COMPRESSED MOTION SENSING
FOR TOMOGRAPHIC PARTICLE IMAGE VELOCIMETRY

STEFANIA PETRA

ABSTRACT. In previous work [PS14, PSS13] we analyzed representative ill-posed scenarios of Tomographic Particle Image Velocimetry (Tomo PIV) [ESWvO07] with a focus on conditions for unique volume reconstruction. Based on sparse random seedings of a region of interest with small particles, the corresponding systems of linear projection equations were probabilistically analyzed in order to determine: (i) the ability of unique reconstruction in terms of the imaging geometry and the critical sparsity parameter, and (ii) sharpness of the transition to non-unique reconstruction with ghost particles when choosing the sparsity parameter improperly. We showed that the sparsity parameter directly relates to the seeding density used for Tomo PIV that is chosen empirically to date. Our results provide a basic mathematical characterization of the Tomo PIV volume reconstruction problem that is an essential prerequisite for any algorithm used to actually compute the reconstruction. Moreover, we have connected the sparse volume function reconstruction problem from few tomographic projections to major developments in compressed sensing (CS) and found out that the predicted critical seeding lies below the theoretical optimal threshold in CS.

In more recent work [DPS16] we complement the standard tomographic sensor, based on few projections, by additional measurements of moving objects at two subsequent points in time. Denoting by $\mathbf{A}$ the Tomo PIV sensor that corresponds to few projections synchronously recorded with few cameras only, the standard approach is to reconstruct an image pair $(\mathbf{u}, \mathbf{u}_t)$ from $\mathbf{A}\mathbf{u} \approx \mathbf{b}$, $\mathbf{A}\mathbf{u}_t \approx \mathbf{b}_t$ and then - in a subsequent step - to estimate the unknown flow transport mapping $T_t(\mathbf{u}) = \mathbf{u}_t$ by cross-correlating $(\mathbf{u}, \mathbf{u}_t)$.

Our approach is to use the available information at time step $t$, to consider the projections $\mathbf{b}_t$ as additional measurements together with $\mathbf{b}$ and to jointly estimate the images and the transformation parameters from the available multi-view measurements. Thus, we solve

$$
\min_{\mathbf{T}_t, \mathbf{u} \geq 0} \| \mathbf{A}\mathbf{u} - \mathbf{b} \|^2 + \| \mathbf{A}\mathbf{T}_t(\mathbf{u}) - \mathbf{b}_t \|^2
$$

and regard $\mathbf{A}\mathbf{T}_t(\cdot)$ as an additional sensor. From the CS viewpoint this raises the key question if and how much the recovery performance of the complemented sensor

$$
\mathbf{A}_{T_t} := \begin{pmatrix} \mathbf{A} \\ \mathbf{A}\mathbf{T}_t(\cdot) \end{pmatrix}, \quad \mathbf{A}_{T_t} \mathbf{u} = \begin{pmatrix} \mathbf{b} \\ \mathbf{b}_t \end{pmatrix}
$$

improves, under the assumption that $\mathbf{T}_t$ is known. We call compressed sensing in connection with the correspondence information $\mathbf{u}_t = \mathbf{T}_t(\mathbf{u})$ compressed motion sensing. We evaluate both theoretically and numerically the recovery performance of $\mathbf{A}$ vs. $\mathbf{A}_{T_t}$ and show that our approach enables highly compressed sensing in dynamic imaging scenarios of practical relevance.

REFERENCES


(S. Petra) MATHEMATICAL IMAGING GROUP, HEIDELBERG UNIVERSITY, GERMANY
E-mail address: petra@math.uni-heidelberg.de