Es 1

$$\lim_{x\to+\infty} \frac{x^{\log x} + \underbrace{\operatorname{arctg}(e^x)}_{\to \pi/2}}{5^x + \log(e^x + \operatorname{sen} x)} =$$

$$\log\left(e^x\left(1+\frac{\operatorname{sen} x}{e^x}\right)\right) = \log(e^x) + \log\left(1+\frac{\operatorname{sen} x}{e^x}\right)$$

$$= x + \underbrace{\log\Big(1 + \underbrace{\frac{\operatorname{sen} x}{e^x}}_{\to 0}\Big)}_{\to 0}$$

$$= \lim_{x \to +\infty} \frac{x^{\log x}}{5^x} = \lim_{x \to +\infty} \frac{e^{\log x \cdot \log x}}{e^{x \log 5}} =$$

$$= \lim_{x \to +\infty} e^{(\log x)^2 - x \log 5} = 0$$

$$\lim_{x \to +\infty} (\log x)^2 - x \log 5 = \lim_{x \to +\infty} -x \left(\log 5 - \underbrace{\frac{(\log x)^2}{x}}_{0} \right) =$$

$$= -\infty$$

$$f(x) = \operatorname{arctg}\left(e^x + \alpha\right), \quad \alpha \in \mathbb{R}$$

Det. il parametro α in maniera che $f(x)$ abbia un flesso in $x = 0$.

$$f'(x) = \frac{1}{1 + (e^x + \alpha)^2}\, e^x$$

$$f''(x) = \frac{e^x\left(1 + (e^x+\alpha)^2\right) - e^x 2\left(e^x + \alpha\right)e^x}{\left(1 + (e^x+\alpha)^2\right)^2}$$

$$f''(0) = 0 \qquad \left(1 + (1+\alpha)^2\right) - 2(1+\alpha) = 0$$

$$\cancel{1} + \cancel{1} + \alpha^2 + \cancel{2}\alpha - \cancel{2} - \cancel{2}\alpha = 0$$

$$\alpha^2 = 0 \quad \Rightarrow \quad \alpha = 0$$

$$f''(0) = 0 \iff \alpha = 0$$

$$\lim_{x\to 0^+} \frac{27x^5 + \log(1+e^{-1/x})}{\sqrt{1+x^8} - 1 + \alpha \operatorname{sen}^5 x} \qquad \alpha \in \mathbb{R}$$

$\underline{N.}$ $\quad 27x^5 + e^{-1/x} + o(e^{-1/x}) = 27x^5 + o(x^5)$

$\underline{D.}$ $\quad \frac{1}{2}x^8 + o(x^8) + \alpha x^5 + o(\alpha x^5)$

$\alpha \neq 0 \quad \underline{D.} \quad \alpha x^5 + o(\alpha x^5) \qquad \frac{N.}{D.} \to \frac{27}{\alpha}$

$N.$

$\alpha = 0 \quad \underline{D.} \quad \frac{1}{2}x^8 + o(x^8)$

$$\frac{N.}{D.} = \frac{27\, x^5 + o(x^5)}{\frac{1}{2}\, x^8 + o(x^8)} \to +\infty$$

Es Dire se converge e in caso affermativo, calcolarlo.

$$\int_0^{1/3} \frac{x}{\sqrt{9-(6x+1)^2}}\,dx =$$

$$(6x+1) = y$$

$$6x = y-1$$

$$x = \frac{1}{6}(y-1)$$

$$dx = \frac{1}{6}\,dy$$

$$= \int_1^3 \frac{y-1}{6\sqrt{9-y^2}}\,\frac{1}{6}\,dy =$$

$$= \int_1^3 \frac{y-1}{36\sqrt{(3-y)}\sqrt{(3+y)}}\,dy$$

ē integrabile

$$\int_0 \frac{1}{\sqrt{t}}$$

ē integrabile

$3 - y = t$

Calcoliamolo:

$$\int_1^3 \frac{y-1}{\sqrt{9-y^2}}\,dy = \int_1^3 \frac{y}{\sqrt{9-y^2}}\,dy - \int_1^3 \frac{1}{\sqrt{9-y^2}}\,dy$$

$$y^2 = t$$

$$2y\,dy = dt$$

$$\int_1^9 \frac{1\,dt}{2\sqrt{9-t}} = -\frac{\cancel{2}}{\cancel{2}}\sqrt{9-t}\,\Big|_1^9 = \sqrt{8}$$

$$\int_1^3 \frac{1}{\sqrt{9-y^2}}\,dy = \int_1^3 \frac{1}{3\sqrt{1-\left(\frac{y}{3}\right)^2}}\,dy =$$

$$\frac{y}{3} = t$$

$$\frac{1}{3}dy = dt$$

$$dy = 3dt$$

$$= \frac{1}{\cancel{3}} \arcsin\left(\frac{y}{3}\right)\cancel{3}\Big|_1^3$$

$$= \arcsin 1 - \arcsin \frac{1}{3}$$

<u>Es.</u> $\sum (-1)^n \frac{\log n}{n}$

$\log n$

$n \geq 3$

Conv. <u>assoluta</u> $\sum_{n=2}^{+\infty} \frac{\log n}{n}$

$\log n \geq 1$

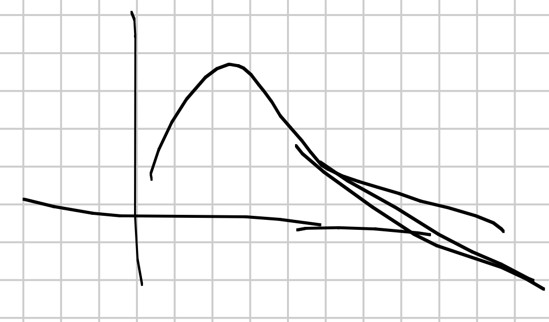
$\frac{\log n}{n} \geq \frac{1}{n}$ poiché $\sum \frac{1}{n}$ diverge

dal principio del confronto

diverge anche $\sum \frac{\log n}{n}$.

No conv. assoluta.

Convergenza? $\sum (-1)^n \frac{\log n}{n}$



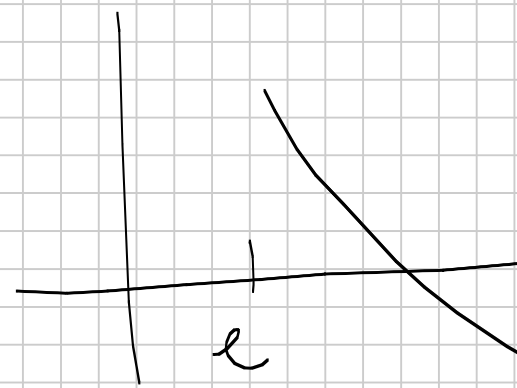
1) $a_n = \frac{\log n}{n} \xrightarrow[n \to +\infty]{} 0$

2) a_n succ. decrescente $\frac{1}{n} \operatorname{sen}\left(\frac{1}{n}\right)$

$f(x) = \frac{\log x}{x}$ voglio dim. che $f(x)$ è decrescente da un certo x in poi

$$f'(x) = \frac{\frac{1}{x}\cdot x - \log x}{x^2} = \frac{1-\log x}{x^2} \leq 0$$

$\log x \geq 1 \Rightarrow x \geq e$



$\Rightarrow \forall n \geq 3 \Rightarrow a_n = \frac{\log n}{n}$

è decrescente.

$$\frac{1}{n} \operatorname{arctg}\left(\frac{1}{n}\right)$$

Es.

$$\int_0^1 \frac{e^x}{1+e^{2x}-e^x}\,dx =$$

$e^x = y$

$e^x\,dx = dy$

$$= \int_1^e \frac{1}{1+y^2-y}\,dy =$$

$\Delta = 1-4<0$

$$y^2 - y + \frac{1}{4} - \frac{1}{4} + 1 =$$

$$= \left(y-\frac{1}{2}\right)^2 + \frac{3}{4}$$

$$= \int_1^e \frac{1}{\frac{3}{4} + \left(y - \frac{1}{2}\right)^2} \, dy = \int_1^e \frac{4}{3} \frac{1}{1 + \frac{\left(y - \frac{1}{2}\right)^2}{\frac{3}{4}}} \, dy$$

$$= \frac{4}{3} \int_1^e \frac{1}{1 + \left(\frac{\left(y - \frac{1}{2}\right)2}{\sqrt{3}}\right)^2} \, dy =$$

$$\frac{2\left(y - \frac{1}{2}\right)}{\sqrt{3}} = t$$

$$\frac{2}{\sqrt{3}} dy = dt$$

$$dy = \frac{\sqrt{3}}{2} dt$$

$$= \frac{4}{3} \int \frac{1}{1+t^2} \frac{\sqrt{3}}{2} \, dt$$

$$= \frac{4}{3} \frac{\sqrt{3}}{2} \operatorname{arctg} \left(\frac{2}{\sqrt{3}} \left(y - \frac{1}{2} \right) \right) \Big|_{1}^{e} = \ldots$$