \Longrightarrow x_1 = x_2$$

sì è iniettiva

$$x_1^3 = x_2^3 \Longrightarrow x_1 = x_2$$

$f(x) = x^2 + 8$ è iniettiva ?

$f(x_1) = f(x_2) \Longrightarrow x_1 = x_2$

$\forall x_1, x_2$ Hp. Ts.

$x_1^2 + \cancel{8} = x_2^2 + \cancel{8}$ $x_1^2 = x_2^2$ $x_1 = x_2$

no ! $x_1 = -x_2$

Def. $f: X \to Y$ si dice SURIETTIVA se

$$\underbrace{f(X)}_{\text{immagine}} = Y \qquad (\text{im} f = Y)$$

es. $f(x) = x^2 \geq 0 \quad f: \mathbb{R} \to \mathbb{R}$

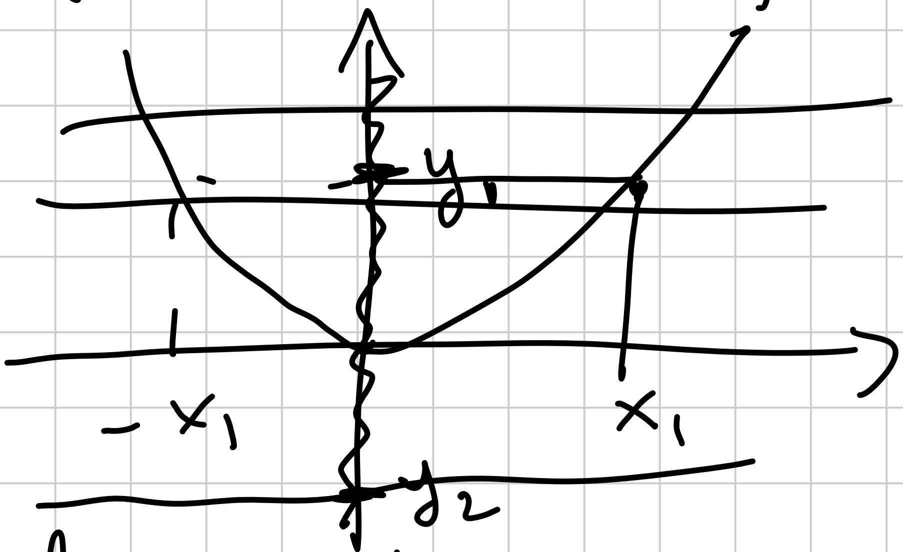
$$\text{im} f = [0, +\infty) \qquad \text{im} f \subset \underset{\text{non è suriettiva}}{Y} \text{ non } =!$$

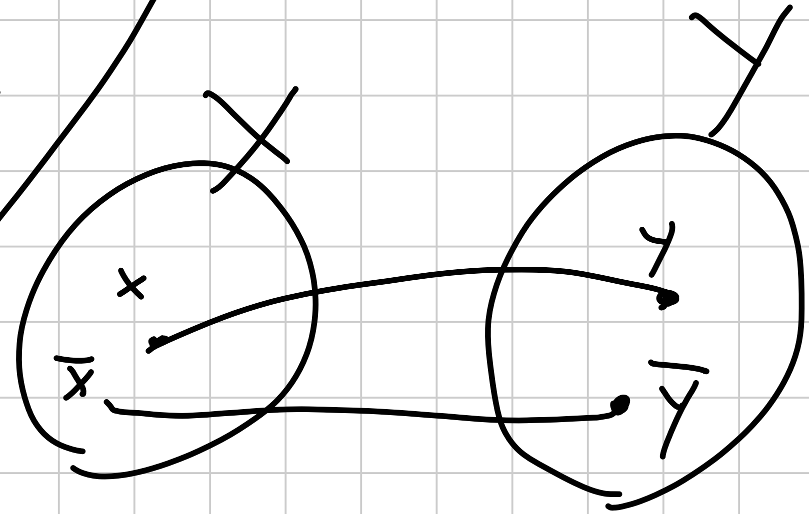
f è suriettiva se

$\forall y \in Y \ \exists x \in X$ t.c. $f(x) = y$

$f(x) = x^2$

$f(x) = x^2$ $\qquad f : \mathbb{R} \to \mathbb{R}$





f suriettiva se $\forall y \in Y \ \exists\, x \in X : f(x) = y$

Def. iniettiva e suriettiva $\Rightarrow$ BIETTIVA

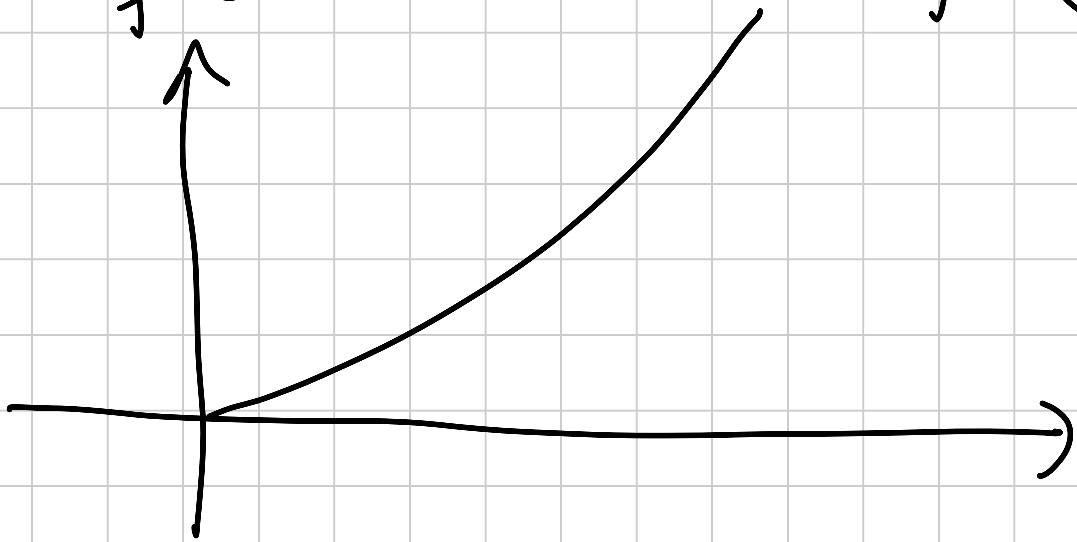
X e Y c'è una corrispondenza biunivoca

OSS. l'iniettività e la suriettività di una funzione dipendono dalla scelta del dominio e del codominio.

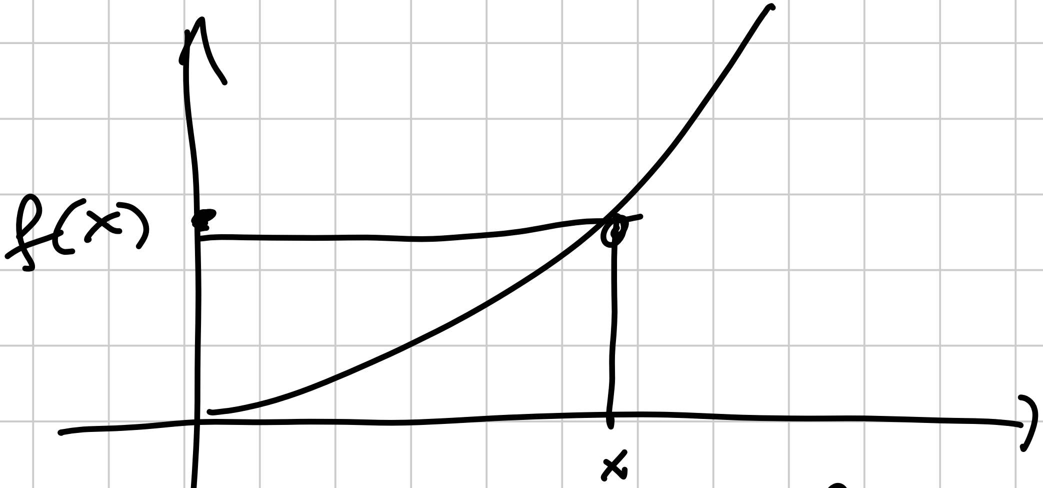
es. $f(x) = x^2$ $\qquad f : \mathbb{R} \to \mathbb{R}$

non è suriettiva né iniettiva

$f(x) = x^2$ $\qquad f : \underbrace{[0, +\infty)}_{X} \to \underbrace{[0, +\infty)}_{Y}$



la mia f è diventata biiettiva



$f(x)=x^2$

$f: \overset{X}{[0,+\infty)} \to \overset{Y}{[0,+\infty)}$

iniettiva? $x_1^2 = x_2^2$, $x_1, x_2 \geq 0$ $\Rightarrow x_1 = x_2$, sì!

suriettiva? $\boxed{\forall y \in Y \;\; \exists\, x \in X : y = f(x)}$

? $\forall y \in [0,+\infty) \;\; \exists\, x \in [0,+\infty)$ t.c. $y = x^2$

$\forall y \geq 0 \;\; \exists\, x \geq 0 : y = x^2$ $\quad (x = \sqrt{y})$

Riassunto

$f : X \to Y$, $y = f(x)$

- $\forall y \in Y \ \exists x \in X$ t.c. $y = f(x)$ (almeno) suriettività
- $\forall y \in Y$ c'è al più un $x \in X : y = f(x)$ iniettività

suriett. + iniett.

$\Rightarrow \forall y \in Y \ \exists! \, x \in X : \ y = f(x)$

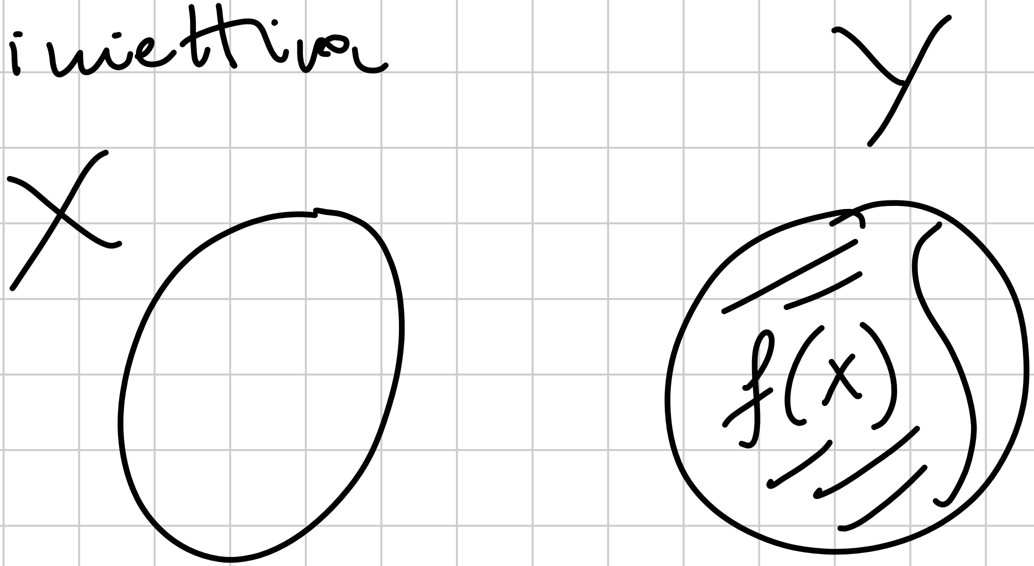
Biettività.

FUNZIONE <u>INVERSA</u>

$f: X \to Y$ iniettiva

$f(X)$

$f: X \to \underbrace{f(X)}$



come codominio prendo l'immagine

quindi è **suriettiva**

$f : X \to Y$ iniettiva

$f : X \to f(X)$ è biettiva

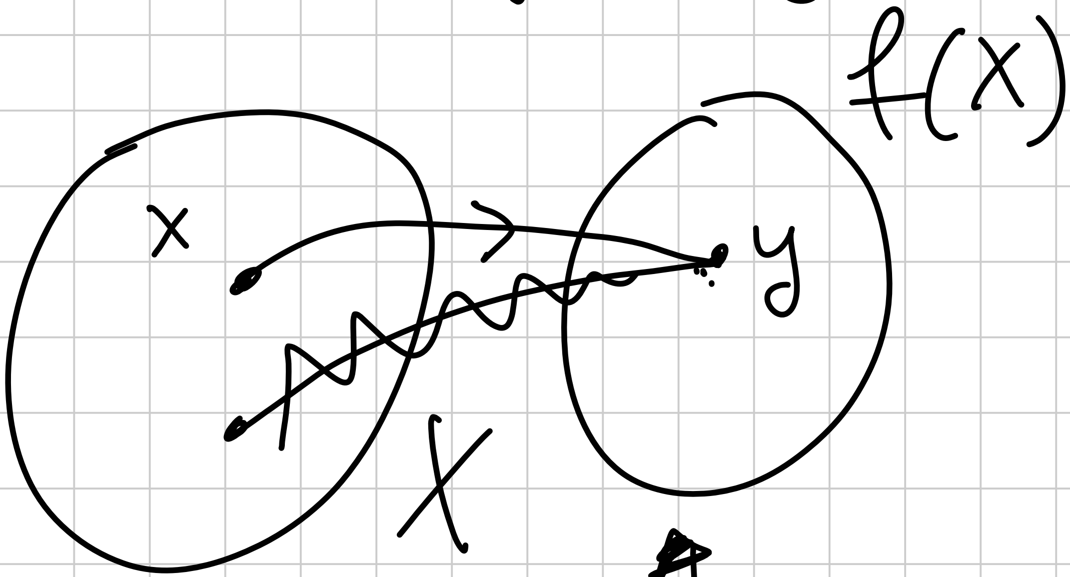
$\forall y \in f(X) \quad \exists!\, x \in X : f(x) = y$

è definita una nuova funzione

$$f^{-1} : f(X) \longrightarrow X$$

$$y \longmapsto x$$

t.c. $y = f(x)$



$$x = f^{-1}(y) \iff y = f(x)$$

oss. x è unico!

Es. $f(x) = x^2 \quad f : \mathbb{R} \to \mathbb{R}$

non è iniettiva

$f(x) = x^2 \quad f : [0, +\infty) \to [0, +\infty)$

è biettiva

quindi $\exists$ la sua funzione inversa

$f^{-1} : [0, +\infty) \longrightarrow [0, +\infty)$

$y \longrightarrow x \quad$ t.c. $\quad x^2 = y$

$f^{-1}(y) =: \sqrt{y} = x \iff y = x^2$

$\sqrt{y} \geqslant 0.$!

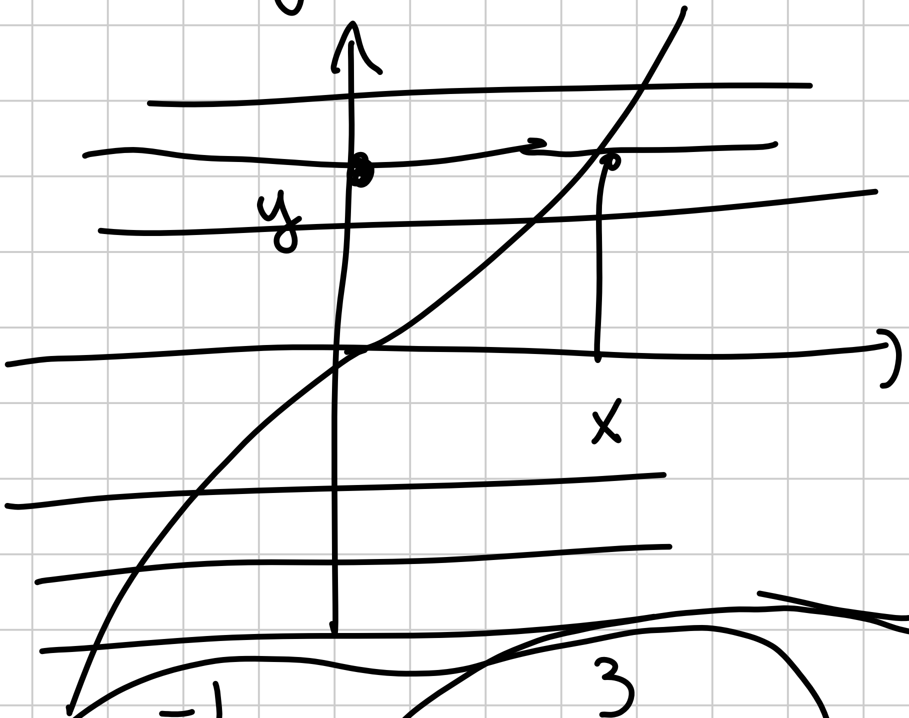
$\sqrt{x^2} = |x|$ Attenzione

$\Longleftrightarrow |x|^2 = x^2$

$\sqrt{4} = 2$ $\qquad$ $\sqrt{(-2)^2} = 2$

≥ 0

Es. $f(x) = x^3$



$f: \mathbb{R} \to \mathbb{R}$ è biettiva

$\forall y \ \exists!\, x : \ y = x^3$

f è invertibile
($\exists$ l'inversa)

$f^{-1}: \mathbb{R} \longrightarrow \mathbb{R}$

$y \longrightarrow x$ t.c.

$y = x^3$

$f^{-1}(y) =: \sqrt[3]{y} = x$ t.c.

$x^3 = y$

$\forall y \in \mathbb{R}$

In generale

$f(x) = x^n$ $\quad$ n pari

$f : [0, +\infty) \to [0, +\infty)$ è biettiva

$\exists\, f^{-1} : [0, +\infty) \to [0, +\infty)$ funzione inversa

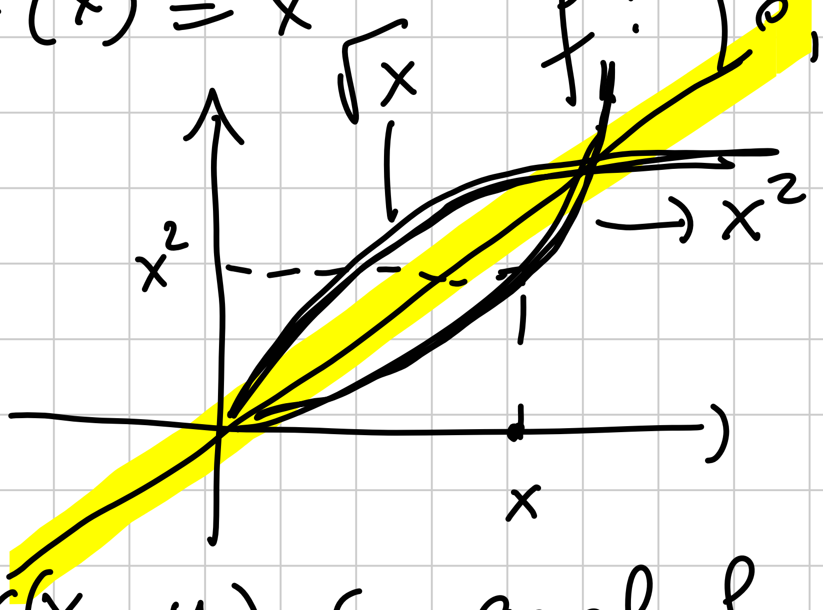
$f^{-1}(y) =: \sqrt[n]{y} \quad \forall y \geq 0.$

$f(x) = x^n$ $\quad$ n dispari

$f^{-1} : \mathbb{R} \to \mathbb{R}$ $\qquad \sqrt[n]{y} := f^{-1}(y)$

$\forall y \in \mathbb{R}$

$f(x) = x^2 \qquad f : [0, +\infty) \to [0, +\infty)$



(x, x^2)

$f^{-1}(y) = x \iff y = f(x)$

$(x, y) \in \text{graf } f \iff (y, x) \in \text{graf } f^{-1}$

$y = x$ bisettrice 1° e 3° quadrante

$f^{-1}(x) = \sqrt{x}$

Funzione composta

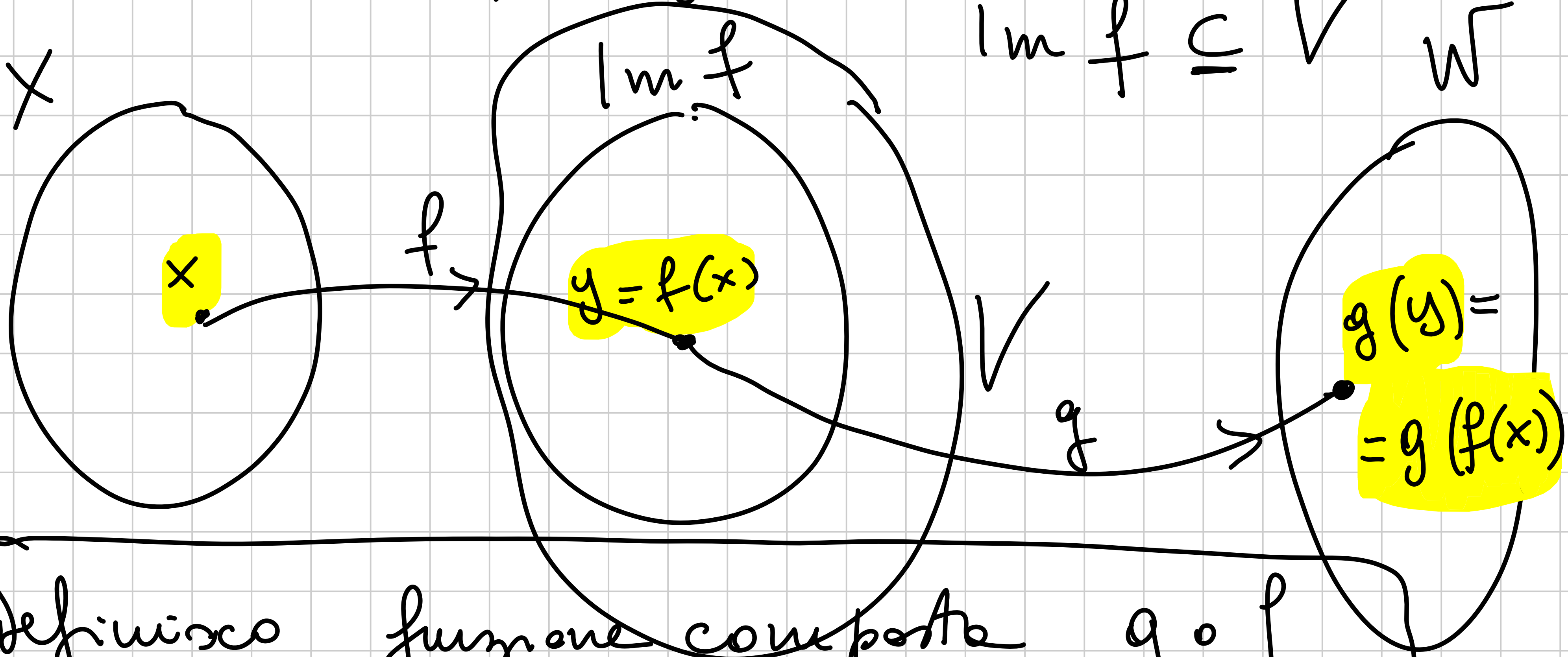
Es. $h(x) = \sqrt{x^2 + 1}$

$$x \longrightarrow x^2 + 1 \longrightarrow \sqrt{x^2+1}$$

$f(x) = x^2 + 1$ $\qquad$ $g(x) = \sqrt{x}$

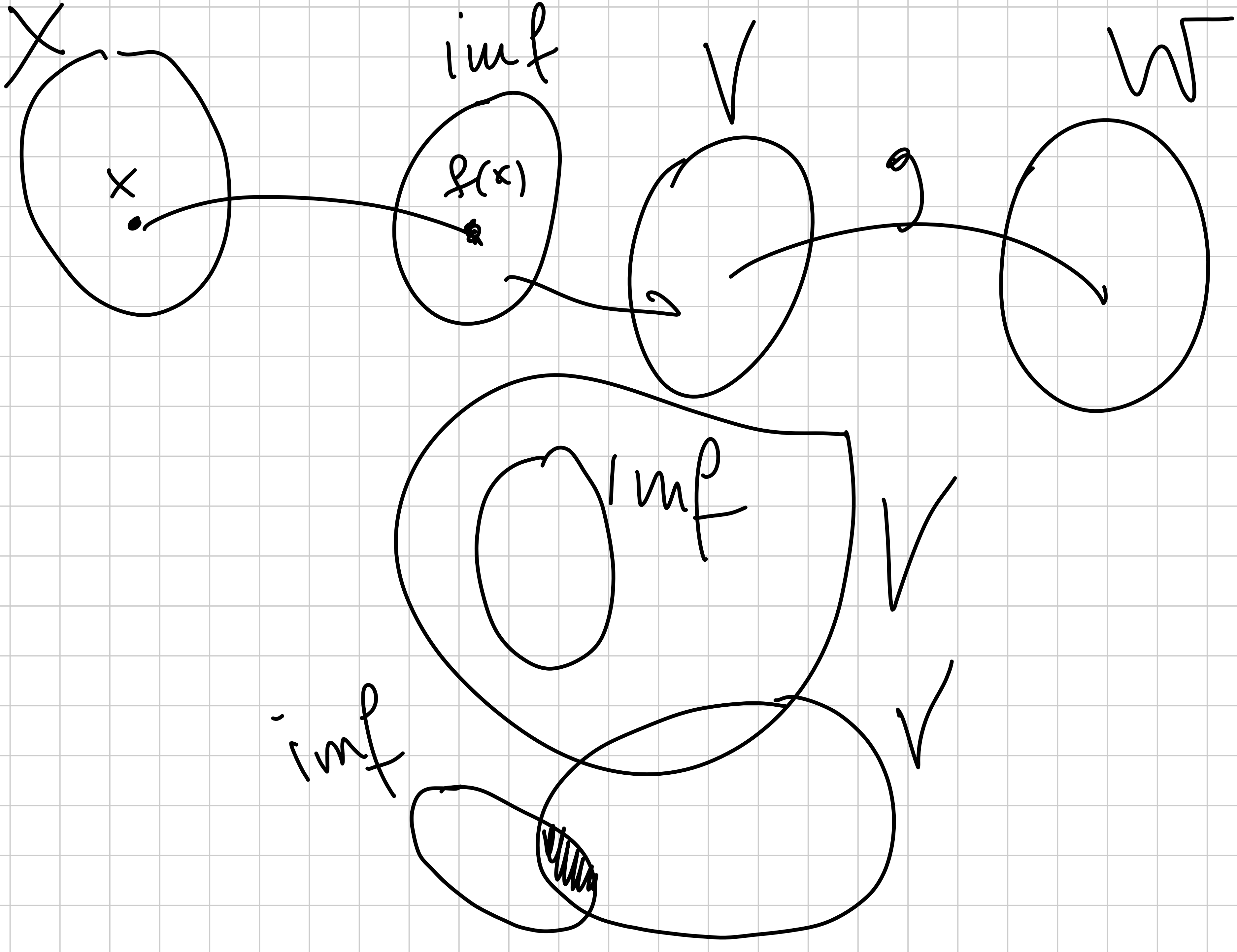
$g \circ f(x)$. $\qquad$ funzione composta

Def. $f : X \to Y$, $g : V \to W$ t.c. $\operatorname{Im} f \subseteq V$



Definisco funzione <u>composta</u> $g \circ f$
("g composto f")

$$g \circ f : X \to W \quad \text{così definita}$$
$$x \longmapsto g(f(x))$$



X
x
im f
f(x)
V
g
W
im f
V
im f
V

Es. $f(x) = x^2+1$ $\quad f: \mathbb{R} \to [1, +\infty)$

$g(x) = \sqrt{x}$ $\quad g: [0, +\infty)$

$$(g \circ f)(x) = g(f(x)) = \sqrt{x^2+1}$$

$$x \xrightarrow{f} \underbrace{x^2+1}_{y} \xrightarrow{g} \sqrt{x^2+1}$$

$\mathrm{Im} f \subseteq V$

sì!

$[1, +\infty) \subseteq [0, +\infty)$

$$(f \circ g)(x) = f(g(x)) = x+1$$

$$x \xrightarrow{g} \underbrace{\sqrt{x}}_{y} \xrightarrow{f} (\sqrt{x})^2 + 1 = x+1$$

$f(x) = -x^2 \qquad f : \mathbb{R} \to (-\infty, 0]$

$g(x) = \sqrt{x} \qquad g : [0, +\infty) \to [0, +\infty)$

$g \circ f(x) = g(f(x))$

$x \longrightarrow \underbrace{-x^2}_{y} \overset{?}{\longrightarrow}$

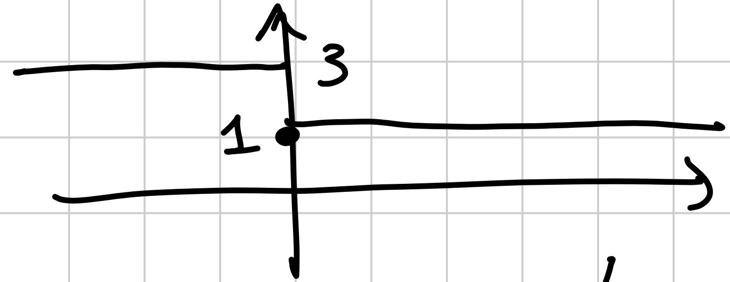
≤ 0 no perché dominio di g non contiene l'immagine di f

$x \neq 0$

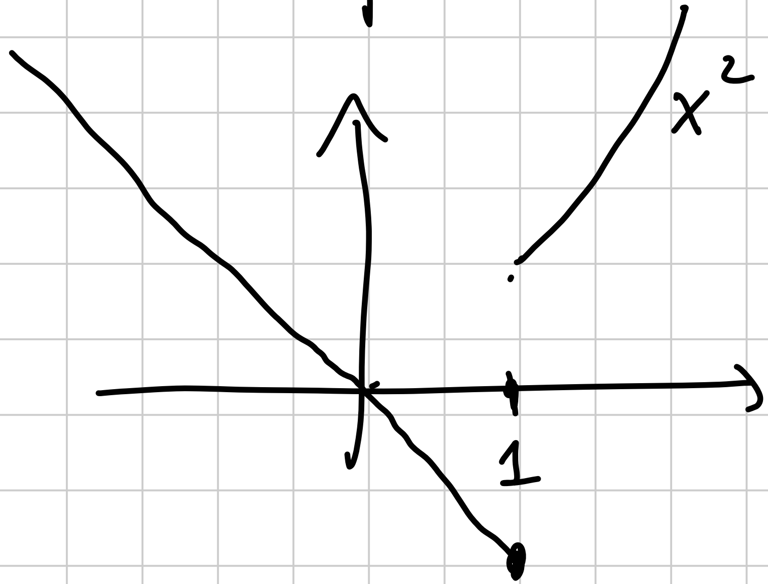
<u>PC.</u> $f \circ g(x)$ o.k.! $\text{im}\, g \subseteq \mathbb{R}$

PC.

$$f(x)=\begin{cases}1 & x\geq 0\\ 3 & x<0\end{cases}$$



$$g(x)=\begin{cases}x^2 & x>1\\ -x & x\leq 1\end{cases}$$



$$g(f(x)) = g\circ f$$

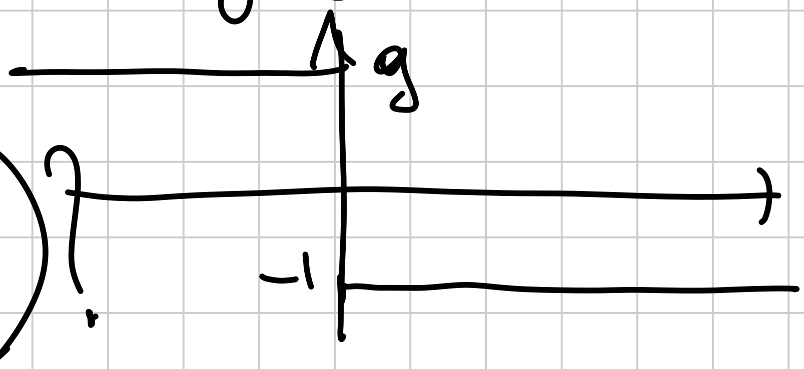
$x\geq 0 \qquad x \longrightarrow \overbrace{f(x)=1}^{y} \longrightarrow g(1) = -1$

$x<0 \qquad x \longrightarrow f(x)=3 \longrightarrow g(3) = 9$

$\operatorname{Im} f = \{1, 3\}$

$\operatorname{dom} g = \mathbb{R}$

$f\circ g$?



Oss. $f : X \to f(X)$ iniettiva

$\exists \; f^{-1} : f(X) \to X$ inversa

$$f^{-1} \circ f = f^{-1}(f(x)) = x$$

$$x \longrightarrow f(x) = y \longrightarrow f^{-1}(y) = x$$

$x \longrightarrow x$ funzione identità Id

$$f \circ f^{-1} = Id$$