Challenges of anisotropic refinement

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The compactly supported solutions (refinable functions) of the functional equation

$$\varphi(x) \;=\; \sum_{k\in\mathbb{Z}^s} c_k\, \varphi\,(Mx \,-\, k), \quad x\in\mathbb{R}^s, \quad c_k\in\mathbb{R}, \quad M\in\mathbb{Z}^{s\times s},$$

can generate systems of multivariate wavelets or frames. Refinable functions are building blocks for the limits of subdivision algorithms widely used in approximation and for generating curves and surfaces. Refinable functions also naturally appear in applications in probability, number theory, and combinatorics.
In the univariate case, $M \geq 2$ is an integer, there are several efficient methods for determining the regularity of refinable functions. The so-called *matrix approach* yields the Hölder exponent of $\varphi \in C(\mathbb{R})$ and, in addition, provides a detailed analysis of its moduli of continuity and of its local regularity. The generalization of the matrix approach to the multivariate case turned out to be a difficult task in the case of general dilation matrices M. The special case of isotropic dilation M (all eigenvalues of M are equal in the absolute value) is currently fully understood. In this talk we discuss the challenges of the anisotropic case, i.e. the dilation matrix M has eigenvalues different in the absolute value. We show how the Hölder exponent of $\varphi \in C(\mathbb{R}^s)$ reflects the influence of the invariant subspaces of M corresponding to its different in modulus eigenvalues.

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