Research activity (a short overview)

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The framework of our research activity is Algebraic Analysis, introduced by the japanese school of M. Sato and M. Kashiwara. Although Algebraic Analysis was originally born to study systems of linear differential equations with analytic coefficients from an algebraic point of view (by associating to any system the corrisponding "$\mathcal{D}$-module", i.e. a coherent module on the ring of analytic differential operators on the manifold) and using "microlocalization" (i.e. the analysis of singularities of the system in the cotangent bundle), thanks to its interdisciplinary nature it progressively enlarged its range, eventually connecting research domains which seemed to be very far from each other: particularly suggestive examples are the contributions of M. Kashiwara to singularity theory coming from the solution of the Riemann-Hilbert problem, and to representation theory with the proof of Kazhdan-Lusztig conjecture.

In the following we provide a short description of our studies (the numbering refers to the below list of publications).

The tangential Cauchy-Riemann system. The first research object has been the tangential Cauchy-Riemann system $\overline{\partial}^b$ associated to a CR submanifold of a complex manifold.

In [**1**] we present a natural generalization of the notion of "generic submanifold" of a complex analytic manifold, where the Cauchy-Riemann system $\overline{\partial}$ is replaced by any coherent $\mathcal{D}$-module on the complexification of the manifold, viewed as real analytic.

In [**2**] we study the solvability of $\overline{\partial}^b$ in the case where the CR submanifold has constant Levi rank, with applications to the vanishing of cohomology of the complex of "microfunctions at the boundary" naturally associated to the geometric framework.

In [**3**] we classify the sheaves on a real analytic manifold whose "microsupport" (the analogous notion for sheaves of the characteristic variety of a coherent $\mathcal{D}$-module) is contained in a non necessarily smooth Lagrangian subset of the cotangent bundle associated to a convex subset.

Integral trasforms for sheaves and $\mathcal{D}$-modules. We then turned our attention to integral trasforms, a classic research subject (Laplace, Fourier, Radon...) which can be fruitfully investigated in the language of derived categories of sheaves and $\mathcal{D}$-modules.

In [**4,5,6,7**] we generalize the known results about "projective duality" (the natural correspondence between a complex projective space and its dual) to the case of "dual" Grassmann manifolds (i.e. formed by subspaces of complementary dimensions), where the singularities of the "incidence relation" between them require the use of group invariance and of b-function theory.

In [**8**] we show how the complex Penrose transform and many real affine or projective integral trasforms (integrations of Radon-type on planes of prescribed dimension, conformal case...) are

induced by a unique complex integral transform; this allows us to recover and improve various classical results in a unified framework. In particular, in **[9]** we perform a more explicit study of the integral kernels which describe the Radon transform for hyperplanes.

Systems of invariant equations on homogeneous varieties and representation theory. The invariance under group action of the problems described above naturally leads to consider them in the more general situation of compact homogeneous varieties of complex semisimple Lie groups (the so-called "generalized flag manifolds").

In **[11]** we sistematically study the integral transforms induced by the closed orbit of any Lie group (hence, a generalization of [8]), and we also provide an algorithm for the computation of the transforms of invariant $\mathcal{D}$-modules.

With **[10]** (which announces the main results of **[12]** and of another work currently under redaction) we similarly generalize the duality between homogeneous manifolds studied in [6], with a particular regard to the applications to representation theory.

Scientific publications

[1] *Varietà generiche rispetto a sistemi differenziali* (Italian). Rend. Sem. Mat. Univ. Padova, 86 (1991), p. 233–237

[2] *Microfunctions along submanifolds with constant Levi rank.* Rend. Acc. delle Scienze 113 (1995), vol. XIX, p. 111–121 [with A. D'Agnolo and Giu. Zampieri]

[3] *Non smooth Lagrangian sets and estimations of micro-support.* J. Math. Soc. Japan, 48/2 (1996), p. 255–258 [with A. D'Agnolo and Giu. Zampieri]

[4] *La dualité de Grassmann pour les $\mathcal{D}$-modules.* C. R. Acad. Sci. Paris, série I, t. 322 (1996), p. 929–933.

[5] *Quantification de la dualité de Grassmann.* C. R. Acad. Sci. Paris, série I, t. 324 (1997), p. 1343–1348.

[6] *Grassmann duality for $\mathcal{D}$-modules.* Ann. Sci. École Normale Sup., 4^e série, t. 31 (1998), p. 459–491.

[7] *Teoria dei fasci e trasformazioni integrali per $\mathcal{D}$-moduli tra varietà di Grassmann* (Italian). Bollettino U.M.I. (8) 1-A Suppl. (1998), p. 129–132.

[8] *Real forms of the Radon-Penrose transform.* Publ. of R.I.M.S., Kyoto University, 36/3 (2000), p. 337–383 [with A. D'Agnolo]

[9] *A topological obstruction for the real Radon transform.* "Harmonic Analysis and Integral Geometry", CRC, Chapman Hall (2000), p. 45–51 [with A. D'Agnolo]

[10] *Generalized Verma modules, b-functions of semi-invariants and duality for twisted $\mathcal{D}$-modules on generalized flag manifolds.* C. R. Acad. Sci. Paris, série I, t. 335 (2002), p. 111–116.

[11] *Radon transforms for quasi-equivariant $\mathcal{D}$-modules on generalized flag manifolds.* Diff. Geom. Appl. 18 (2003), no. 2, p. 147–176 [with T. Tanisaki]

[12] *Sheaves and $\mathcal{D}$-modules on dual flag manifolds and generalized Verma modules* (2008); available on the web site `www.math.unipd.it/~maraston/cvpub/Dmod_dualflag.pdf`