



Universita' degli Studi di PADOVA

Progetti di Ricerca di Ateneo

Anno: 2014 - prot. CPDA143275

## 1.0 Macroarea di Afferenza del Responsabile Scientifico del Programma di Ricerca

1 - Matematica, scienze fisiche, dell'informazione e della comunicazione, ingegneria e scienze della Terra

## 1.1 Area Scientifica del Responsabile Scientifico del Programma di Ricerca

01 - Scienze Matematiche

## 1.2 Responsabile Scientifico del Programma di Ricerca

SOMMARIVA	Alvise	M
(Cognome)	(Nome)	(sesso)
PROFESSORE ASSOCIATO	MAT/08	11/10/1968
(Qualifica)	(Settore Scientifico Disciplinare)	(Data di Nascita)
SMMLVS68R11L736D		DIP. MATEMATICA
(Codice fiscale)	(Facoltà)	(Dipartimento/Istituto)
041-5205176	049-8271499	alvise@math.unipd.it
(Prefisso e Telefono)	(Numero Fax)	(Indirizzo di Posta Elettronica)

## Lingua di compilazione del progetto (inglese o lingua veicolare)

English

## 1.3 Area Scientifica del Programma di Ricerca

Area Scientifica Prevalente	Scienze Matematiche	(% di afferenza)	100
Area Scientifica		(% di afferenza)	
Area Scientifica		(% di afferenza)	

## 1.4 Titolo del Programma di Ricerca

Conditioning issues in multivariate approximation and image reconstruction

## 1.5 Abstract del Programma di Ricerca

Our research plan is to introduce new techniques to treat ill conditioning issues in multivariate approximation and image reconstruction.

We will mainly consider three directions.

In the first one, we observe that in many problems of multivariate polynomial interpolation and approximation, the determination of a well-conditioned polynomial basis. e.g. an orthogonal basis w.r.t. to a weighted discrete scalar product, allows to obtain better numerical results. It is the case of hyper interpolation [S95] that requires a polynomial basis orthogonal w.r.t. to a scalar product and tailored cubature rules, as well as the extraction of Approximate Fekete Points or Discrete Leja Points on a compact multivariate domain from (weakly-) admissible meshes that for high degrees may provide poor interpolation sets if the polynomial basis is not properly chosen.

A second direction concerns Radial Basis Functions (RBF), a frequently used method to handle scattered data approximation problems on different structures by linear combinations of a single positive definite kernel. The kernel is responsible for the quality of the approximation error and the stability of the system. As well known, the a trade-off principle [S95] is a key tool for understanding the behavior of the RBF interpolation process and it suggests when the pertinent Vandermonde matrix is ill-conditioned. Our aim is to study this issue and introduce new stable bases [DMS13, DMS14]. Applications to medical image reconstruction [DMIS13] and partition of unity methods [DR14] seems promising.

A third direction, regards the image restoration problem, often formulated as a large linear system in which the coefficient matrix representing a blur/distorsion operator can be heavily ill-conditioned, making necessary some sort of regularization to get a significant approximation of the solution. We intend here to follow two approaches, i.e. the regularization by Krylov methods, eventually coupled with the well-known Tikhonov regularization (hybrid methods) as well as the Kaczmarz, Landweber, SIRT methods. We will devote special attention to new parameter selection strategies, the numerical solution of problems arising from nonlinear

regularization and application of extrapolation techniques to improve the aforementioned methods.

Note: The cited references are in section 2.4.

## 1.6 Caratteri di innovatività del progetto e del gruppo

Our approach to the treatment of ill-conditioning goes in different directions, depending on the topic.

In the framework of multivariate polynomial approximation theory, we intend to determine good polynomial basis on some standard and nonstandard domains. This result can be achieved for instance by considering a polynomial basis orthogonal w.r.t. a suitable discrete scalar product. In this sense, we aim to propose algorithms alternative to those presented in [HL02], [VC10], [VBHS14] that tend to loose orthogonality even at low degrees.

Once such results are achieved, we can overcome some problems in the extraction of Approximate Fekete Points and Discrete Leja Points from (weakly) admissible meshes due to the ill-conditioning of the Vandermonde matrix when the polynomial basis is not carefully chosen. This is relevant since it allows to compute good interpolation point sets, on rather general domains, only by linear algebra routines. As by-product, we can construct new spectral and high-order methods for PDEs on nonstandard geometries.

The idea to resort to orthogonality to cope ill-conditioning in multivariate polynomial basis, can be pursued also along other directions. Orthogonalizing polynomials of degree  $n$  with respect to the discrete measure generated by a positive cubature formula of exactness degree  $2n$ , gives an orthogonal basis for the underlying absolutely continuous measure, so opening the path to compute hyperinterpolation polynomials on these arc-related domains.

In the RBF setting, very often interpolation based on translates of the kernel, say  $\Phi$ , is numerically unstable due to exceedingly large condition number of the kernel matrix. Better results can be achieved by computing new orthonormal basis of the native space associated to  $\Phi$  (cfr. [DMS13, DMS14]).

Our innovation consists in providing better and tighter stability estimates using a different metric for RKHS (cf. [ARSW11]), allowing us to study new formulas for the power function. Moreover, we are trying to build algorithms based on this characterization of the power function, which aim at constructing good sets of interpolation points. Applications to medical image reconstruction are planned.

The use of Krylov type methods for the solution of inverse problems in image restoration is nowadays a very active research topic. In this proposal we plan to investigate new techniques based on these methods and study some theoretical aspects that are still vaguely known.

The use of sequence transformations for accelerating the convergence of iterative methods for solving linear systems is an appealing technique not very widely known. In this sense, Shanks transformation (and its theoretical link with several iterative methods) has been taken in high consideration and the topological epsilon-algorithm, used for implementing such a transformation, received much attention in the literature. But up to now, the use of general elements of a vector space and of its dual vector space was considered as a difficult problem.

In [BRZ14], it has been achieved a major improving, suggesting new simple algorithms, more stable and requiring less storage. We intend to apply these techniques in practical problems of image restoration, typically ill-conditioned.

The research group joins together researchers, presenting a well established experience from different subjects of numerical analysis (multivariate interpolation, inverse problems, extrapolation methods, regularization, matrix functions) with the common aim of studying reliable methods that can be used as tools for particular approximation theory and inverse problems. The interdisciplinary nature of the group seems to be a good opportunity to share knowledge and experience, and to produce useful results.

Our intention is to collaborate with international well-known researchers, as G.Meurant, M.Van Barel, A.Cuyt, R.Schaback, G.Fasshauer, A.Iske, J.G.Nagy for the solution of the described project.

## 1.7 Settori scientifico-disciplinari interessati dal Programma di Ricerca

MAT/08

## 1.8 Parole chiave

1. AREA 01 - Mathematics - Numerical Analysis - Numerical Linear Algebra - ILL-POSEDNESS, REGULARIZATION
2. AREA 01 - Mathematics - Approximations And Expansions - APPROXIMATION BY POLYNOMIALS
3. AREA 01 - Mathematics - Approximations And Expansions - MULTIDIMENSIONAL PROBLEMS
4. AREA 01 - MSC - Numerical Analysis - INVERSE PROBLEMS

## 1.9 Curriculum scientifico del Responsabile Scientifico del programma di ricerca

\* Born in Venice, October 11, 1968.

### SCIENTIFIC CURRICULUM

- \* Degree in Mathematics, University of Padua (1993).
- \* Ph. D. in Computational Mathematics, University of Padua (1996-1999).
- \* Research Associate, project "Numerical Analysis of integral and differential models in applied sciences", University of Padua (1999-2002).
- \* Post-doc fellowship, University of Padua (2002-2004).
- \* Research Associate, University of New South Wales, Australia ( September 1, 2004 -November 31, 2005).
- \* Assistant Professor in Numerical Analysis: University of Padua (March 2006-).
- \* National scientific qualification as Associate professor of Numerical Analysis, December 2013

### RESEARCH RESULTS

\* Author of more than 40 scientific papers in the fields of Numerical Analysis, Approximation Theory and Mathematical Analysis

For a complete list of publications, reprints, software and for other details see the web page: <http://www.math.unipd.it/~alvise/>

### RESEARCH COLLABORATION AND COORDINATION EXPERIENCE

- \* International collaborations: J.-P. Calvi (Toulouse), W. Gautschi (Purdue), N. Levenberg (Bloomington), G. Meurant (Paris), F. Rapetti (Nice), I.H. Sloan (Sydney), R.S. Womersley (Sydney)
- \* Member of the CAA: Padova-Verona Research Group on Constructive Approximation and Applications

### CONFERENCE ORGANIZATION AND EDITORIAL EXPERIENCE

- \* Member of the organizing committees of the Dolomites Workshops (DWCAA) and Research Weeks (DRWA) on Constructive Approximation and Applications, Alba di Canazei (Trento, Italy), since 2006
- \* Referee for "Numerische Mathematik", "IMA Journal Numerical Analysis", "Journal of Integral Equations and Applications", "Electronic Transactions in Numerical Analysis", "Computing", "Applied Numerical Mathematics", "Numerical Algorithms", "Journal Of Computational and Applied Mathematics".

## MAIN RESEARCH TOPICS

- \* Polynomial interpolation on multivariate domains.
- \* Polynomial cubature and hyperinterpolation on multivariate domains.
- \* RBF interpolation and cubature on multivariate domains.
- \* Computation of well-conditioned basis in multivariate domains.

### 1.10 Pubblicazioni scientifiche più significative del Responsabile Scientifico del Programma di Ricerca

- | n° | Pubblicazione  |
|----|--|
| 1. | BOS L, CALVI, J.P, LEVENBERG N, SOMMARIVA A, M. VIANELLO (2011). Geometric Weakly Admissible Meshes, Discrete Least Squares Approximations and Approximate Fekete Points. MATHEMATICS OF COMPUTATION, vol. 80, p. 1601-1621, ISSN: 0025-5718, doi: 10.1090/S0025-5718-2011-02442-7 |
| 2. | Matteo Briani, Alvisè Sommariva, Marco Vianello (2012). Computing Fekete and Lebesgue points: Simplex, square, disk. JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS, vol. 236, p. 2477-2486, ISSN: 0377-0427, doi: 10.1016/j.cam.2011.12.006 -Impact Factor .989                 |
| 3. | Gaspare Da Fies, Alvisè Sommariva, Marco Vianello (2013). Algebraic cubature by linear blending of elliptical arcs. APPLIED NUMERICAL MATHEMATICS, vol. 74, p. 49-61, ISSN: 0168-9274, doi: 10.1016/j.apnum.2013.08.003 -Impact Factor 1.152                                       |
| 4. | Alvisè Sommariva (2013). Fast construction of Fejér and Clenshaw-Curtis rules for general weight functions. COMPUTERS & MATHEMATICS WITH APPLICATIONS, vol. 65, p. 682-693, ISSN: 0898-1221, doi: 10.1016/j.camwa.2012.12.004 -Impact Factor 2.069                                 |
| 5. | M. CALIARI, S. DE MARCHI, A. SOMMARIVA, M. VIANELLO (2011). Padua2DM: fast interpolation and cubature at the Padua points in Matlab/Octave. NUMERICAL ALGORITHMS, vol. 56, p. 45-60, ISSN: 1017-1398, doi: 10.1007/s11075-010-9373-1 -Impact Factor 1.042                          |

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### 1.11 Componenti il Gruppo di Ricerca

#### 1.11.0 Professori e ricercatori anche a tempo determinato dell'Università di Padova

n°	Cognome	Nome	Dipartimento/Istituto	Area scientifica di ateneo	Qualifica	Settore	Mesi/Persona(*) Primo anno	Mesi/Persona(*) Secondo anno	Stato della risposta
1.	DE MARCHI	Stefano	DIP. MATEMATICA	01 - Mathematics	Professore Associato confermato	MAT/08	6	6	
2.	NOVATI	Paolo	DIP. MATEMATICA	01 - Mathematics	Ricercatore confermato	MAT/08	6	6	
3.	REDIVO ZAGLIA	Michela	DIP. MATEMATICA	01 - Mathematics	Professore Associato confermato	MAT/08	6	6	
4.	VIANELLO	Marco	DIP. MATEMATICA	01 - Mathematics	Professore Associato confermato	MAT/08	6	6	
5.	SOMMARIVA	Alvisè	DIP. MATEMATICA	01 - Mathematics	Ricercatore confermato	MAT/08	8	8	

#### 1.11.1 Professori a contratto di cui all'art. 23 della legge 240/2010, altro Personale dell'Università di Padova anche a tempo determinato (personale tecnico-amministrativo, Dirigenti e CEL)

n°	Nome	Dipartimento/Istituto	Qualifica	Mesi/Persona(*) Primo anno	Mesi/Persona(*) Secondo anno
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#### 1.11.2 Titolari di assegni di ricerca dell'Università di Padova

n°	Cognome	Nome	Dipartimento/Istituto	Area scientifica di ateneo	Mesi/Persona(*) Primo anno	Mesi/Persona(*) Secondo anno
1.	GAZZOLA	Silvia	DIP. MATEMATICA	01 - Scienze Matematiche	4	4

### 1.11.3 Studenti di Dottorato di Ricerca dell'Università di Padova

n°	Cognome	Nome	Dipartimento/Istituto	Area scientifica di ateneo	Qualifica	Mesi/Persona(*) Primo anno	Mesi/Persona(*) Secondo anno
1.	PIAZZON	Federico	DIP. MATEMATICA	01 - Scienze Matematiche	Dottorando	4	4
2.	POZZA	Stefano	DIP. MATEMATICA	01 - Scienze Matematiche	Dottorando	4	4
3.	SANTIN	Gabriele	DIP. MATEMATICA	01 - Scienze Matematiche	Dottorando	4	4

### 1.11.4 Professori, ricercatori anche a tempo determinato di altre Università

n°	Cognome	Nome	Università	Area scientifica di ateneo	Dipartimento/Istituto	Qualifica	Settore	Mesi/Persona(*) Primo anno	Mesi/Persona(*) Secondo anno
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### 1.11.5 Dipendenti di altre amministrazioni pubbliche, di enti pubblici o privati, di imprese, di istituzioni straniere, soggetti esterni in possesso di specifiche competenze nel campo della ricerca

n°	Cognome	Nome	Ente	Qualifica	Mesi/Persona(*) Primo anno	Mesi/Persona(*) Secondo anno
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### 2.1.0 Pubblicazioni scientifiche più significative dei componenti il gruppo di ricerca (docenti dell'ateneo di Padova)

n°	Pubblicazioni
1.	Bos Len, S. DE MARCHI, Hormann Kai, Klein Georges (2012). On the Lebesgue constant of barycentric rational interpolation at equidistant nodes. NUMERISCHE MATHEMATIK, vol. 121, p. 461-471, ISSN: 0029-599X, doi: 10.1007/s00211-011-0442-8 Impact factor 1.329
2.	Stefano De Marchi, Gabriele Santin (2013). A new stable basis for radial basis function interpolation. JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS, vol. 253, p. 1-13, ISSN: 0377-0427
3.	L. Bos, S. De Marchi, K. Hormann, J. Sidon (2013). Bounding the Lebesgue constant of Berrut's rational interpolant at general nodes. JOURNAL OF APPROXIMATION THEORY, vol. 169, p. 7-22, ISSN: 0021-9045
4.	BOS L, CALVI, J.P, LEVENBERG N, SOMMARIVA A, M. VIANELLO (2011). Geometric Weakly Admissible Meshes, Discrete Least Squares Approximations and Approximate Fekete Points. MATHEMATICS OF COMPUTATION, vol. 80, p. 1601-1621, ISSN: 0025-5718, doi: 10.1090/S0025-5718-2011-02442-7
5.	F. Piazzon, M. Vianello (2013). Small perturbations of polynomial meshes. APPLICABLE ANALYSIS, vol. 92, p. 1063-1073, ISSN: 0003-6811, doi: 10.1080/00036811.2011.649730
6.	L. Bos, M. Vianello (2012). Subperiodic trigonometric interpolation and quadrature. APPLIED MATHEMATICS AND COMPUTATION, vol. 218, p. 10630-10638, ISSN: 0096-3003, doi: 10.1016/j.amc.2012.04.024 Impact factor 1.349
7.	Claude Brezinski, Michela Redivo-Zaglia (2013). Interpolation, approximation et extrapolation rationnelles. In: Mathématiques pour l'ingénieur: Méthodes numériques. vol. AF 1 390, p. 1-9, Paris:Editions Techniques de l'ingénieur, ISBN: 9782850596070
8.	BREZINSKI C, HE Y, HU X-B, M. REDIVO ZAGLIA, SUN J-Q (2012). Multistep epsilon-algorithm, Shanks' transformation, and the Lotka-Volterra system by Hirota's method. MATHEMATICS OF COMPUTATION, vol. 81, p. 1527-1549, ISSN: 0025-5718, doi: 10.1090/S0025-5718-2011-02554-8 Impact factor 1.366
9.	BREZINSKI C, REDIVO ZAGLIA M (2013). Padé-type rational and barycentric interpolation. NUMERISCHE MATHEMATIK, vol. 125, p. 89-113, ISSN: 0029-599X, doi: 10.1007/s00211-013-0535-7
10.	S. Gazzola, P. Novati, M.R. Russo (2014). Embedded techniques for choosing the parameter in Tikhonov regularization. NUMERICAL LINEAR ALGEBRA WITH APPLICATIONS, ISSN: 1070-5325, doi: 10.1002/nla.1934
11.	Silvia Gazzola, Paolo Novati (2014). Automatic parameter setting for Arnoldi-Tikhonov methods. JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS, vol. 256, p. 180-195, ISSN: 0377-0427, doi: 10.1016/j.cam.2013.07.023
12.	Paolo Novati, Maria Rosaria Russo (2014). A GCV based Arnoldi-Tikhonov regularization method. BIT, ISSN: 0006-3835, doi: 10.1007/s10543-013-0447-z

## 2.1.1 Pubblicazioni scientifiche più significative dei componenti il gruppo di ricerca (altri partecipanti al progetto)

1. S. Gazzola, P. Novati, M. R. Russo. Embedded techniques for choosing the parameter in Tikhonov regularization. Numer. Linear Algebra Appl., in stampa.
2. S. Gazzola, J. Nagy. Generalized Arnoldi-Tikhonov method for sparse reconstruction. SIAM J. Sci. Comput., in stampa.
3. S. Gazzola, P. Novati. Automatic parameter setting for Arnoldi-Tikhonov methods. J. Comput. Appl. Math., 256:180--195, 2014.

## 2.2 Curriculum scientifico dei Componenti il Gruppo di Ricerca

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Stefano De Marchi  
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\* born in Candiana (Italy) on December 17, 1962

### SCIENTIFIC CURRICULUM

- \* Laurea Degree in Mathematics at the Univ. of Padua in 1987
- \* Ph.D. in Computational Mathematics and Computer Science, Padova, 1994
- \* Assistant professor in Numerical Analysis (1995-2005)
- \* Associate professor in Numerical Analysis (2005-)

### RESEARCH RESULTS

- \* 53 papers on referred journals, 14 papers on referred proceedings, 4 monographs
- \* Supervisor of 1 post-doc fellowships and 1 Ph.D. student

For a complete list of publications, reprints, software and for other details see the web page: [www.math.unipd.it/~demarchi](http://www.math.unipd.it/~demarchi)

### RESEARCH COLLABORATION AND COORDINATION EXPERIENCE

- \* 10 degree theses in Mathematics (Univ. of Udine, Verona and Padova).
- \* 1 degree thesis in Computer Science (Univ. of Verona).
- \* 1 master's thesis in Applied Mathematics (Univ. of Padova).
- \* co-supervisor of 3 degree theses in Mathematics (Univ. of Verona, Padova and Calabria).
- \* Founder of the CAA: Padova-Verona Research Group on Constructive Approximation and Applications
- \* Visiting professor in many Universities of Europe and USA
- \* Coordinator of a INdAM-GNCS project on "Multivariate approximation by polynomial and radial bases" (2010, funded with 5K euros)
- \* Coordinator of a "Progetto di Ateneo" project on "Multivariate approximation with applications to image reconstruction" (2013-14, funded with 28K euros)

### CONFERENCE ORGANIZATION AND EDITORIAL EXPERIENCE

- \* Organizer and member of the scientific committee of the "Dolomites Workshop on Constructive Approximation and Applications", in 2006, 2009 and 2012 and of the "Dolomites Research Week on Approximation", in 2007, 2008, 2010, 2011, 2013 and 2014.
- \* Managing editor of the electronic journal Dolomites Research Notes on Approximation
- \* Guest Editor of the Proceedings of DWCAA06 and DWCAA09, "Kernel Functions and Meshless Methods", special issue of DRNA Vol. 4, 2011.
- \* Referee for: SIAM Matrix Analysis and Applications, Journal of Approximation Theory, Advances in Computational Mathematics, Numerische Mathematik, Journal of Computational and Applied Mathematics, Numerical Algorithms, Proceedings A Royal Mathematical Society.
- \* Reviewer of the AMS Mathematical Review.

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Silvia Gazzola  
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\* born in Piacenza (Italy) on February 9, 1985

### SCIENTIFIC CURRICULUM

- \* B.S. in Mathematics at the Univ. of Parma in 2008
- \* M.S. in Mathematics at the Univ. of Parma in 2010
- \* Ph.D. in Mathematics, Padua, 2014
- \* Post-Doctoral Researcher in Mathematics (2014-)

### RESEARCH RESULTS

- \* 6 papers on referred journals, 1 submitted paper.
- \* SIAM UKIE Prize for the best student talk at the 25th Biennial Numerical Analysis Conference, University of Strathclyde (UK), 2013.

For a complete list of publications, reprints, software and for other details see the web page: [http://www.math.unipd.it/~gazzola/cvENG\\_2014.pdf](http://www.math.unipd.it/~gazzola/cvENG_2014.pdf)

### RESEARCH COLLABORATION AND COORDINATION EXPERIENCE

- \* Visiting Scholar at the Department of Mathematics and Computer Science, Emory University, Atlanta (USA); hosted by Prof. J. Nagy; October 18, 2013 - November 8, 2013.
- \* Visiting Scholar at the Department of Mathematics and Computer Science, Emory University, Atlanta (USA); hosted by Prof. J. Nagy; September 19, 2012 - January 29, 2013.

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Paolo Novati  
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\* born in Gorizia (Italy) on December 13, 1971

### SCIENTIFIC CURRICULUM

- \* Laurea Degree in Applied Mathematics at the Univ. of Trieste in 1996 and his Ph.D. in Computational Mathematics in 2000, at the Univ. of Padova with the thesis "Polynomial methods for the computation of functions of large unsymmetric matrices" (Advisor: Prof. I. Moret, Univ. of Trieste; Reviewer: Prof. L. Dieci, Georgia State Univ., USA)
- \* Post-doc position at the Univ. of Trieste.
- \* Assistant Professor of Numerical Analysis at the Univ. of L'Aquila (December 2001-December 2009).

- \* Assistant Professor of Numerical Analysis at the Univ. of Padua (December 2009-).
- \* Teaching Assistant of Analysis, Linear Algebra, Geometry at the Nettuno Univ., Rome (2003-2010).

#### RESEARCH RESULTS

- \* author of about 30 papers in several high qualified international journals
- \* B.Sc., M.Sc. and Ph.D. thesis supervisor
- \* member of the scientific board of the Ph.D. in Computational Mathematics of the Univ. of Padova (2010-)
- \* member of the Selection Committee for the Ph.D. program above and for other Research Fellowships.

For a complete list of publications, reprints, software and for other details see the web page: <http://www.math.unipd.it/~novati/>

#### RESEARCH COLLABORATION AND COORDINATION EXPERIENCE

- \* participant to various national research projects such as PRIN and GNCS
- \* scientific responsible of a two-year (2010-2011) University Research Project (University of Padova); of a research program for a 3-year (2011-2013) Ph.D. position (Univ. of Padova); of a GNCS research proposal (2013)
- \* currently responsible of a research proposal for a two-year (2013-2014) Post-doc position at the Univ. of Padova.

#### CONFERENCE ORGANIZATION AND EDITORIAL EXPERIENCE

- \* referee in several qualified journals

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 Michela Redivo Zaglia  
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- \* born in Padua (I), April 19, 1953

#### SCIENTIFIC CURRICULUM

- \* Master in Mathematics, July 1975 (Padua Univ., I)
- \* PhD degree, May 1992 (Univ. of Lille I, F)
- \* Employed at the Electronic Center of Scientific Computing (Padua Univ.) 1976-1984
- \* Head of the Computing Center (Dept of Electronics and Computer Sciences, Padua Univ.) 1984-1998
- \* Associate professor of Numerical Analysis at the Dept. of Mathematics (Calabria Univ. 1998-2002, Padua Univ. 2002-)
- \* National scientific qualification as full professor of Numerical Analysis, Dec 2013

#### MAIN RESEARCH TOPICS

- \* Biorthogonality and formal orthogonal polynomials, Extrapolation methods, Rational interpolation and approximation, Numerical linear algebra, Numerical Software.

#### RESEARCH RESULTS

- \* Author or coauthor of about 60 papers and 7 books. Another book in preparation.
- \* Invited Conferences: 11 (3 as Plenary speaker)
- \* Invited Seminars: 35
- \* Short-term invitations to Italian and foreign countries: 24
- \* Long-term invitations to foreign countries (visiting professor or senior research fellow): 14
- \* Contributed presentations: 50

#### RESEARCH COLLABORATION AND COORDINATION EXPERIENCE

- \* Vice-Director of the Dept of Mathematics (Univ. of Padova 2008-2011)
- \* Responsibility in Dept and Doctoral School: 8
- \* PhD thesis committees: 11
- \* Committees for scientific positions: 8
- \* Scientific Responsible at the Padua Univ. of scientific project (2006-2008), of 2-year post-doc research grants (2011, 2014) and of a 3-year Cariparo scholarship (2011) for the Doctoral School.
- \* Participant in scientific projects: 15
- \* PhD students supervision: 2

#### CONFERENCE ORGANIZATION AND EDITORIAL EXPERIENCE

- \* Member of the Scientific and Organizing Committee of 16 Conferences and Editor of 12 conference proceedings
- \* Member of the editorial board of 4 international journals: Int. J. of Appl. Math. and Eng. Sciences (2006-), J. of Appl. Math., (2000-2012) Numerical Algorithms (also Software Editor 1991-), DRNA (2008-)

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 Marco Vianello  
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- \* born in Venice on October 26, 1961

#### SCIENTIFIC CURRICULUM

- \* Laurea in Mathematics, 1987 at the Univ. of Padova
- \* Ph.D. in Computational Mathematics, 1992 (university consortium: Bologna-Padova-Trieste-Udine)
- \* INdAM research fellowship (1991-1993)
- \* Lecturer of Numerical Analysis at the Dept. of Mathematics of the Univ. of Padova (1993-2000)
- \* Associate professor of Numerical Analysis at the Dept. of Mathematics of the Univ. of Padova (2000-)
- \* National scientific qualification as full professor of Numerical Analysis, December 2013

#### RESEARCH RESULTS

- \* Author of more than 100 scientific papers in the fields of Numerical Analysis, Approximation Theory and Mathematical Analysis, published in 44 different international journals and book series with 24 coauthors

For a complete list of publications, reprints, software and for other details see the web page: <http://www.math.unipd.it/~marcov/>

## RESEARCH COLLABORATION AND COORDINATION EXPERIENCE

- \* Advisor of 3 Ph.D.s and of 3 post-docs in Computational Mathematics
- \* International collaborations: J.-P. Calvi (Toulouse), W. Gautschi (Purdue), N. Levenberg (Bloomington), F. Rapetti (Nice), S. Waldron (Auckland), Y. Xu (Eugene)
- \* Local coordinator of a national project PRIN 2003 on "Approximation of matrix functions in the numerical solution of differential equations" (2004-2005, grant: 28.6K euros)
- \* Member of the CAA: Padova-Verona Research Group on Constructive Approximation and Applications
- \* Member of the Council of the Ph.D. program in Computational Mathematics, Univ. of Padova (2005-2010)

## CONFERENCE ORGANIZATION AND EDITORIAL EXPERIENCE

- \* Member of the organizing and scientific committees of the Dolomites Workshops (DWCAA) and Research Weeks (DRWA) on Constructive Approximation and Applications, Alba di Canazei (Trento, Italy), since 2006
- \* Managing editor of the electronic journal Dolomites Research Notes on Approximation (DRNA)
- \* Guest Editor of the Proceedings of DWCAA06 and DWCAA09, special issues of Numerical Algorithms and Calcolo

## 2.3 Stato dell'Arte: base di partenza scientifica nazionale ed internazionale

Recently, there has been a renewed interest on multivariate polynomial interpolation and approximation in view of many open issues even for standard geometries, see e.g. [BBCL12]. Relevant steps have been made by hyperinterpolation [S95] and polynomial meshes [CL08].

Hyperinterpolation avoids the main difficulties of interpolation through cubature formulas and the availability of orthogonal basis, while polynomial meshes are near-optimal for least square polynomial approximation, and contain discrete extremal sets that are near-optimal for polynomial interpolation. They can be extracted by standard numerical linear algebra techniques on any compact, provided that a sufficiently well-conditioned polynomial basis is available [BCLNSV11], [BDMSV10], [SV09], often a challenging problem even on standard geometries [DX01]. A promising approach is to compute numerically discrete orthogonal polynomials w.r.t. suitable weighted sets but the possible loss of orthogonality asks for further studies [HL02], [VBC10], [VBHS14].

On hyperinterpolation, some new possibilities have been opened by subperiodic Gaussian trigonometric rules that allow to construct formulas exact on polynomials on many planar and surface sections or subregions of the sphere and of the torus [BV12], [DFSV13], [GSV14]. Again, the relevant orthogonal bases can be numerically computed and the node cardinality reduced [SV14].

The theory of Radial Basis Functions has been widely studied and applied in various fields of Mathematics.

In Approximation Theory, it gives rise to some methods which are emerging as promising and effective since they allow to interpolate functions using samples at scattered points, differently from polynomial interpolation. Moreover, the reconstructed function satisfies some optimality conditions among all the possible interpolants.

To be more precise, we can consider a bounded set  $S$  subset of  $\mathbb{R}^d$ ,  $d > 1$ , and a positive definite, symmetric and continuous kernel  $K: S \times S \rightarrow \mathbb{R}$ . Usually the kernel is radial, i.e. there exist a function  $\phi: [0,1] \rightarrow \mathbb{R}$  such that  $K(x,y) = \phi(c|x-y|)$ , for some shape parameter  $c > 0$ .

The interpolant is easily constructed by using the basis of translates  $T = \{K(\cdot, x_1), \dots, K(\cdot, x_n)\}$  s.t.  $\text{span}\{K(\cdot, x_1), \dots, K(\cdot, x_n)\}$  is a subset of the native space  $N_K$ .

Unfortunately this basis is known to be ill-conditioned.

Two main questions arise

- Are there better bases than  $T$  to produce a more stable reconstruction method?
- Are there optimal subspaces  $V_n$  of  $N_K$  so that the error functional is the smallest?

We have studied these questions in [DMS13, DMS14] but we wish to give more complete answers.

The image restoration problem is often formulated as a large linear system in which the coefficient matrix represents a blur/distorsion operator and is typically heavily ill-conditioned [CKN11], making necessary some sort of regularization to find a meaningful approximation of the solution [H98].

Due to the dimensions, the regularization technique must be taken with some care, in order to keep the computational work reasonably low w.r.t. the requirements of the applications. In this view, the regularization by Krylov methods, eventually coupled with the well-known Tikhonov regularization (hybrid methods), is commonly considered an effective approach for this kind of problems [CMRS00, CNO08, GN14, GNR14, OS81, LR09, NR14, NR14-2].

Alternatively one can use Kaczmarz's method [K37], [GBH70], applied nowadays in tomographic imaging. It is a particular case of row projection methods (suited for parallel computations and large-scale problems) as well as of the method of alternating projections (MAP) [ER11]. Its convergence is often slow and several methods for its acceleration have been proposed [BRZ13] and between them the topological epsilon-algorithms [B75] and those proposed in [BRZ14] seem to be the most appealing. They need less storage and stability is improved, allowing their use also in the case of large vectors or matrices.

## 2.4 Descrizione del Programma di Ricerca

### A) Conditioning in polynomial approximation theory.

The availability of well-conditioned (possibly orthogonal) polynomial bases has a fundamental role in solving many multivariate problems of approximation theory also allowing the construction of new spectral and high-order methods for PDEs on nonstandard geometries.

Often, orthogonal polynomials w.r.t. absolutely continuous measures are not known analytically, so orthogonal polynomials w.r.t. discrete measures are an appealing alternative. On the other hand, in order to have good mathematical properties the choice of the measure support is essential.

A possible idea is that of precomputing, for example on an admissible mesh [CL08, BDMSV11], a set of unisolvent discrete extremal points, e.g. Discrete Leja Points [BDMSV10], and then to orthogonalize a starting polynomial basis, w.r.t. the discrete probability measure with equal weights supported at the extracted points. This approach has a theoretical basis in the fact that such a probability measure converges weakly to the pluripotential equilibrium measure of the compact set [BDMSV10].

In such a way, we orthogonalize on a support with the least cardinality, that is the dimension of the polynomial space. The first computational results are promising but some of the linear algebra techniques that can be adopted suffer of loss of orthogonality at increasing degrees [HL02, VBC10], though the computed bases are much better conditioned than the original ones up to degrees in the order of 50-60. By suitable reorthogonalization steps, we are confident that orthogonality could be preserved up to such degrees and even beyond.

The idea to resort to orthogonality to cope ill-conditioning in multivariate polynomial basis, can be pursued also along other directions.

For example, we have recently constructed several positive algebraic cubature formulas on domains related to circular arcs, such as sections of the disk, the sphere and the torus, by resorting to "subperiodic" trigonometric approximation [DFSV13], [GSV14], [BV12].

Orthogonalizing polynomials of degree  $n$  with respect to the discrete measure generated by a positive cubature formula of exactness degree  $2n$ , gives an orthogonal basis for the underlying absolutely continuous measure, and opens the way to compute hyperinterpolation polynomials on these arc-related domains.

Subperiodic trigonometric orthogonality deserves itself some attention: once we had at hand an orthogonal trigonometric basis on a subinterval of the period, along with a subperiodic trigonometric Gaussian formula, we can compute discretized Fourier expansions on circular, spherical and toroidal sections that could have a computational role in geomathematics, in applications to local phenomena.

We have been collaborating on multivariate polynomial approximation for many years with A.Cuyt, L.Bos, J.P.Calvi, N.Levenberg and Y. Xu and recently started new collaborations with G. Meurant and M. Van Barel on the computation of discrete multivariate orthogonal polynomials. The funding of the present project would allow to establish a real collaboration by reciprocal visits to connect the different competences.

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## B) Conditioning in RBF theory.

It is often observed that interpolation based on translates of radial basis functions or non-radial kernels is numerically unstable due to exceedingly large condition number of the kernel matrix. But if stability is assessed in function space without considering special bases, we would like to prove that kernel-based interpolation is stable. Provided that the data are not too wildly scattered, the  $L_2$  or  $L_\infty$  norms of interpolants can be bounded above by discrete  $l_2$  and  $l_\infty$  norms of the data. Furthermore, Lagrange basis functions are uniformly bounded and Lebesgue constants grow at most like the square root of the number of data points. However, this analysis applies only to kernels of limited smoothness [DMS10].

Recently we have introduced a new orthonormal basis of the native space  $N$  of the kernel  $\Phi$ . The basis arises from a weighted singular value decomposition of the kernel matrix associated to the interpolation by radial kernels. As been also observed the basis is related to a discretization of the compact operator, self adjoint and trace-class  $T: N \rightarrow N$

$$T_\Phi(x) = \int_\Omega \Phi(x,y) f(y) dy,$$

for all  $x$  in  $\Omega$  subset of  $\mathbb{R}^d$ . Using the approximation of the eigenvalues of this operator, we have provided convergence estimates and stability bounds for interpolation and discrete least-squares approximation [DMS13]. The approximations are indeed the singular values of the WSVD. Recently we have found a way to speed-up the computation of these singular values, in order to compute the most significant. This is based on Krylov spaces and Arnoldi method, as outlined in [DMS14].

This first part of the proposed research will mainly focus on these aspects

- provide a better and tighter stability estimates;
- study the approximate Lebesgue constant based on the new basis;
- understand the stability of the fast algorithm for the construction of the new basis
- applications to medical image reconstruction and partition of unity of the new stable basis with fast algorithms.

Point d) is motivated by a few master's theses given recently whose results seems very promising.

We wish to collaborate with R.Schaback on a), b), c), G. Fasshauer on b) and c) and A.De Rossi, A.Iske on d).

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## C) Conditioning in image reconstruction.

The iterative approach is a well known strategy for solving numerically large-scale linear inverse problems, e.g. those stemming in image restoration. More precisely, one can employ a purely iterative method (in this case, the regularization is achieved by early termination of the iterations [H98]) or an hybrid scheme (in this case, at each iteration some additional direct regularization is imposed [CMRS00, OS81]). In this proposal we plan to focus the attention on Krylov type methods and to the possibility to use adapted acceleration and extrapolation methods.

In detail we intend to:

- gain some theoretical insight on the regularization properties of some Krylov subspace methods, based on the Arnoldi and Lanczos algorithms, similarly to what has already been done in [CLR02, GNR14, H95, NR14]. We also plan to study the use of some recent flexible, augmented and inexact variants [SS07] of the classical Krylov subspace methods for the solution of linear ill-posed problems.
- perform a deeper study of the parameter selection strategies. When considering hybrid schemes, two parameters have to be suitably and simultaneously chosen (number of iterations and amount of regularization). Some work in this direction has already been done in [CNO08, GN14, GNR14, NR14]. See also [BRZRS03].
- use extrapolation techniques to define the continuous regularization parameter whenever the iterative process shows an asymptotically stable behavior. Indeed, many experiments have revealed that some iterative processes such as the Arnoldi based methods typically allow to gain information about the noise level, since the residual stabilizes around it. In order to avoid the computation of many iterations we plan to use extrapolation to simulate this asymptotic behavior and then to approximate the noise level, that is required, for instance, when using the Discrepancy Principle. This approach was successfully already used with Tikhonov regularization [BRZRS98].
- propose new iterative methods for the solution of problems arising from non linear regularization, as for instance the Tikhonov-like formulation, whose regularization term is given by a non-differentiable functional such as the Total Variation or the  $l_1$ -norm. Moreover, in medical imaging, often the original problem to be solved is itself nonlinear [CKN11]; it is interesting to assess the performance of methods still based on Krylov subspaces, after the problem has been linearized



[GNa14].

5. since with the new algorithms of the simplified topological epsilon-transformations [BRZ14] it is possible to manage also large-scale linear and non-linear problems, we intend to accelerate the convergence of the sequences obtained with our iterative methods similarly to what we made with Kaczmarz's method [BRZ13].

6. use the above acceleration techniques with other row-projection iterative methods often used in image reconstruction: variants of Kaczmarz and Cimmino, Landweber, methods of the SIRT (Simultaneous Iterative Reconstruction Technique) class [G72], and so on.

We wish to continue our collaboration on all these topics with J.G. Nagy.

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## 2.5 Obiettivo del Programma di Ricerca ed indicazione dei risultati previsti alla fine del primo anno e a conclusione della ricerca

The proposed program has the following main goals.

### A) Conditioning in multivariate polynomial approximation

1. Introduce new methods for the computation of multivariate orthogonal polynomials with respect to a fixed scalar product;
2. Continue the study of multivariate polynomial meshes, with the intend of increasing the quality of the (quasi-)extremal point sets, e.g. almost-Lebesgue, almost-Fekete, Approximate Fekete Points, Discrete Leja Points, by the fundamental advantage of using a well-conditioned basis as orthogonal ones w.r.t. to a suitable scale product;
3. Introduce new cubature formulas so that, with multivariate orthogonal polynomials at hand, we can substitute the difficult computation of interpolants (due to possible ill-conditioning) with that of hyperinterpolants.

### B) Conditioning in RBF theory

In virtue of the results in [DMS13, DMS14] relatively to a new orthogonal basis of a native space, we intend to

1. provide a better and tighter stability estimates;
2. study the new distance for the power function;
3. build algorithms based on this "new" power function for constructing good sets of interpolation points;
4. applications of the new fast algorithms to medical image reconstruction and partition of unity methods.

### C) Conditioning in image reconstruction

1. gain some theoretical insight on the regularization properties of some Krylov subspace methods
2. study the use of some recent flexible, augmented and inexact variants of the classical Krylov subspace methods for the solution of linear ill-posed problems.
3. perform a deeper study of the parameter selection strategies.
4. use extrapolation techniques to define the continuous regularization parameter whenever the iterative process shows an asymptotically stable behavior.
5. propose new iterative methods for the solution of problems arising from non linear regularization, as for instance the Tikhonov-like formulation, whose regularization term is given by a non-differentiable functional such as the Total Variation or the 1-norm.
6. accelerate the convergence of the sequences obtained with our iterative methods similarly to what we made with Kaczmarz's method.
7. use the above acceleration techniques with other row-projection iterative methods often used in image reconstruction: variants of Kaczmarz and Cimmino, Landweber, methods of the SIRT (Simultaneous Iterative Reconstruction Technique) class [G72].

A brief tentative temporal subdivision of the two years research program follows:

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- numerical computation of well-conditioned multivariate polynomial basis; computation of new multivariate cubature rules;
- provide a better and tighter RBF stability estimates; study the approximate Lebesgue constant based on the new orthogonal basis;
- gain some theoretical insight on the regularization properties of some Krylov subspace methods; study the use of some recent flexible, augmented and inexact variants of the classical Krylov subspace methods; use extrapolation techniques to define the continuous regularization parameter whenever the iterative process shows an asymptotically stable behavior.

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- study of multivariate polynomial meshes for high polynomial degrees (using well-conditioned polynomial basis);
- understand the stability of the fast algorithm for the construction of the new basis; applications to medical image reconstruction of the new stable basis with fast algorithms.

- perform a deeper study of the parameter selection strategies. accelerate the convergence of the sequences obtained with our iterative methods similarly to what we made with Kaczmarz's method.

We will also extend the already available numerical software to improve the results obtained from the mathematical theory.

## 2.6 Elementi e modalità per la valutazione dei risultati finali

Our evaluation of the achieved results, will be mainly based in exposing our research in papers to well-known national and international journals and the publication of (free) numerical software that shows, whenever is necessary, the peculiarities of the developed mathematical theory. As done in the past, we will present our results to international workshops and conferences.

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## 3.0 Costo del Programma

Il finanziamento complessivo biennale, richiesto e assegnato, deve essere compreso fra il 50% e il 150% del costo medio dei PRAT finanziati negli ultimi due anni. Vedi Bando Tabella 1: Importo medio dei PRAT finanziati nell'ultimo biennio (periodo 2012 - 2013) Il costo per Assegni di Ricerca non può essere inferiore a Euro 23.333 per annualità di un assegno di ricerca.

### 3.1 Assegni di ricerca da attivare in questo Programma di Ricerca

n°	Attività specifica nel progetto e competenze	Durata complessiva	Costo assegno annuo (euro)	Costo totale (euro)
1.	Conditioning issues in multivariate approximation and image reconstruction	ANNUALE	23.500	23.500
	<b>TOTALE</b>			<b>23.500</b>

### 3.2 Richiesta di attrezzature di importo superiore a 5.000 Euro

n°	Descrizione attrezzatura da acquistare	Costo previsto (euro)
	<b>TOTALE</b>	<b>0</b>

### 3.3 Costo complessivo del Programma di Ricerca

	Descrizione	Costo totale (euro)
<b>Materiale inventariabile</b>	High performance computers.	1.500
<b>Materiale di consumo e funzionamento</b>	DRWA and DWCAA conference organization (see below).	1.000
<b>Congressi e missioni</b>	Organization of "Dolomites Research Week on Approximation" (DRWA) in 2015 and "4th Dolomites Workshop on Constructive Approximation and Applications" (DWCAA) in 2016. Missions abroad (conferences, visiting periods) of the local participants. Invitation to the non-local participants.	16.000
<b>Servizi esterni</b>		
<b>Assegni di ricerca</b>	(vedi punto 3.1)	23.500
<b>Attrezzature scientifiche di importo superiore a 5.000 Euro</b>	(vedi punto 3.3)	
<b>TOTALE</b>		<b>42.000</b>

Il presente progetto NON prevede sperimentazione animale

## SI DICHIARA INOLTRE QUANTO SEGUE:

1) È stata presentata richiesta di finanziamento, per lo stesso o analogo progetto, anche ad altro Ente, da parte del Responsabile o dei componenti il gruppo di ricerca:  
Se sì indicare:

NO

- a quale Ente:

NO

**2) La realizzazione del presente progetto sarà sovrapposta alla realizzazione di altri rilevanti progetti di ricerca:**

**Se sì, indicare quali:**

Per la copia da depositare presso l'Ateneo e per l'assenso alla elaborazione e diffusione delle informazioni riguardanti i programmi di ricerca presentati;  
decreto legislativo 196/03 sulla "Tutela dei dati personali".

Il Responsabile della Ricerca: .....

Padova li, 15/07/2014 13:38

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