

A simple multi-objective optimization problem



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modeFRONTIER
the multi-objective optimization and design environment

Introduction

Let's introduce a geometrical optimization problem, named **cones problem**, with the following characteristics:

- **multi-objective** problem (two objective functions): the solution is not a single optimum design, but instead it is represented by the set of designs belonging to the *Pareto frontier*
- **simple** mathematical formulation: easy and quick implementation from scratch of the relevant modeFRONTIER project
- **constrained** problem: objectives space and designs space present *feasible* and *unfeasible* regions



Problem definition

Right circular cone:

r = base radius

h = height

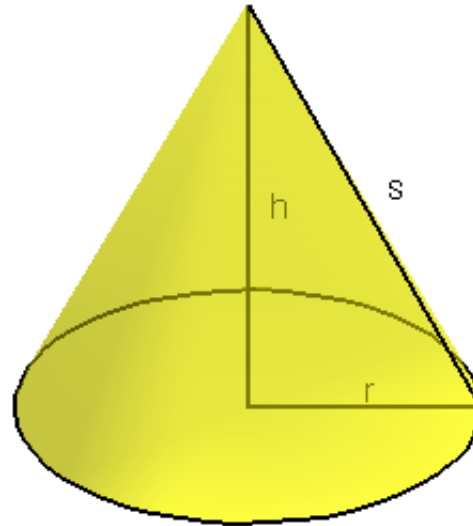
s = slant height

V = volume

B = base area

S = lateral surface area

T = total area



$$s = \sqrt{r^2 + h^2}$$

$$V = \frac{\pi}{3} r^2 h$$

$$B = \pi r^2$$

$$S = \pi r s$$

$$T = B + S = \pi r (r + s)$$



Cones problem

- two input variables: r , h

$$r \in [0, 10] \text{ cm} , \quad h \in [0, 20] \text{ cm}$$

The cone shape (i.e. the design) is defined univocally when both r and h are given.

- two objectives:

$$\min S$$

$$\min T$$

We want to minimize both the lateral surface area and the total surface area

- one constraint:

$$V > 200 \text{ cm}^3$$

A constraint for the cone volume is given, in order to guarantee a minimum volume.



Project building

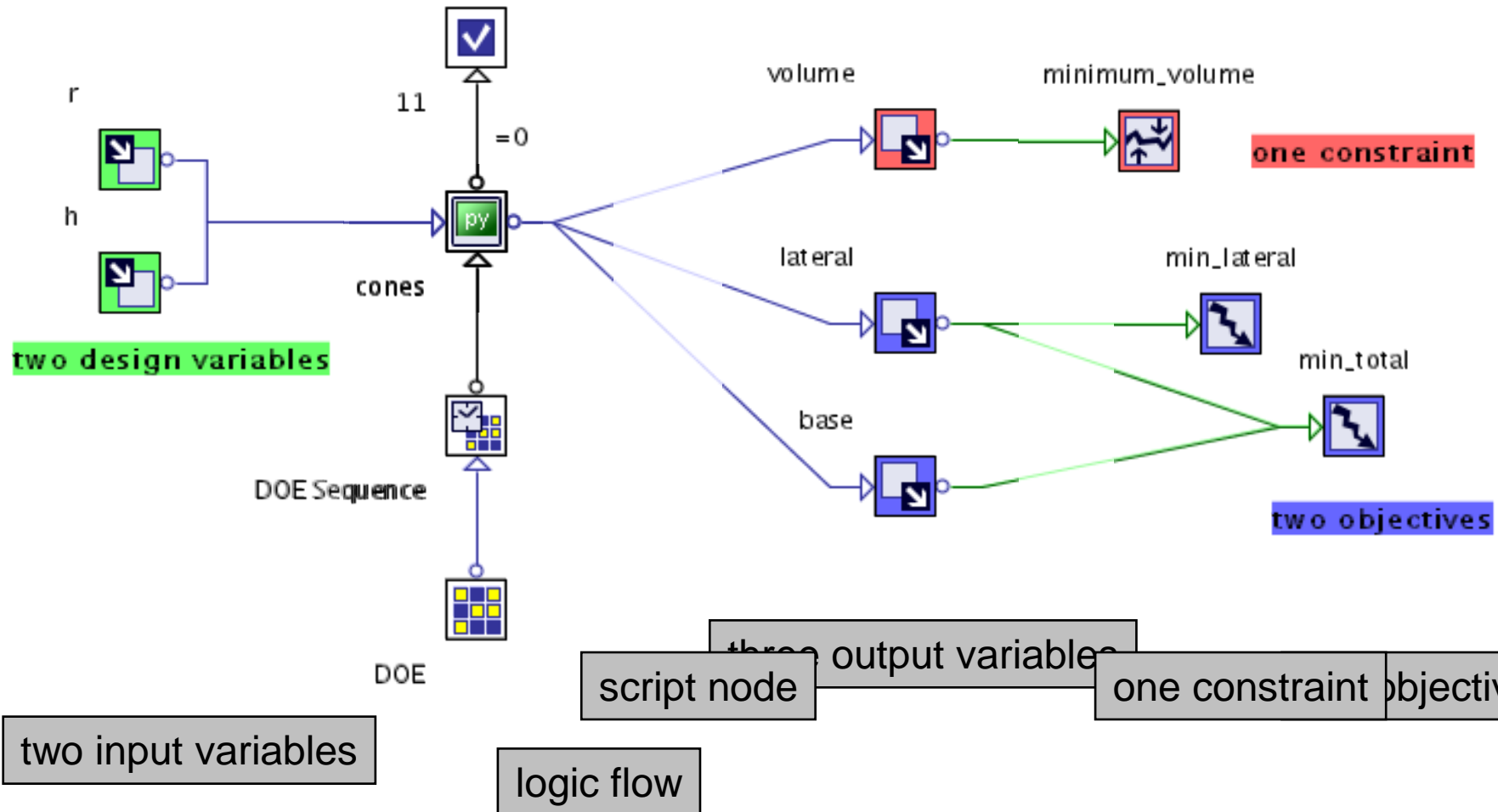
Let's build from scratch the pertinent modeFRONTIER project:

1. Work Flow setup: fill the work canvas with the project's building blocks
2. Script Node setup: use your favourite math tool
 - Jython script
 - Matlab node
 - Excel Workbook node
 - OpenOffice Spreadsheet node



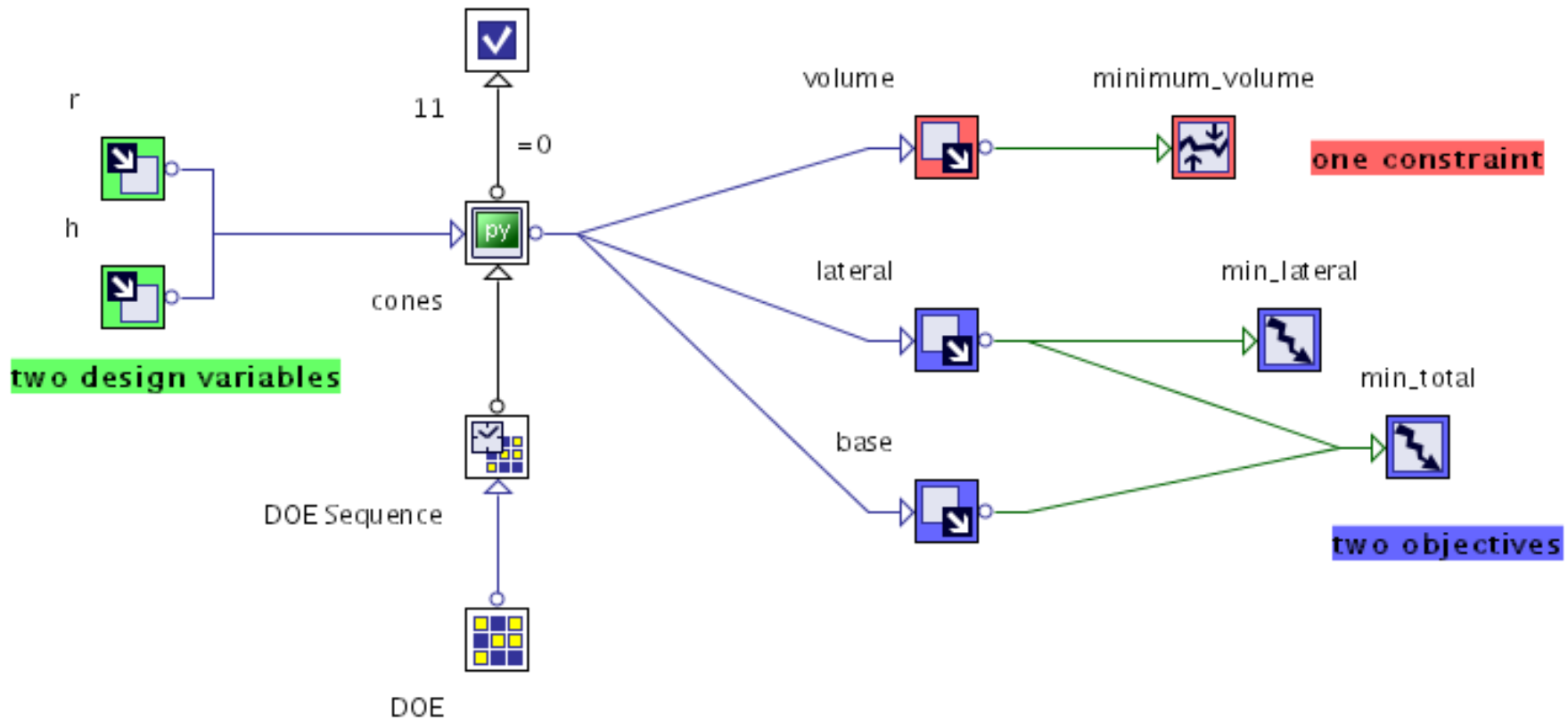
Work Flow setup

cones: two-objective optimization problem



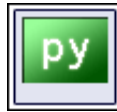
Work Flow setup

cones: two-objective optimization problem



Script node: Jython

Jython (Python) script case:



Write down the formulae

```
1 import math
2
3 volume = math.pi/3*r*r*h
4 s = math.sqrt(r*r+h*h)
5 lateral = math.pi*r*s
6 base = math.pi*r*r
7
```

$$V = \frac{\pi}{3} r^2 h$$

$$s = \sqrt{r^2 + h^2}$$

$$S = \pi r s$$

$$B = \pi r^2$$

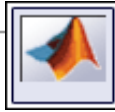
Load math module

Note the syntax of mathematical functions and constants



Script node: Matlab

Matlab case:



Write down the formulae

$$V = \frac{\pi}{3} r^2 h$$

$$s = \sqrt{r^2 + h^2}$$

$$S = \pi r s$$

$$B = \pi r^2$$

```
1 - volume = pi/3*r*r*h;  
2 - s = sqrt(r*r+h*h);  
3 - lateral = pi*r*s;  
4 - base = pi*r*r;  
5
```

The screenshot shows the Matlab Editor window with the title bar 'Editor - C:\users\enrico\projects\simple optim...'. The menu bar includes File, Edit, Text, Cell, Tools, Debug, Window, and Help. The toolbar contains various icons for file operations and execution. The script area shows five lines of code for calculating the volume, slant height, lateral surface area, and base area of a cone. The status bar at the bottom indicates 'script', 'Ln 5', 'Col 1', and 'OVR'.

The screenshot shows the 'Matlab Script Properties' dialog box. The 'Script' field contains 'cones' and the 'Description' field contains 'C:\users\enrico\projects\simple...'. The 'Process Input Connector' section has checkboxes for 'h', 'r', 'base', 'lateral', and 'volume', with 'h' and 'r' checked. The 'Process Output Connector' section has checkboxes for 'base', 'lateral', and 'volume', with 'base', 'lateral', and 'volume' checked. The 'Data Input Connector' section has checkboxes for 'h', 'r', 'base', 'lateral', and 'volume', with 'h' and 'r' checked. The 'Data Output Connector' section has checkboxes for 'base', 'lateral', and 'volume', with 'base', 'lateral', and 'volume' checked. The 'OK', 'Preview', 'Cancel', and 'Help' buttons are at the bottom.

The screenshot shows the 'modeFRONTIER Preferences' dialog box. The 'Components' list on the left includes 'General', 'Work Flow', 'External Applications', 'Matlab', 'Excel', 'Design Space', 'Run Logs', 'Print To File Settings', 'Print Settings', and 'AutoReport'. The 'General' component is selected, showing 'Matlab Version' as '7.0.x' and 'Matlab Max Reuse Count' as '100'. The 'Check Matlab version' text box is overlaid on the dialog.

Check Matlab version

Load the matlab file

Script node: Excel

Excel Workbook case:



Insert the formulae

Microsoft Excel - cones.xls

File Modifica Visualizza Inserisci Formato Strumenti Dati Finestra ? Adobe PDF

Arial 10 G C S

J6 fx

	A	B	C	D	E	F	G	H
1	Input variables			Output variables				
2	r	h	s	volume	lateral	base		
3	5	10	11.18034	261.7994	175.6204	78.53982		
4								

Sheet1 Sheet2 Sheet3

Pronto

$$=SQRT(A1*A1+B1*B1) \quad s = \sqrt{r^2 + h^2}$$

$$=PI()/3*A1*A1*B1 \quad V = \frac{\pi}{3} r^2 h$$

$$=PI()*A1*C1 \quad S = \pi r s$$

$$=PI()*A1*A1 \quad B = \pi r^2$$

Build the spreadsheet

Load the xls file

Link variables to cells

Excel workbook Properties

Name cones

Description

Workbook C:\users\henrico\projects\simple...

Macro Name

Excel Workbook Advanced Properties

Excel Default Cells Properties

Process Input Connector

Scheduler ☒

Data Input Connector

h	<input checked="" type="checkbox"/>	Cell Address...	Sheet1!B3
r	<input checked="" type="checkbox"/>	Cell Address...	Sheet1!A3
base	<input type="checkbox"/>	Cell Address...	
lateral	<input type="checkbox"/>	Cell Address...	
volume	<input type="checkbox"/>	Cell Address...	

Process Output Connector

EndOK11 ☒ Condition =0

Data Output Connector

base	<input checked="" type="checkbox"/>	Cell Address...	Sheet1!F3
lateral	<input checked="" type="checkbox"/>	Cell Address...	Sheet1!E3
volume	<input checked="" type="checkbox"/>	Cell Address...	Sheet1!D3

OK Preview Cancel Help



Script node: OpenOffice

OpenOffice Spreadsheet case:



Insert the formulae

cones.sxc - OpenOffice.org 1.1.0

File Edit View Insert Format Tools Data Window Help

//home/enrico/projects/simple_optimization/1

Bitstream Vera Sa 10 B i U A

A6

	A	B	C	D	E	F
1	Input variables		Output variables			
2	r	h	s	volume	lateral	base
3	5	10	11.18	261.8	175.62	78.54
4						

Sheet1 / Sheet2 / Sheet3

Sheet 1 / 3 Default 100% STD Su

$$=SQRT(A3*A3+B3*B3) \quad s = \sqrt{r^2 + h^2}$$

$$=PI()/3*A3*A3*B3 \quad V = \frac{\pi}{3} r^2 h$$

$$=PI()*A3*C3 \quad S = \pi r s$$

$$=PI()*A3*A3 \quad B = \pi r^2$$

Build the spreadsheet

Load the sxc file

Link variables to cells

OpenOffice Spreadsheet Properties

Properties

cones

/home/enrico/projects/simple...

Interactive Selection

OpenOffice Spreadsheet Advanced Properties

OpenOffice Default Cells Properties

Process Input Connector		Process Output Connector	
Scheduler	<input checked="" type="checkbox"/>	EndOK11	<input checked="" type="checkbox"/>
Condition	=0		
Data Input Connector		Data Output Connector	
h	<input checked="" type="checkbox"/> Cell Address Sheet1.B3	base	<input checked="" type="checkbox"/> Cell Address Sheet1.F3
r	<input checked="" type="checkbox"/> Cell Address Sheet1.A3	lateral	<input checked="" type="checkbox"/> Cell Address Sheet1.E3
base	<input type="checkbox"/> Cell Address	volume	<input checked="" type="checkbox"/> Cell Address Sheet1.D3
lateral	<input type="checkbox"/> Cell Address		
volume	<input type="checkbox"/> Cell Address		

OK Preview Cancel Help



Runs examples

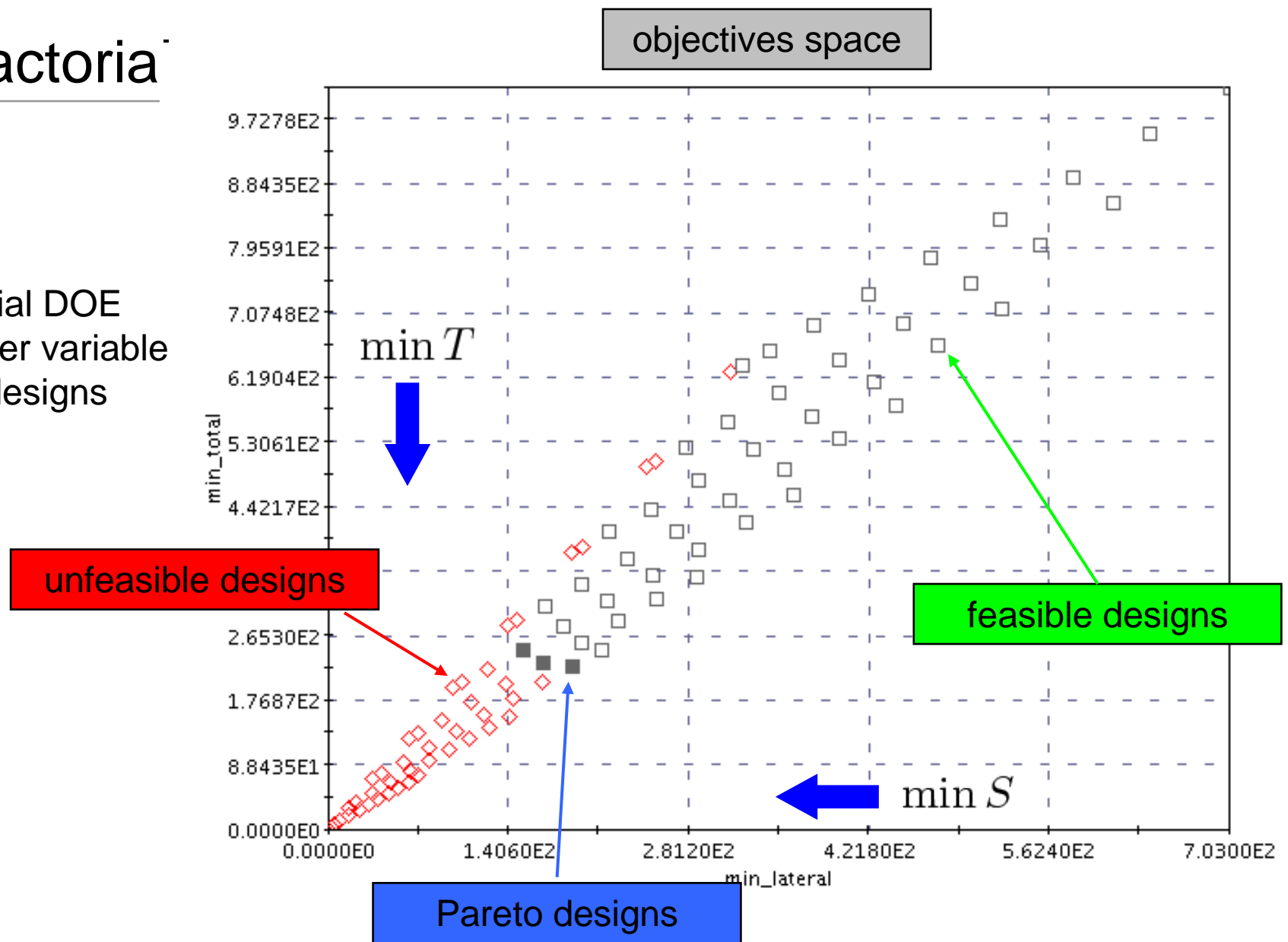
Let's see some examples of runs with different DOEs and/or schedulers:

- **Full Factorial DOE**
- random samplings: **Random Sequence** and **Sobol DOEs**
- genetic algorithms: **MOGA-II, NSGA-II**
- **MOSA**
- **NBI-NLPQLP**



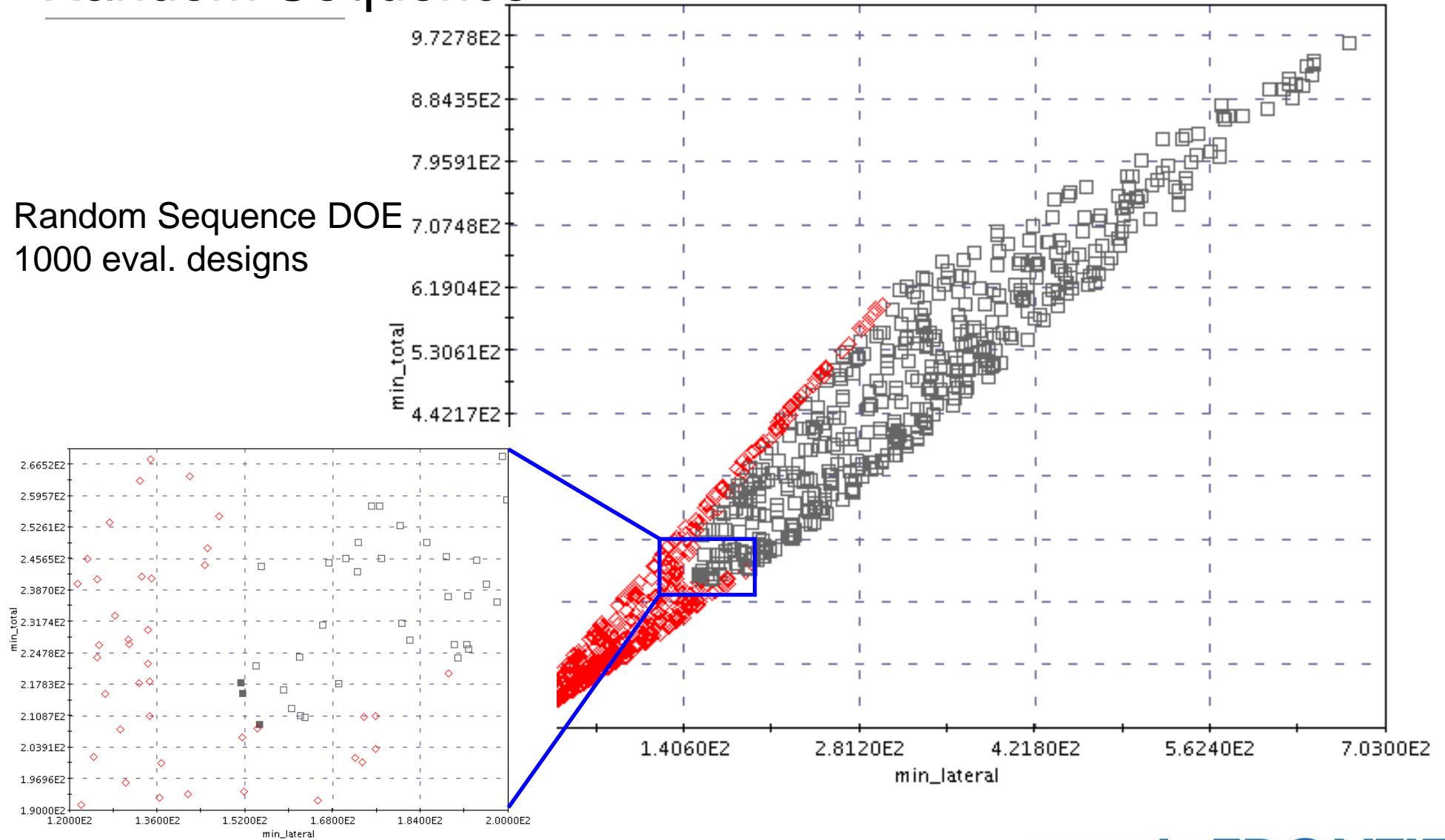
Full Factorial

Full Factorial DOE
10 levels per variable
100 eval. designs



Random Sequence

Random Sequence DOE
1000 eval. designs



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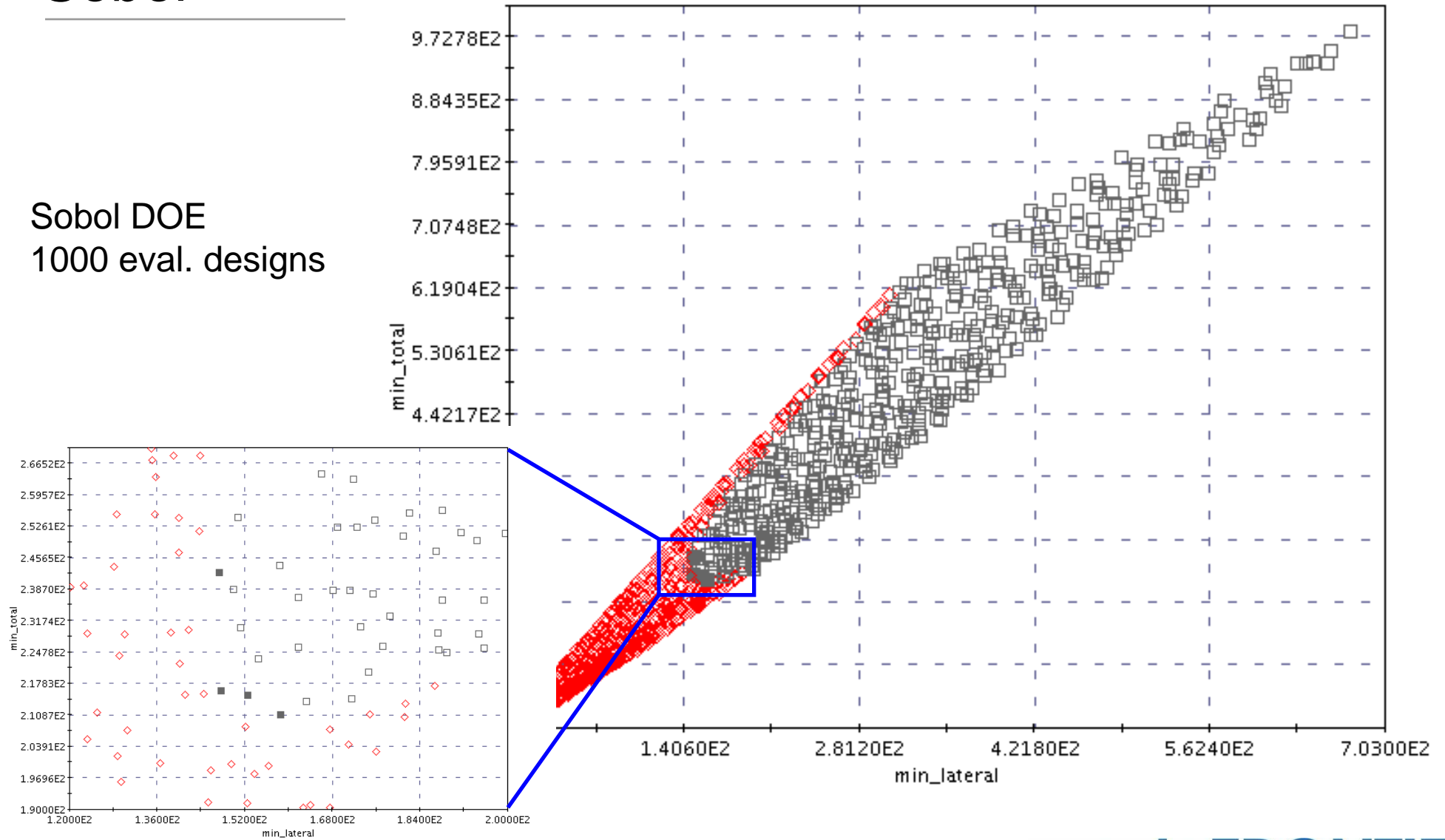


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Sobol

Sobol DOE
1000 eval. designs



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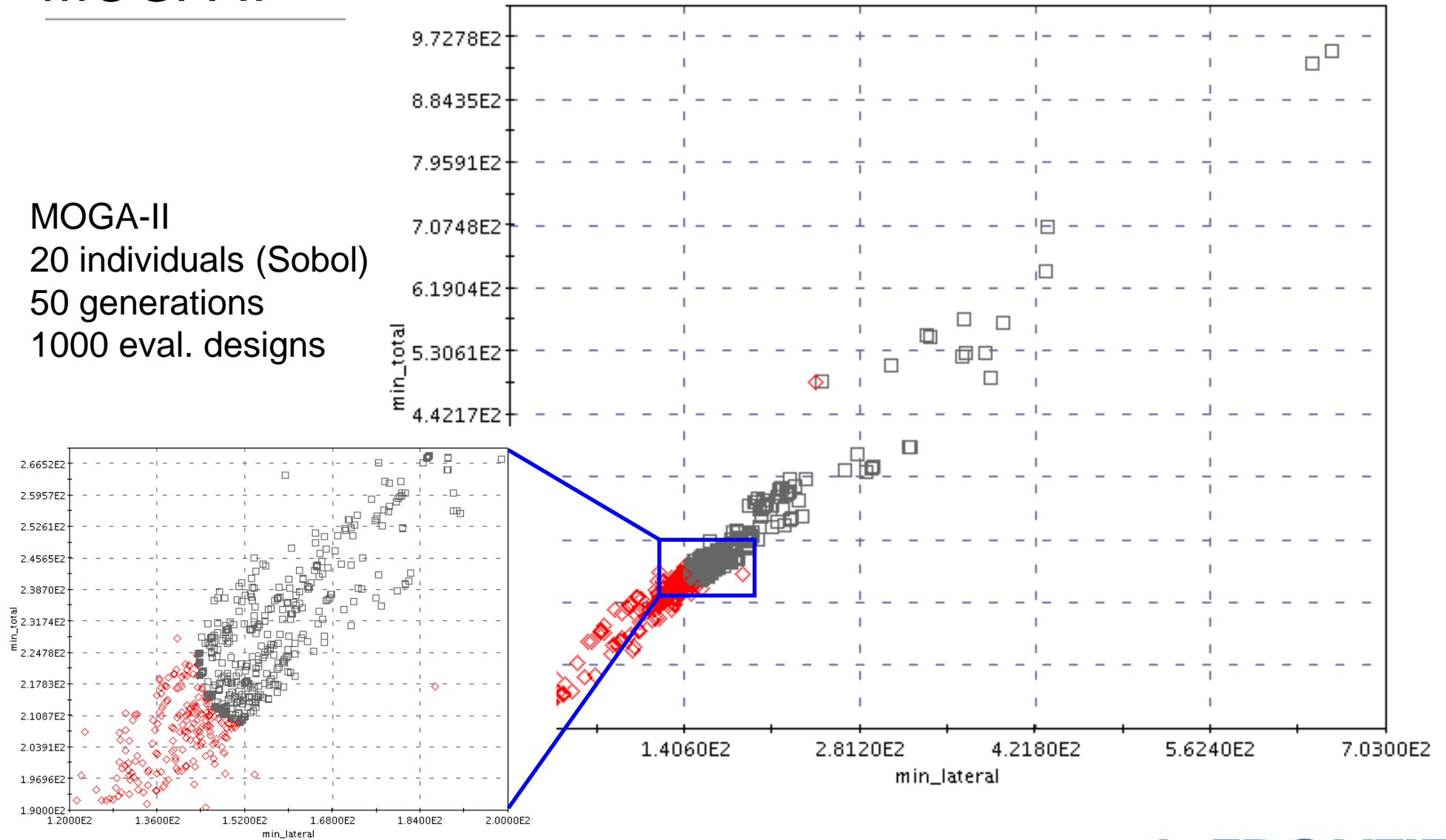


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MOGA-II

MOGA-II
20 individuals (Sobol)
50 generations
1000 eval. designs



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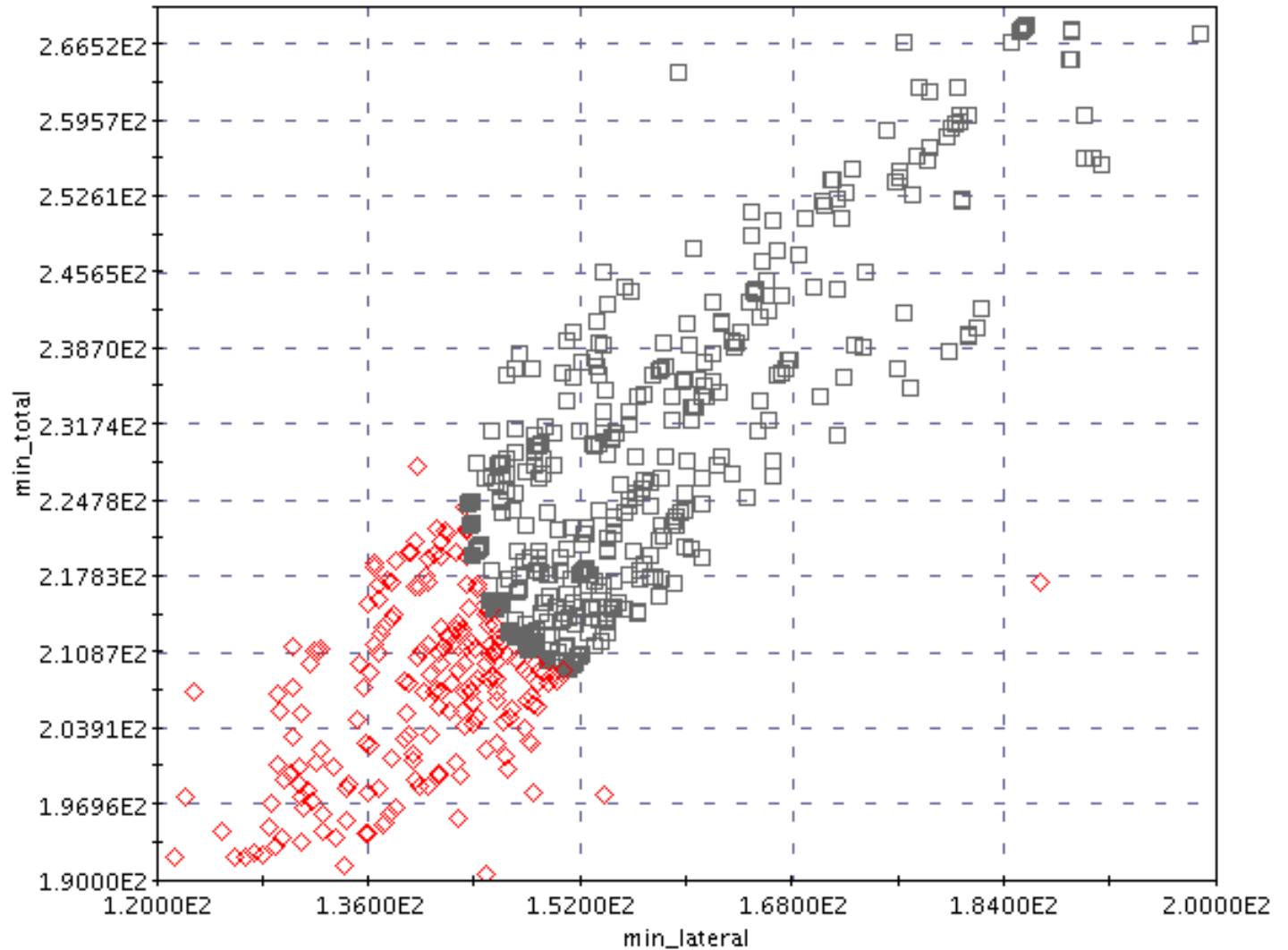


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MOGA-II

MOGA-II
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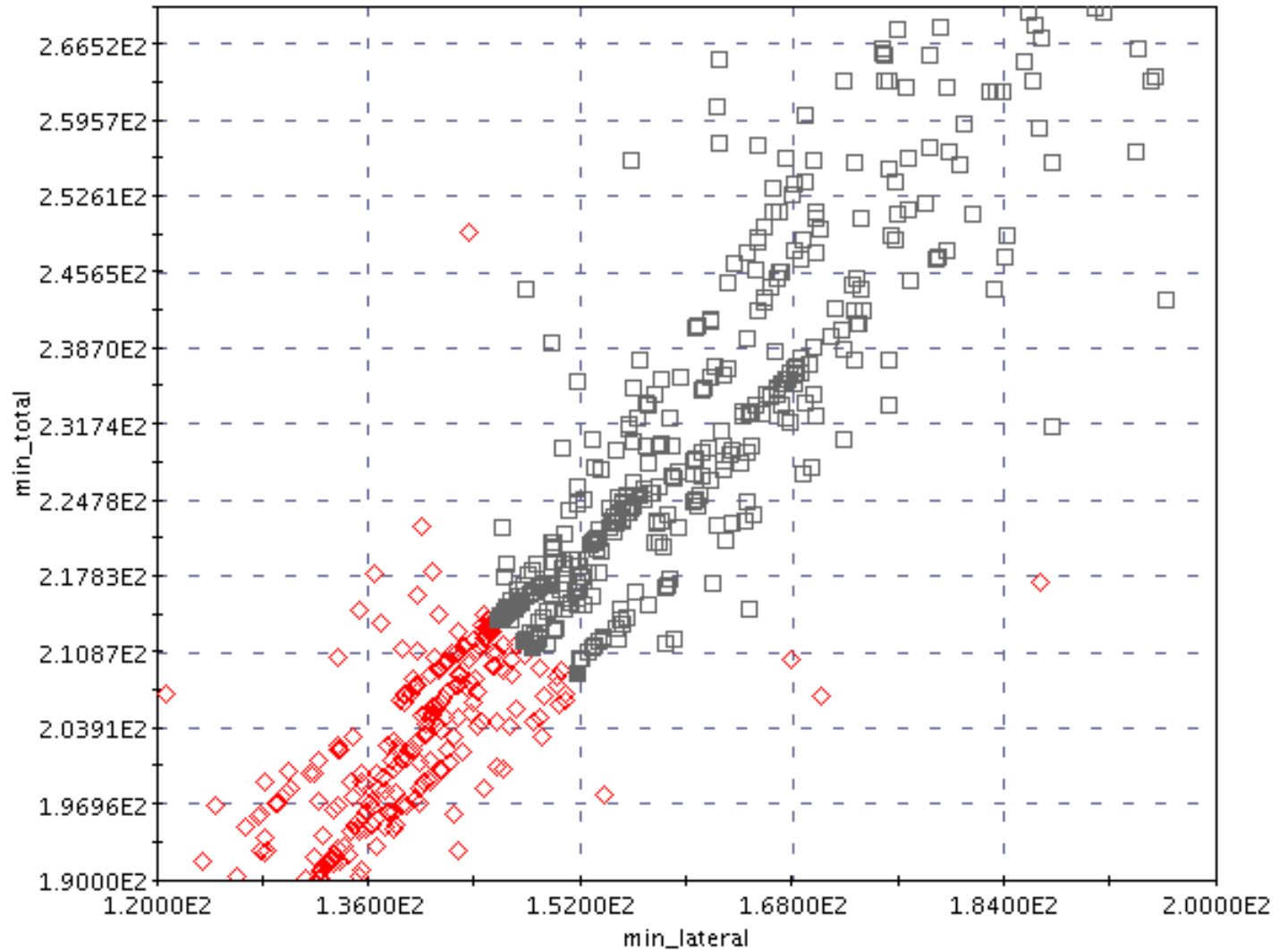


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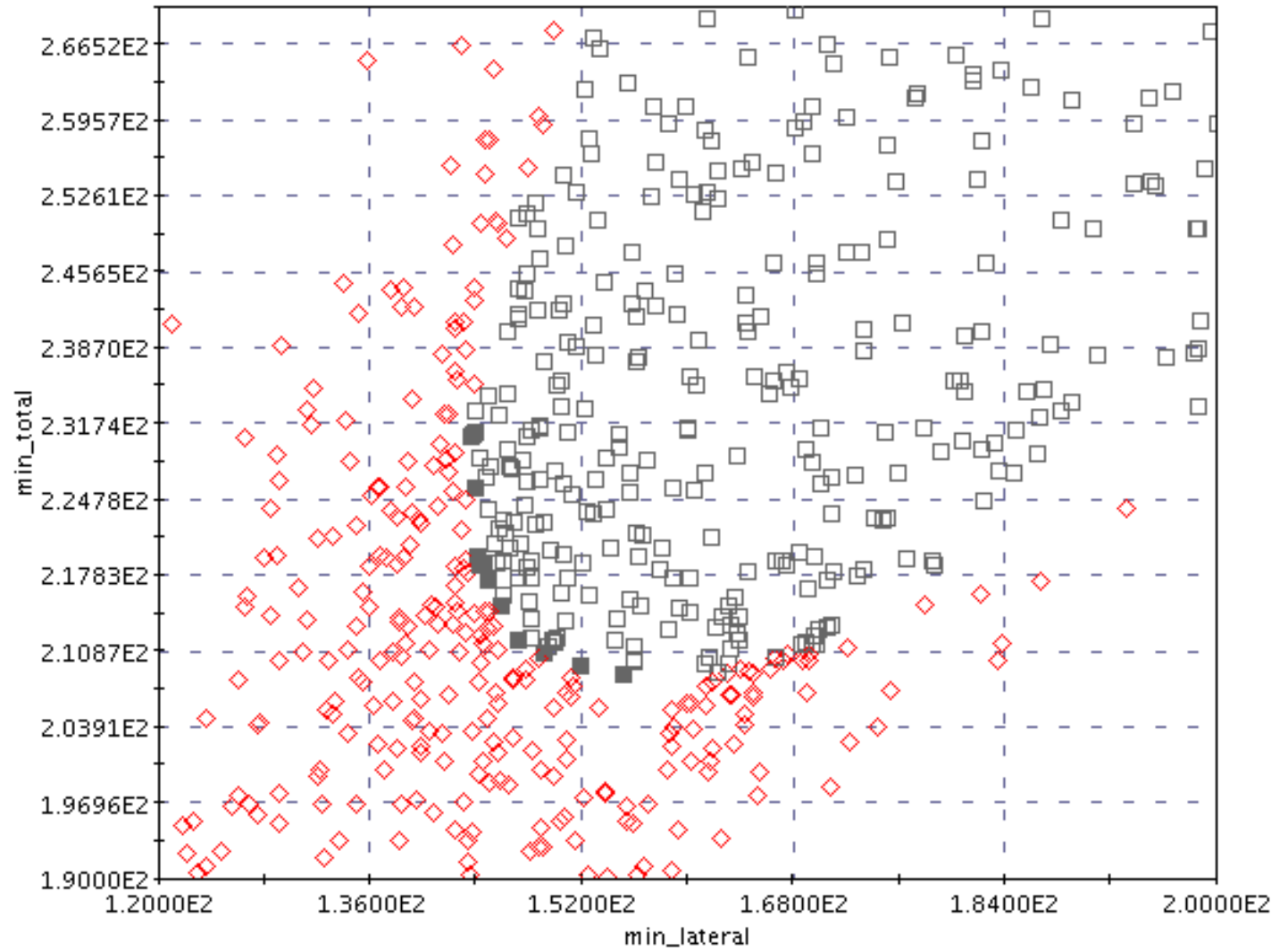
NSGA-II

NSGA-II
20 individuals (Sobol)
50 generations
1000 eval. designs



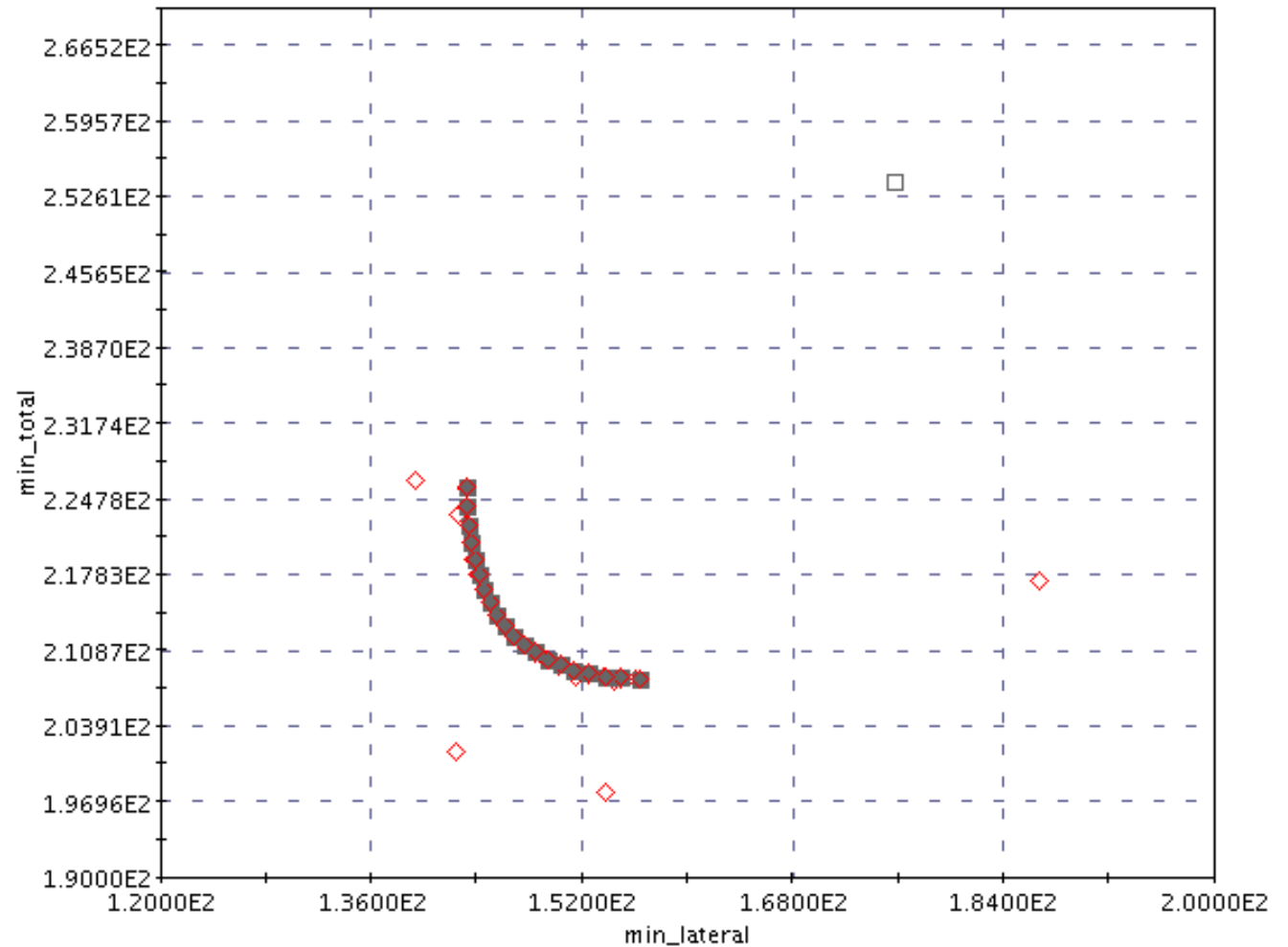
MOSA

MOSA
10 points (Sobol)
100 iterations
1000 eval. designs



NBI-NLPQLP

NBI-NLPQLP
(DOE: 10 Sobol)
20 NBI-subproblems
346 eval. designs



Final considerations

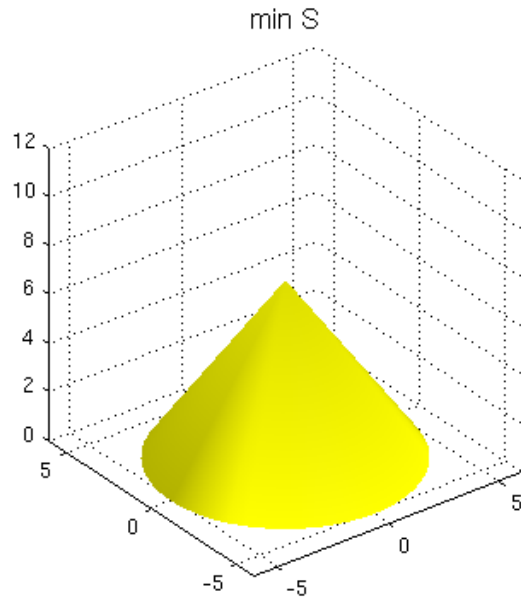
Let's consider the difference between

- single-objective problem solutions: two different minima
- multi-objective problem solutions: the Pareto frontier



Single-objectives minima

Each design represents the optimum solution for its corresponding single-objective problem.

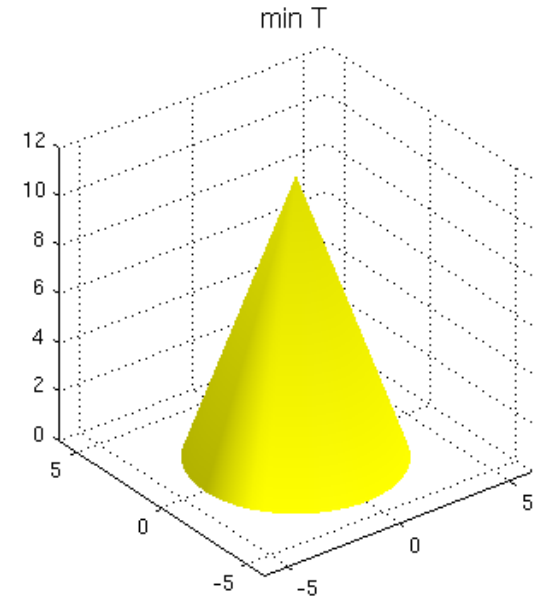


$$\begin{aligned}\min S \\ r &= 5.131 \text{ cm} \\ h &= 7.256 \text{ cm} \\ V &= 200 \text{ cm}^3 \\ S &= 143.23 \text{ cm}^2 \\ T &= 225.92 \text{ cm}^2\end{aligned}$$



...but what about the in between designs?

...we would like to get a compromise solution. A trade-off of the two objectives...



$$\begin{aligned}\min T \\ r &= 4.072 \text{ cm} \\ h &= 11.518 \text{ cm} \\ V &= 200 \text{ cm}^3 \\ S &= 156.28 \text{ cm}^2 \\ T &= 208.38 \text{ cm}^2\end{aligned}$$

What we want is the **Pareto frontier!**



The Pareto frontier

