Design of Experiment

For more information visit:
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What does DOE mean?

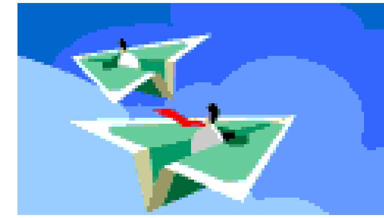
DOE stands for Design Of Experiments

DOE techniques are used to generate a series of designs which satisfy different requisites according to the objective of the analysis, which can be however always summarized as:

“have the best with the smallest effort”

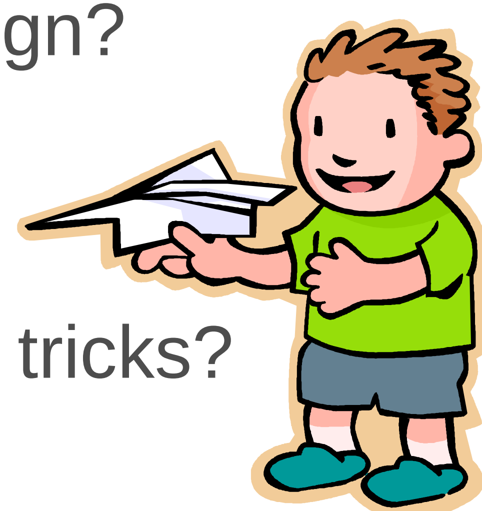
The main concern is to determine the relationship between factors (inputs) affecting a process and the output of that process with the lowest number of experiments as possible.

In other words….



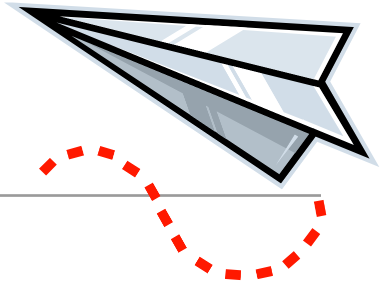
- Have you flown a paper airplane before? (Hopefully not in this class)

- Do you always use the same type of paper?
- Do you always use the same design?



- Do you want it to fly straight or do tricks?

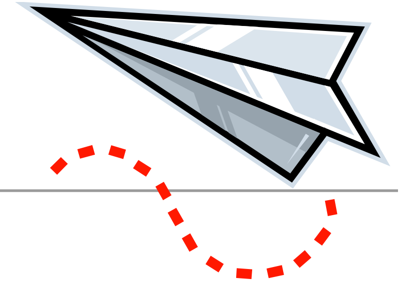
What does DOE mean in this case?



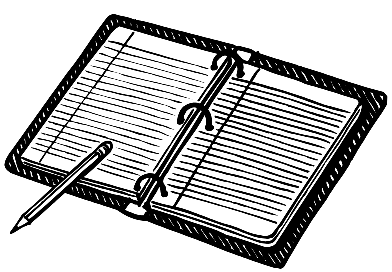
- Design of experiment is used here to test paper airplane flight distance
- We want the planes to fly as far as they can.
- We need to think about how we are going to design and perform the experiment.
- What things do we need to think about? (Think about the steps of the Scientific Method)

- What question are we trying to answer?
  - We want to design an experiment to test how the addition of paper clips will affect the flight distance of the paper airplane.
  - How does adding paper clips to a paper airplane affect its flight?

Procedure

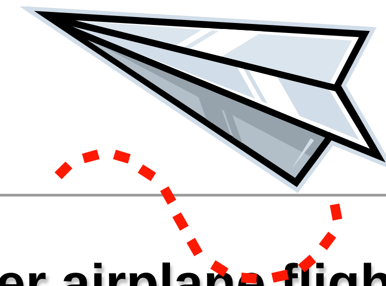


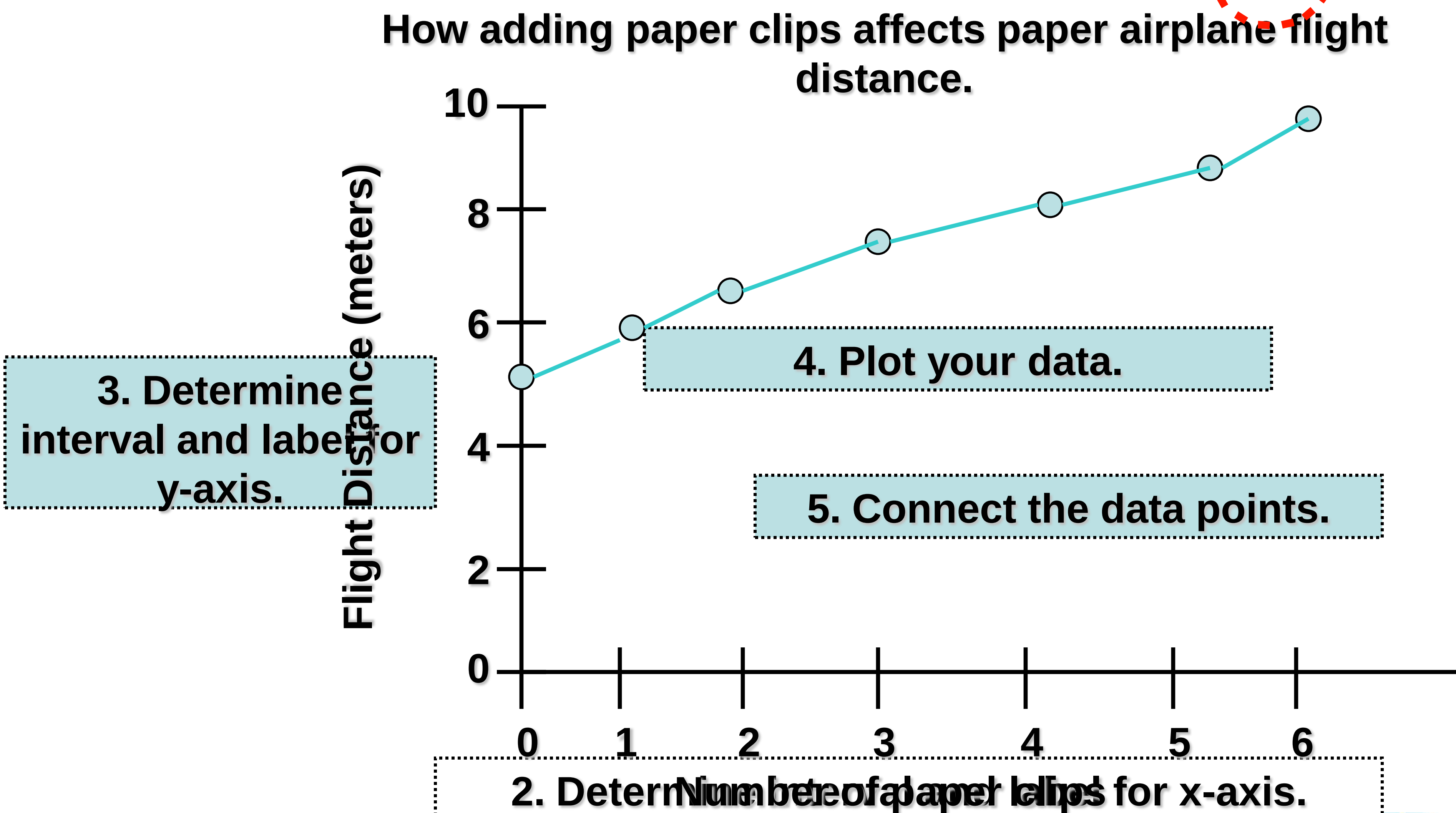
- How are we going to perform the experiment?
  - What do we need to do?
  - What needs to be kept constant?
  - What is our control?
  - Which are our independent variables?
  - What are we going to observe? How?



# of paper clips	Flight Distance (m)
0	
1	
2	
3	
4	
5	
6	

Making a graph of your data

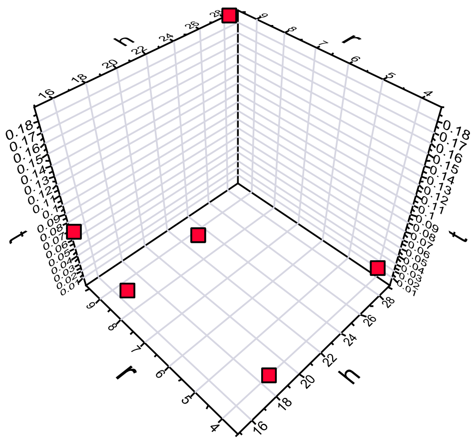


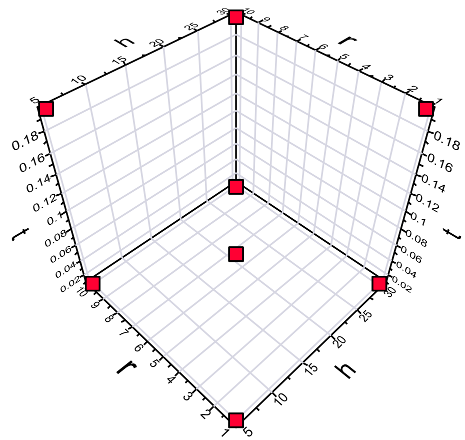


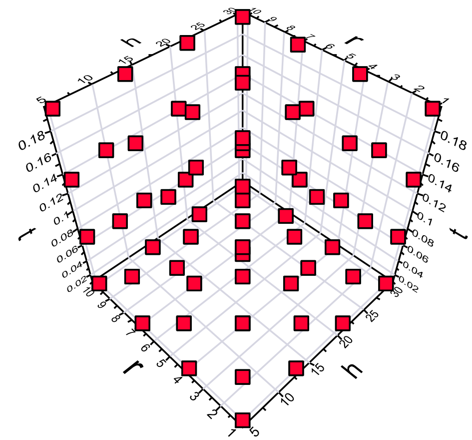
The choice on experiments

Aim of the Activity: have a good sample from laboratory tests for statistic study

Cost per Experiment: 1000 $







6 Random entries

Cost of the Campaign = 6,000 $

Cost

Quality

8 Full Factorial entries

Cost of the Campaign = 8,000 $

Cost

Quality

64 Full Factorial entries

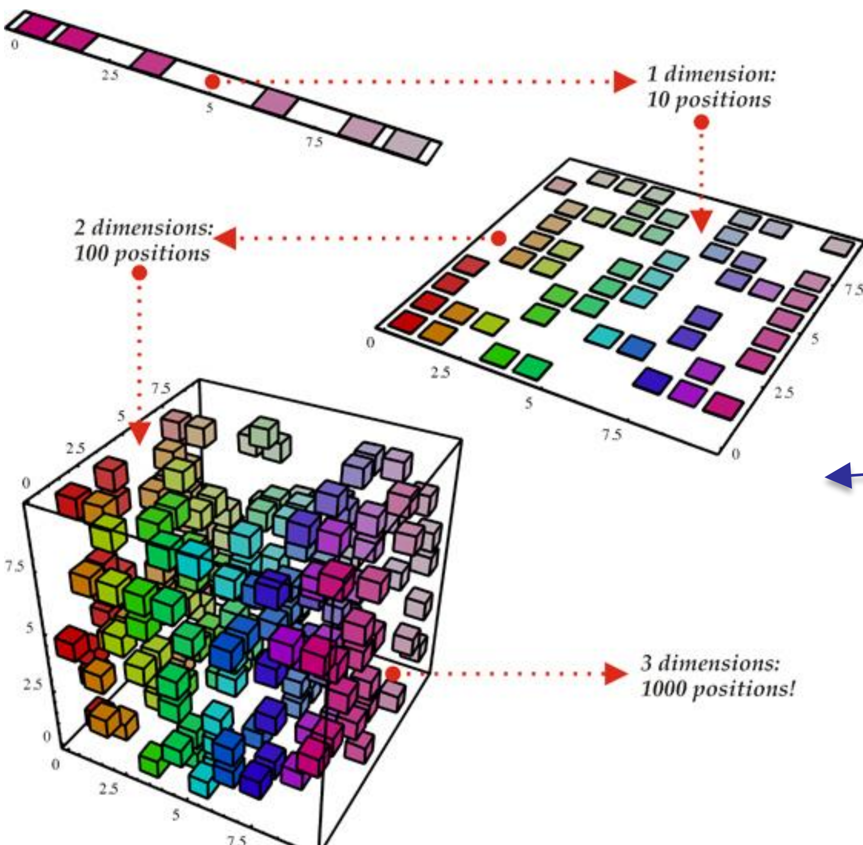
Cost of the Campaign = 64,000 $

Cost

Quality

Curse of dimensionality

- is the problem caused by the exponential increase in volume associated with adding extra dimensions to a space.



For example, 100 evenly-spaced sample points suffice to sample a unit interval with no more than 0.01 distance between points; an equivalent sampling of a 10-dimensional unit hypercube with a lattice with a spacing of 0.01 between adjacent points would require 10^{20} sample points.

Random & SOBOL

- The DOE Random & Sobol Sequences cover sufficiently the domain of the functions.

- The mathematical theory is the Random Number Generation.

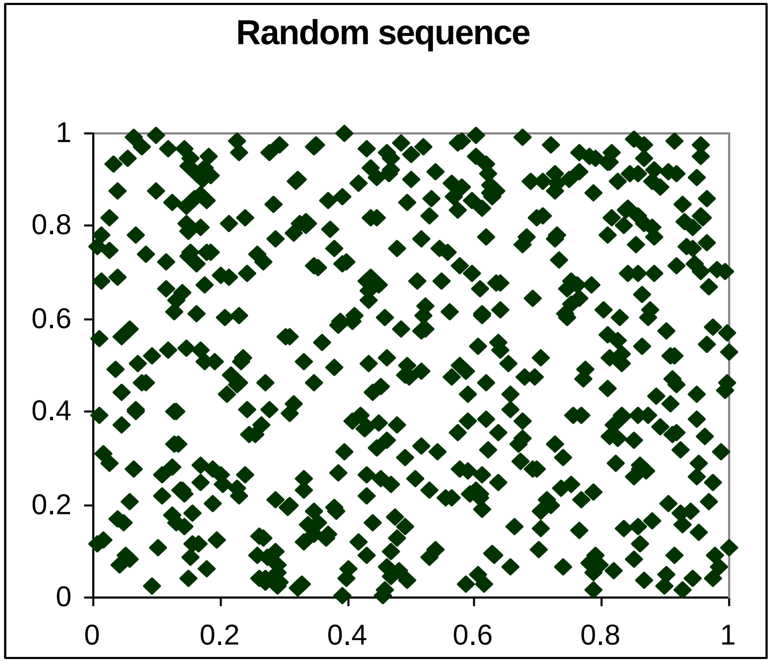
    - Sequence Random (function with “many” variables)
    - Sequence Sobol (function with “few” variables < 10)

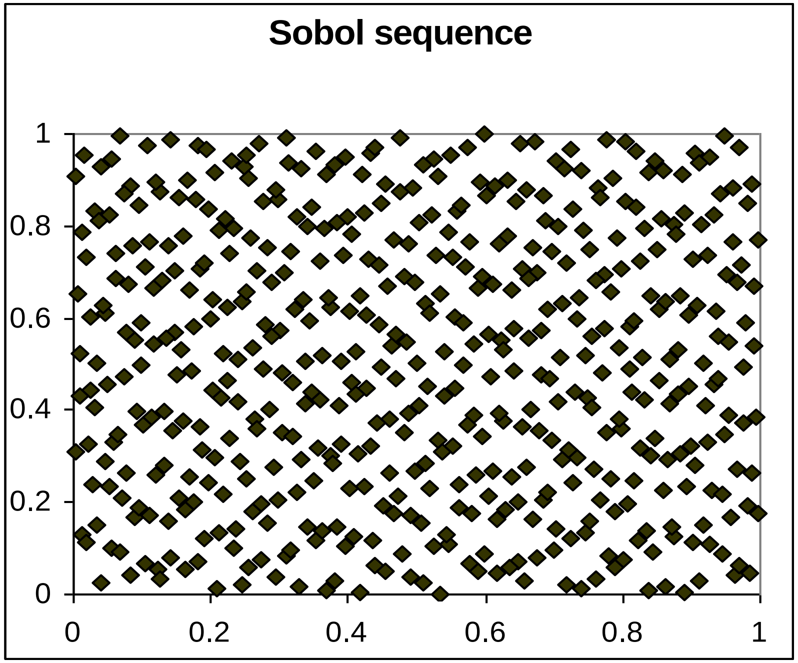
- Random sequences of experiments allow the sampling of a configuration space with continuous and discrete variables without pre-defined interactions

- The use of random sequences avoids the risk of “correlated sampling” even in the case of limited sampling

DOE: space fillers

Random and Sobol





Pseudo-random

High number of variables

Suitable for GA

Better distributed designs

Suitable for a low number of variables (<10)

Suitable for RSM, GA

Factorial DOE

Full factorial

Number of generated designs: m^n

m = variable level

(number of “possible” states of a variable)

n = variables number

Pascal triangle

$$a_0$$

$$a_1x \quad a_2y$$

$$a_3x^2 \quad a_4xy \quad a_5y^2$$

$$a_6x^3 \quad a_7x^2y \quad a_8xy^2 \quad a_9y^3$$

$$a_{10}x^4 \quad a_{11}x^3y \quad a_{12}x^2y^2 \quad a_{13}xy^3 \quad a_{14}y^4$$

$$a_{15}x^5 \quad a_{16}x^4y \quad a_{17}x^3y^2 \quad a_{18}x^2y^3 \quad a_{19}xy^4 \quad a_{20}y^5$$

Full Factorial 2 levels

2^n designs allow to correctly capture the first order interaction (e.g. x*y)

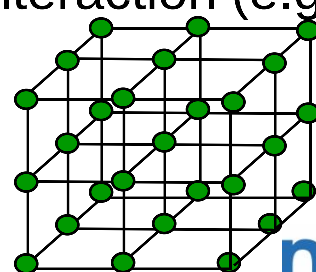
Full Factorial 3 levels

3^n designs allow to correctly capture the second order interaction (e.g. x^2*y)

Full Factorial

3 variables

3 levels



DOE: space fillers

Reduced factorial

Number of generated designs = 2^p

$p < n$ (number of variables)

n. design	x1	x2	x3	x4 (=x1*x2)
1	+	+	+	+
2	+	+	-	+
3	+	-	+	-
4	+	-	-	-
5	-	+	+	-
6	-	+	-	-
7	-	-	+	+
8	-	-	-	+

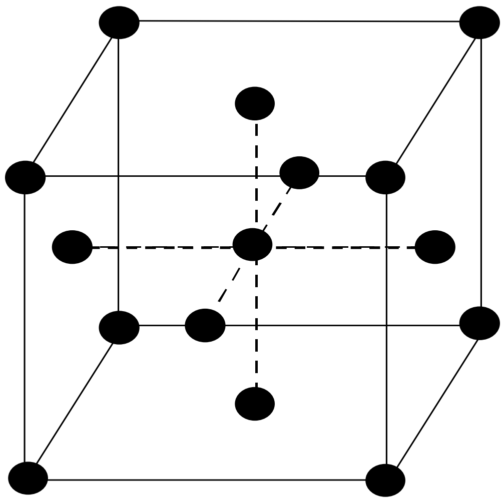
Removing the second order information for x1*x2

DOE: space fillers

Cubic face centered

$2^n + 2*n + 1$ designs

allow to correctly capture the second order interaction (e.g. x^2*y)

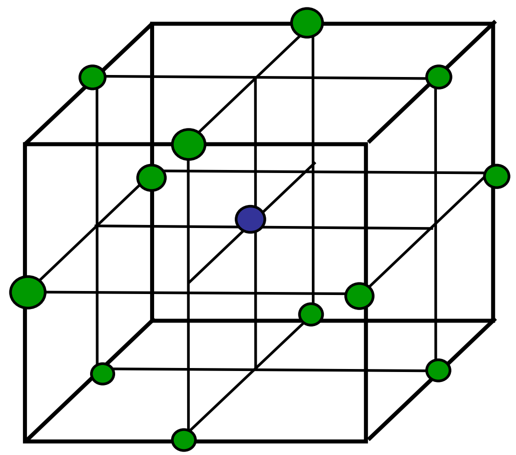


3 variables
15 designs

DOE: space fillers

Box Behnken

Very similar to the face centered algorithm, it uses the mid-side nodes and the center of the (hyper-) cube



3 variables
13 designs

DOE: space fillers

Latin Square

The designs number (m^2, where m is the required level) does not depend on the number of variables

Suitable for statistical analysis

Only the zero order interactions can be captured

Example: Latin Square with 3 variables (x_1, x_2, x_3) and 3 levels

- x_1 (1,2,3)
- x_2 (A,B,C)
- x_3 (a,b,c)

1	2	3
3	1	2
2	3	1

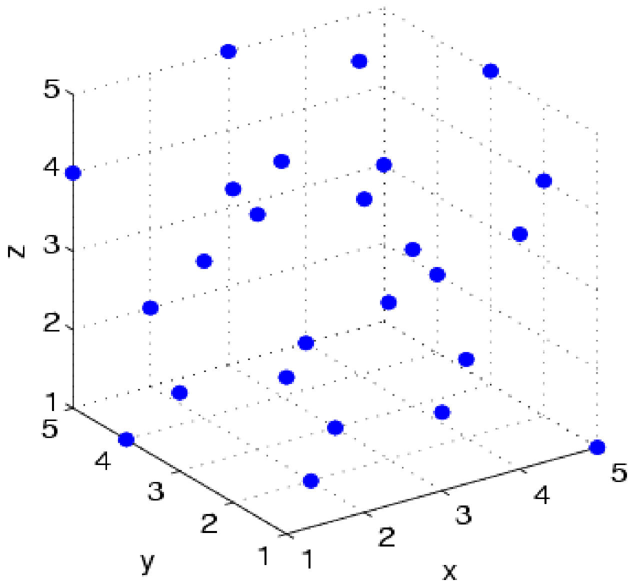
a	b	c
c	a	b
b	c	a

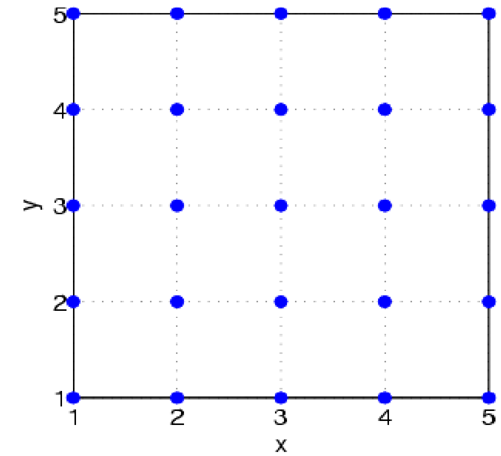
A	B	C
B	C	A
C	A	B

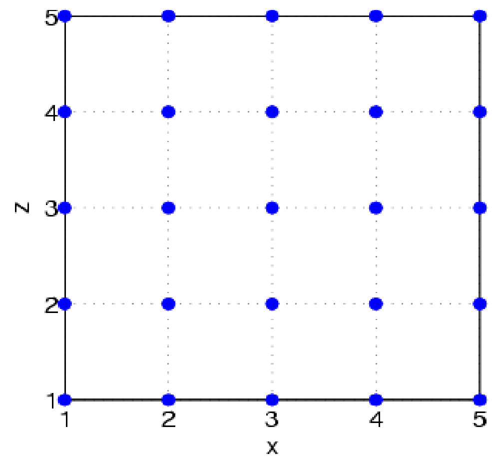
a1A	b2B	c3C
c3B	a1C	b2A
b2C	c3A	a1B

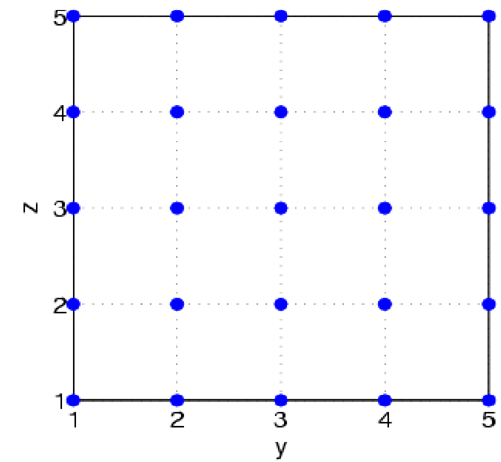
DOE: space fillers

Latin Square





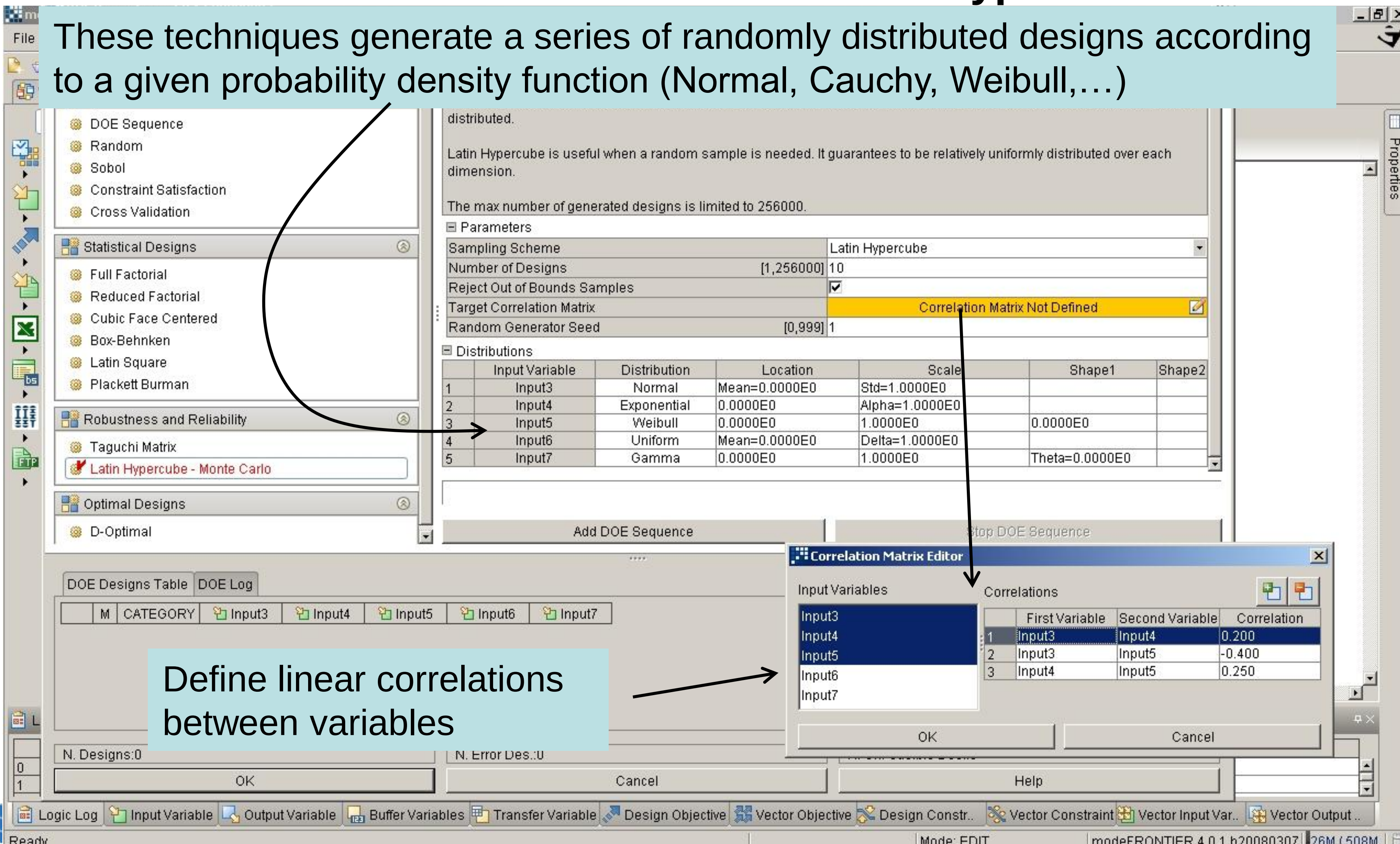




DOE: Reliability and Robustness

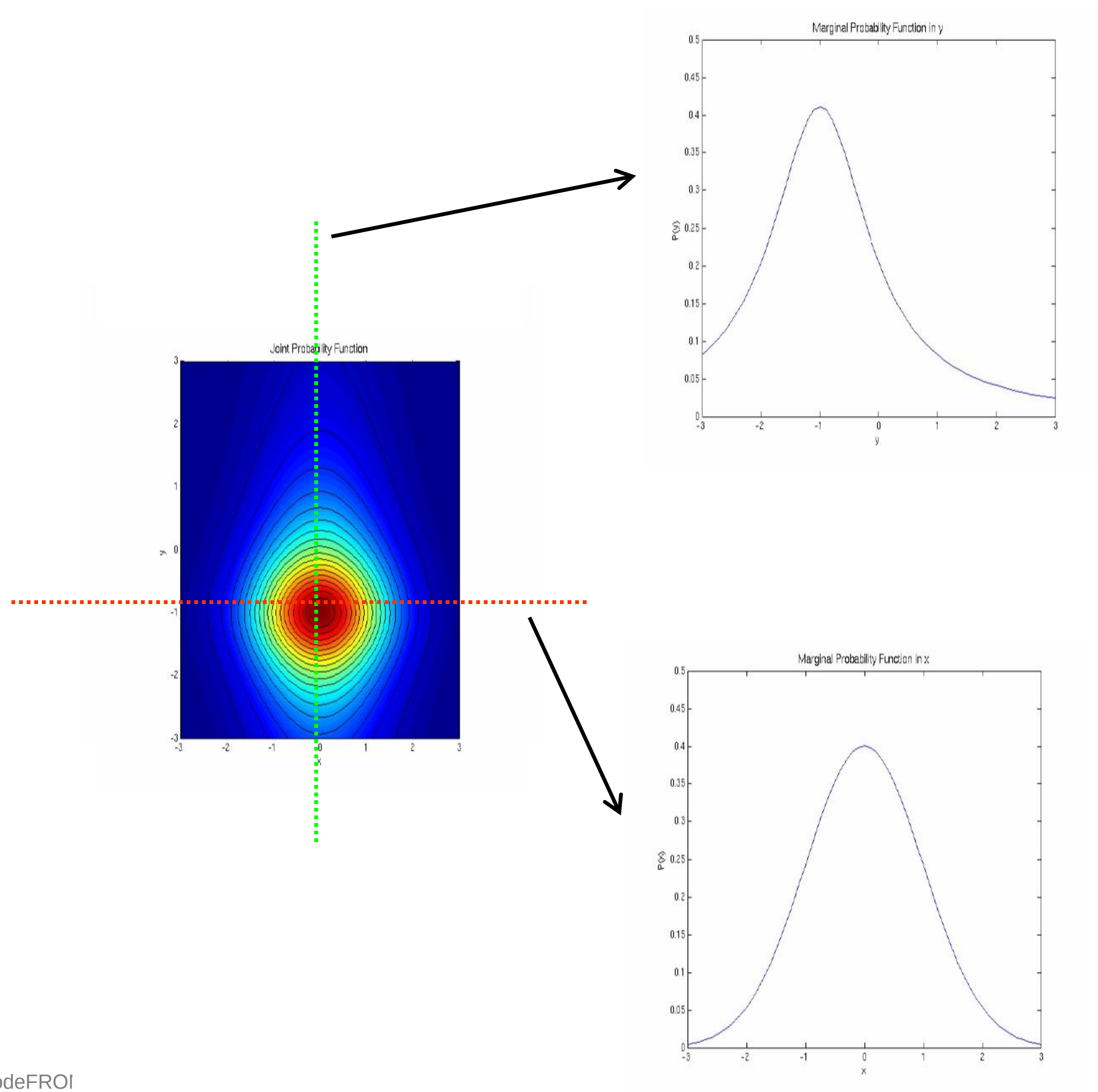
Montecarlo and Latin Hypercube

These techniques generate a series of randomly distributed designs according to a given probability density function (Normal, Cauchy, Weibull,…)



Define linear correlations between variables

DOE: Reliability and Robustness

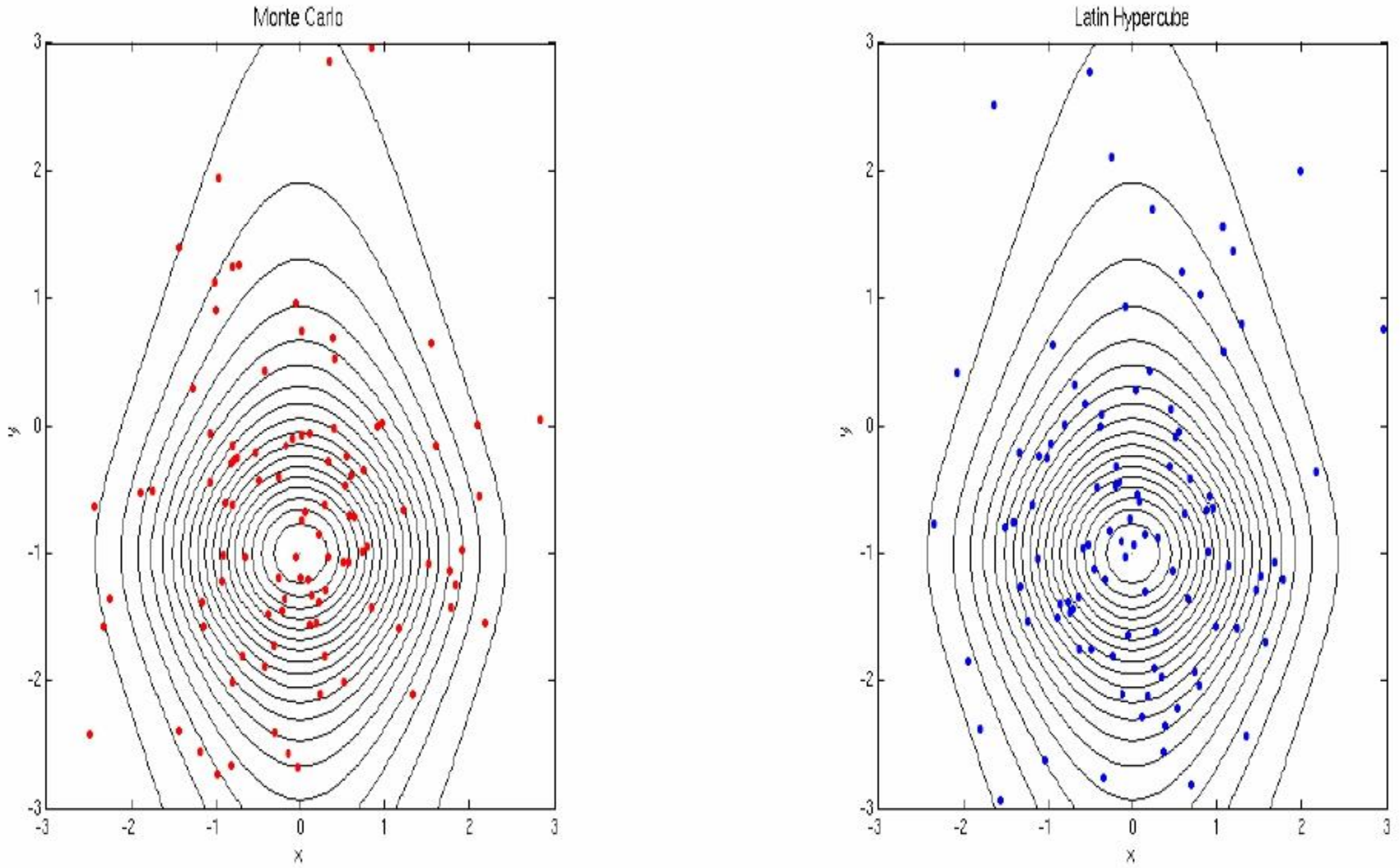


Cauchy

Normal

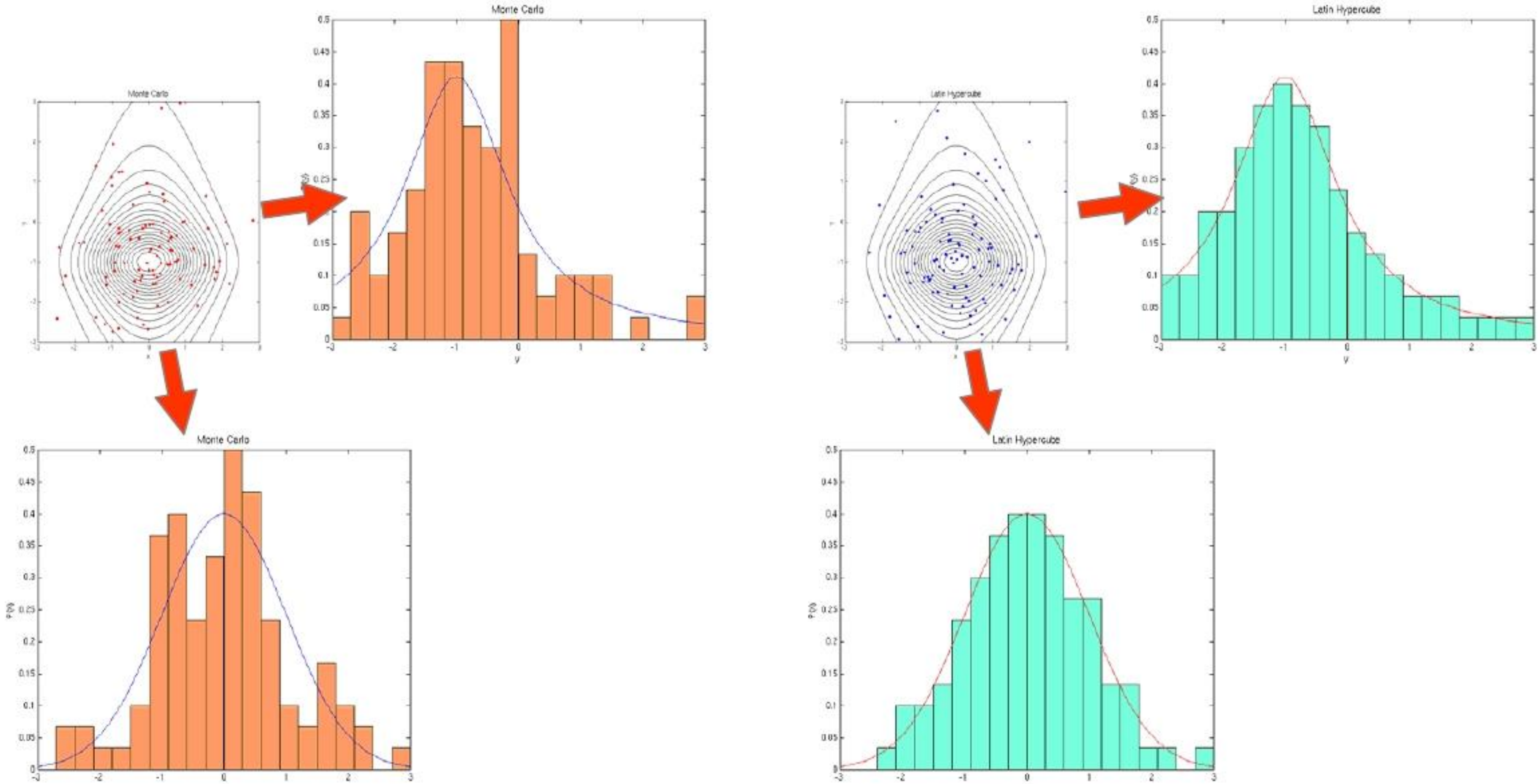
DOE: Reliability and Robustness

Montecarlo versus Latin Hypercube



DOE: Reliability and Robustness

Montecarlo versus Latin Hypercube



DOE: hands on

Try to show the differences between Montecarlo and latin Hypercube

For a two variables problem

Generate a Montecarlo DOE

Generate a Latin Hypercube DOE

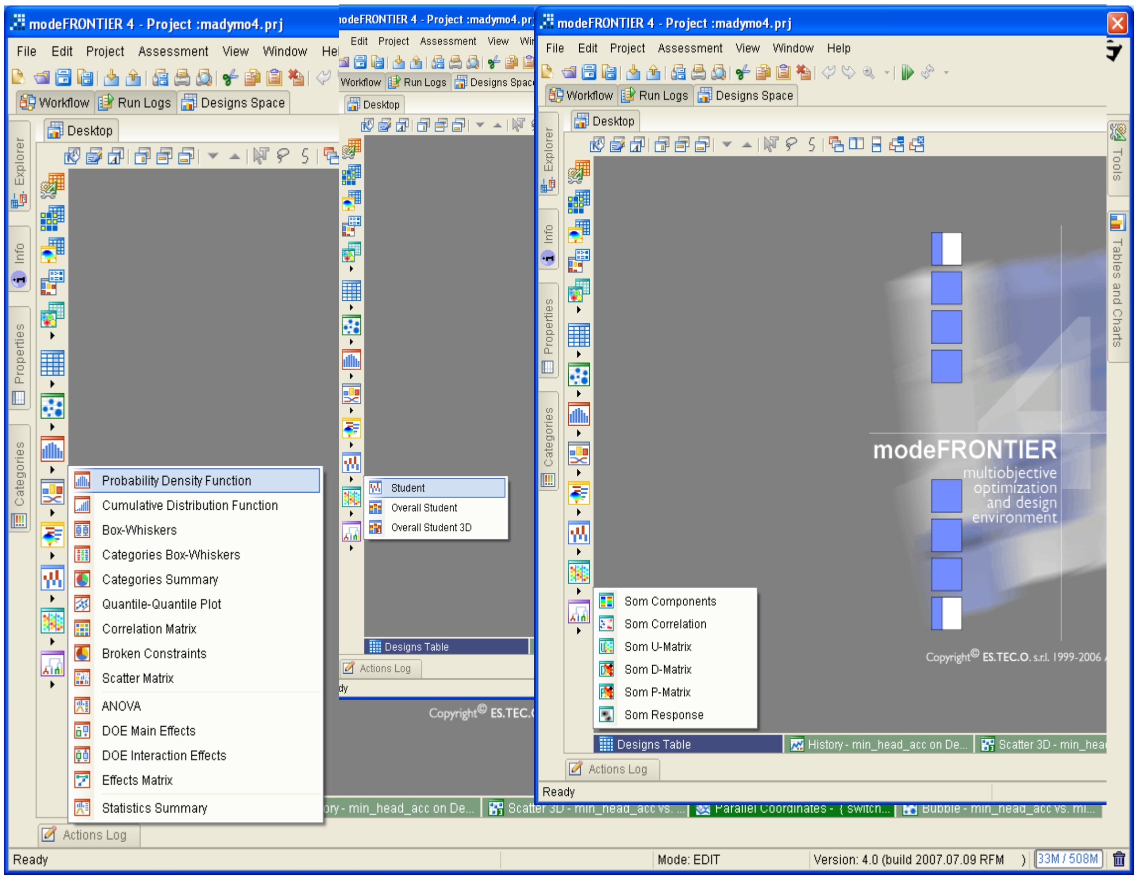
Plot hystograms, curve fitting, correlation matrix, Q-Q plots, compare results

Statistical analysis

- Statistical tools can be used to analyze distributions, coming from experiments or from DOE, to obtain information from the system (e.g., what is the most responsible cause of failures)

- Statistical tools can be used to find correlations, in particular which input variables have most influences in the system outputs; these results can be obtained from an available database, from a DOE, or after an optimisation phase

Statistical analysis



Several statistical tools are available:

- Density and cumulative Distribution
- Box-Whiskers
- Quantile Plot
- Statistics summary
- ANOVA
- Broken Constraints
- Main and interaction effects
- Student
- Correlation Matrix
- Scatter Matrix
- SOM

Tools for distribution analysis

Tools for correlation analysis

For more information visit: www.esteco.com or send an e-mail to: modeFRONTIER@esteco.com

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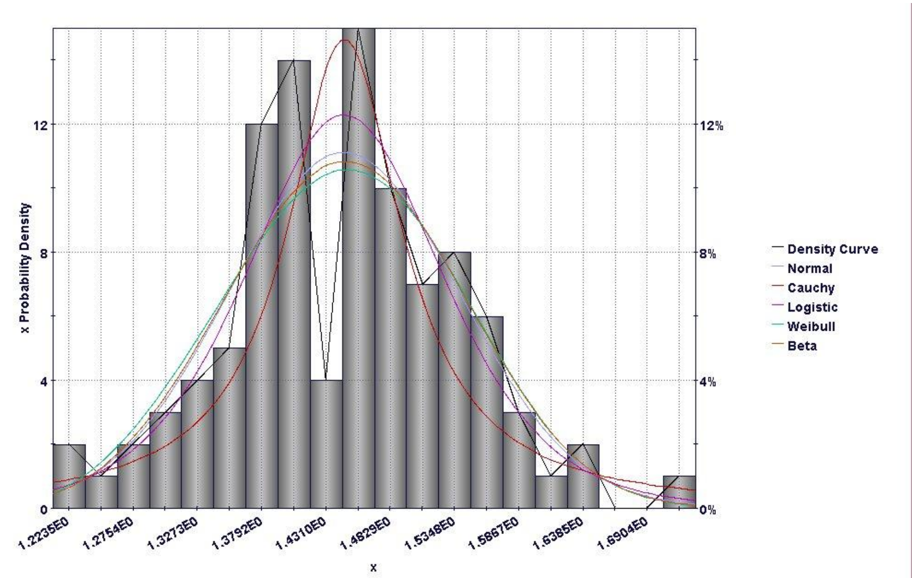
Probability Density Plot

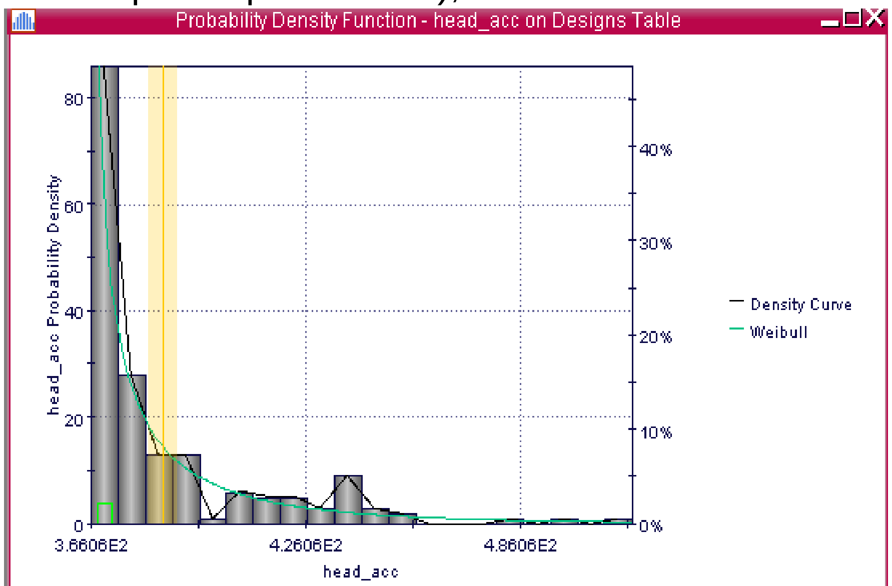
- The Probability Density chart summarizes the distribution of a data set (min, max, mean, variance,…)
- This plot is obtained by splitting the range of the data into equal sized classes
- The number of points that fall into each class are counted
- It reveals:
  - the kind of distribution
  - where the data is located
  - how spread out the data are



N. of Designs	100	Std. Deviation	9.3683E-2
N. of Classes	20	Skewness	4.4698E-2
Min	1.2106E0	Kurtosis	3.2765E0
Max	1.7293E0	Confidence Int. 95%	(1.4272E0,1.4644E0)
Arithmetic Mean	1.4458E0	Confidence Int. 99%	(1.4212E0,1.4704E0)
Variance	8.7766E-3	Variation Coeff.	6.4796E-2

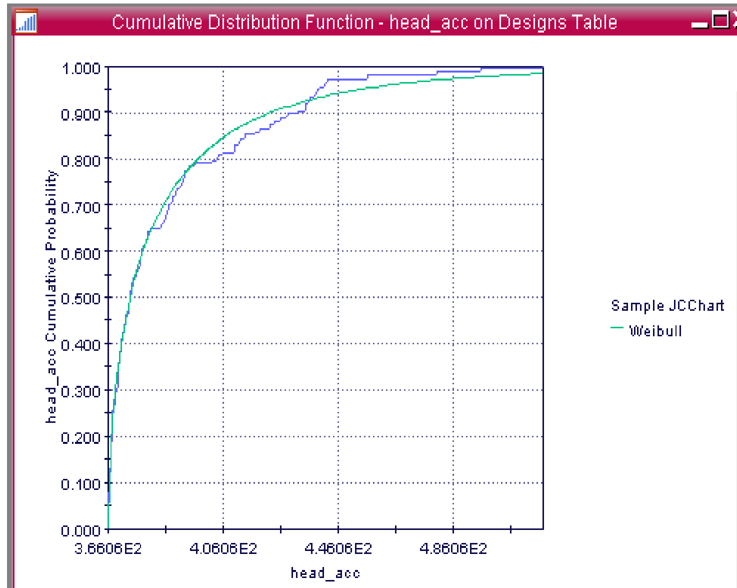
Statistical analysis for distributions (**Density and Cumulative Distribution**)

Probability density function (for any input/output variable);

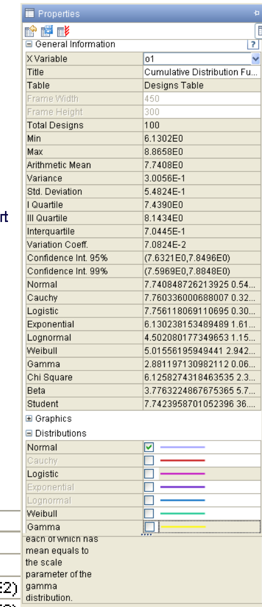


N. of Designs	177	Std. Deviation	2.7138E1
N. of Classes	20	Skewness	1.9939E0
Min	3.6606E2	Kurtosis	7.3924E0
Max	5.1718E2	Confidence Int. 95%	(3.8220E2,3.9025E2)
Arithmetic Mean	3.8622E2	Confidence Int. 99%	(3.8091E2,3.9153E2)
Variance	7.3646E2	Variation Coeff.	7.0265E-2

Cumulative Distribution function (for any input/output variable);

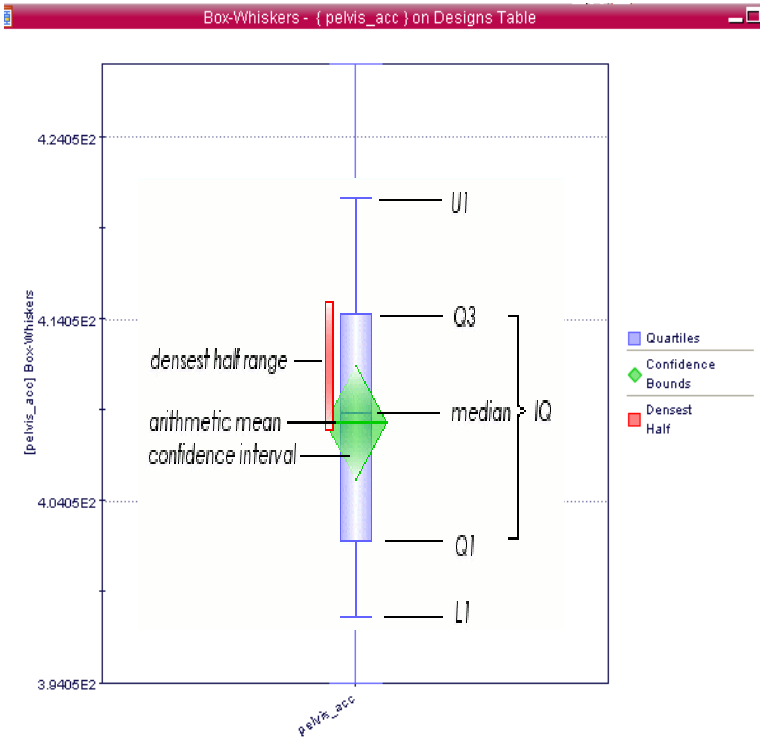


N. of Designs	177	I Quartile	3.6794E2
Min	3.6606E2	III Quartile	3.9254E2
Max	5.1718E2	Interquartile	2.4597E1
Arithmetic Mean	3.8622E2	Confidence Int. 95%	(3.8220E2,3.9025E2)
Variance	7.3646E2	Confidence Int. 99%	(3.8091E2,3.9153E2)
Std. Deviation	2.7138E1	Variation Coeff.	7.0265E-2



- Select theoretical distribution from Properties>Distribution
- Distribution parameters are available in Properties>General Information

Statistical analysis for distributions (**Box Whiskers**)



Std. Deviation	Std. Error	I Quartile	Median	III Quartile	Interquartile ...	Confidence I...	Confidence I...
9.7864E0	7.3559E-1	3.9958E2	4.0309E2	4.1672E2	1.7138E1	(4.0595E2,4....	(4.0548E2,4....

Q1 (first quartile): cut off lowest 25% of data

Q3 (third quartile): cut off highest 25% of data

U1 (upper fence) = Q3+1.5*(Q3-Q1) : it is the limit over which points are considered as outliers

L1 (lower fence) = Q1-1.5*(Q3-Q1) : it is the limit below which points are considered as outliers

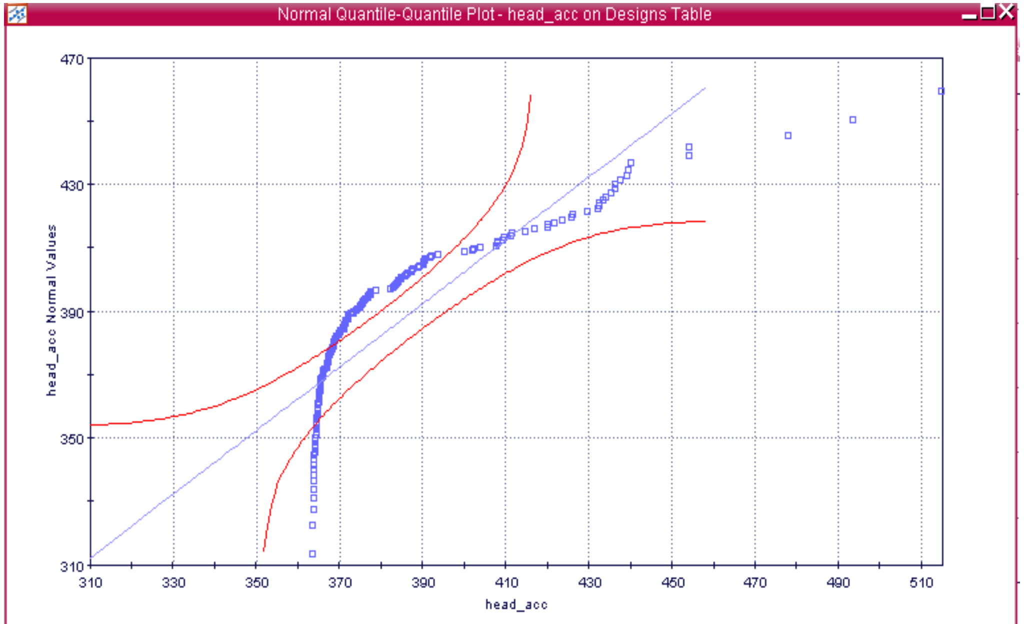
MEDIAN: 50% of the distribution data are expected to be lower (or greater) than the Median value.

DENSEST HALF RANGE: smallest range that contains half of the distribution samples

CONFIDENCE INTERVAL: 95% of confidence that the mean is inside this range

Reports statistical data (symbols and summary table) for any input/output variable

Statistical analysis for distributions (**quantile-quantile plot**)



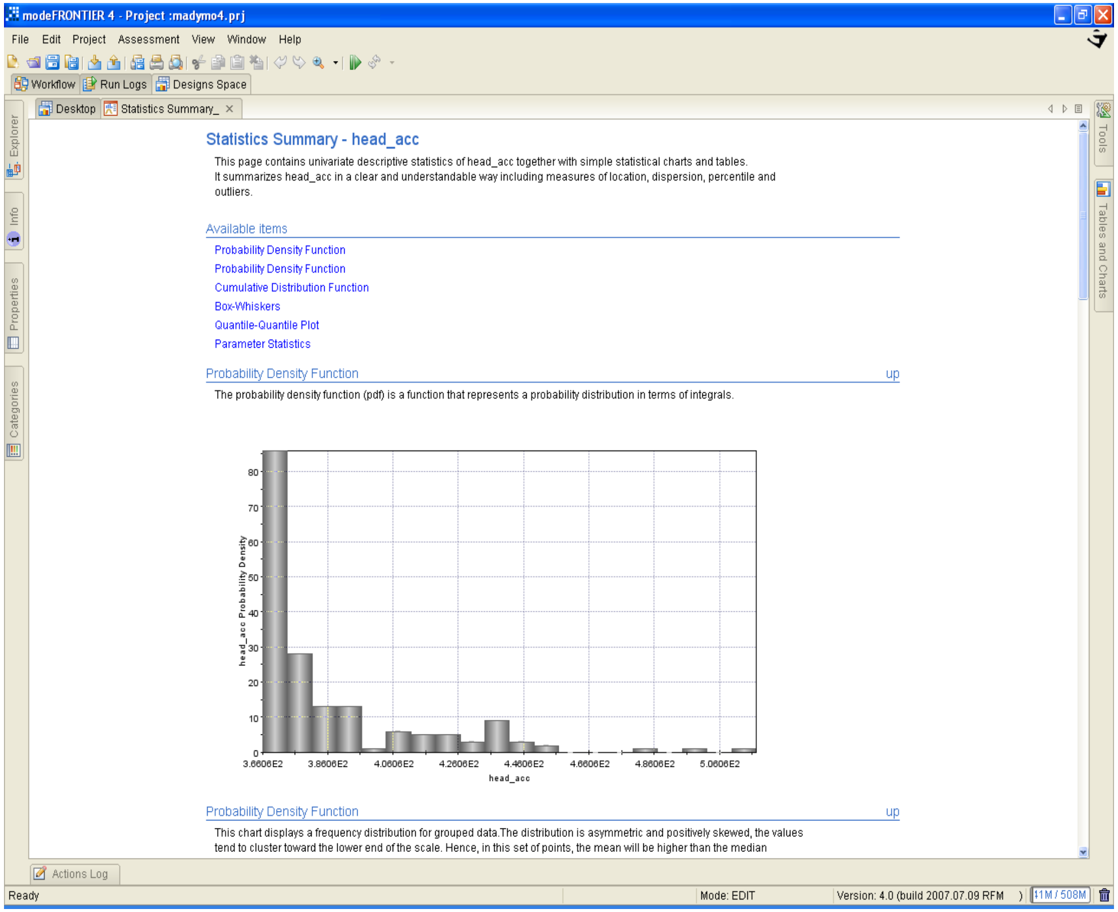
N. of Designs	177	I Quartile	2.4597E1
Min	3.6606E2	III Quartile	3.6794E2
Max	5.1718E2	Interquartile	3.9254E2
Arithmetic Mean	3.8622E2	Confidence Int. 95%	(3.8223E2,3.9022E2)
Variance	7.3646E2	Confidence Int. 99%	(3.8097E2,3.9148E2)
Std. Deviation	2.7138E1	Variation Coeff.	7.0265E-2

Quantile-quantile plot for any input/output variable: It is used to determine if the data points (abscissa) can be represented by an analytic distribution (ordinate, you can change the type in properties)

• best fitting distribution would have points on the green 45° line

• if some points are outside the region bounded by red lines (Lillenfor's test), distributions are different

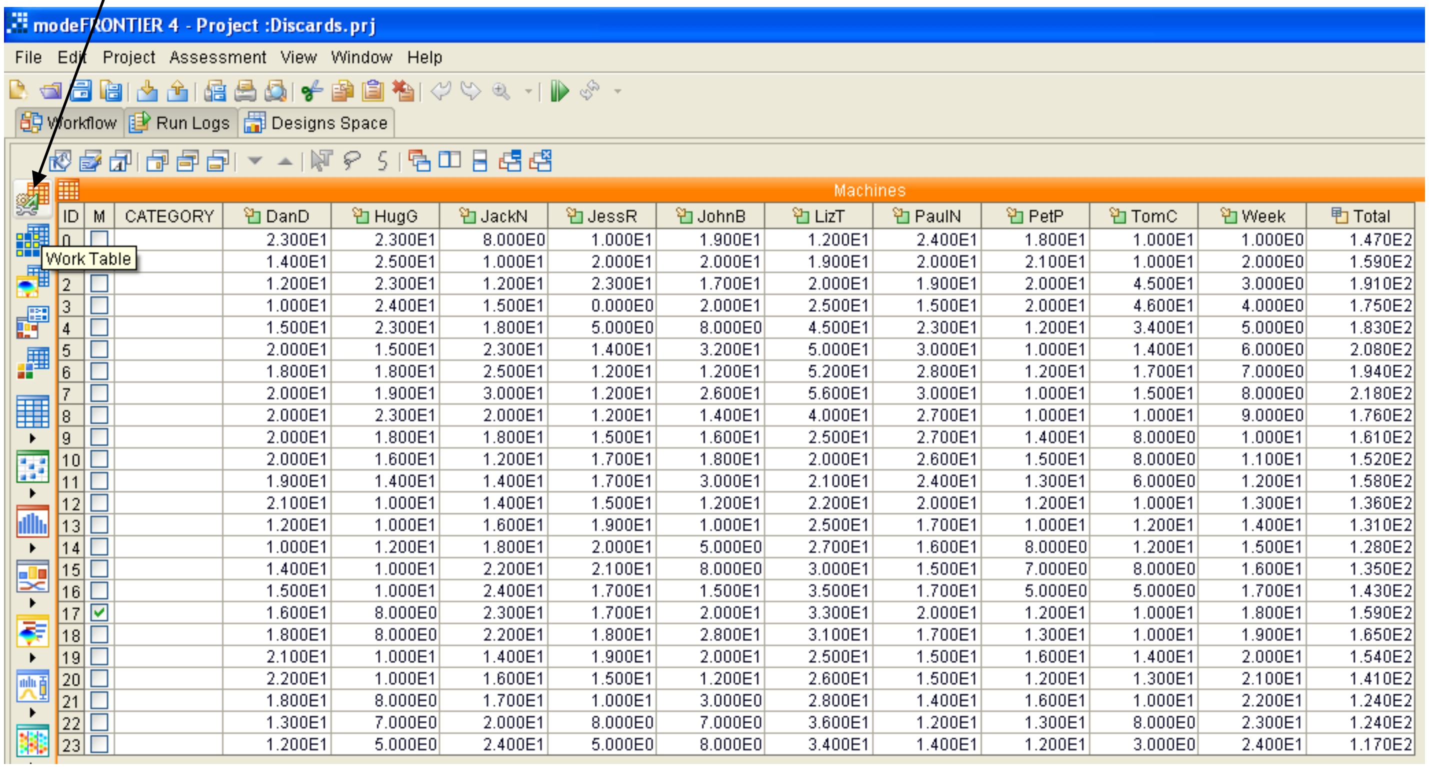
Statistical analysis for distributions (**Statistic summary**)



Automatic creation of all the previous chart on the selected input/output variables

ANOVA: import database

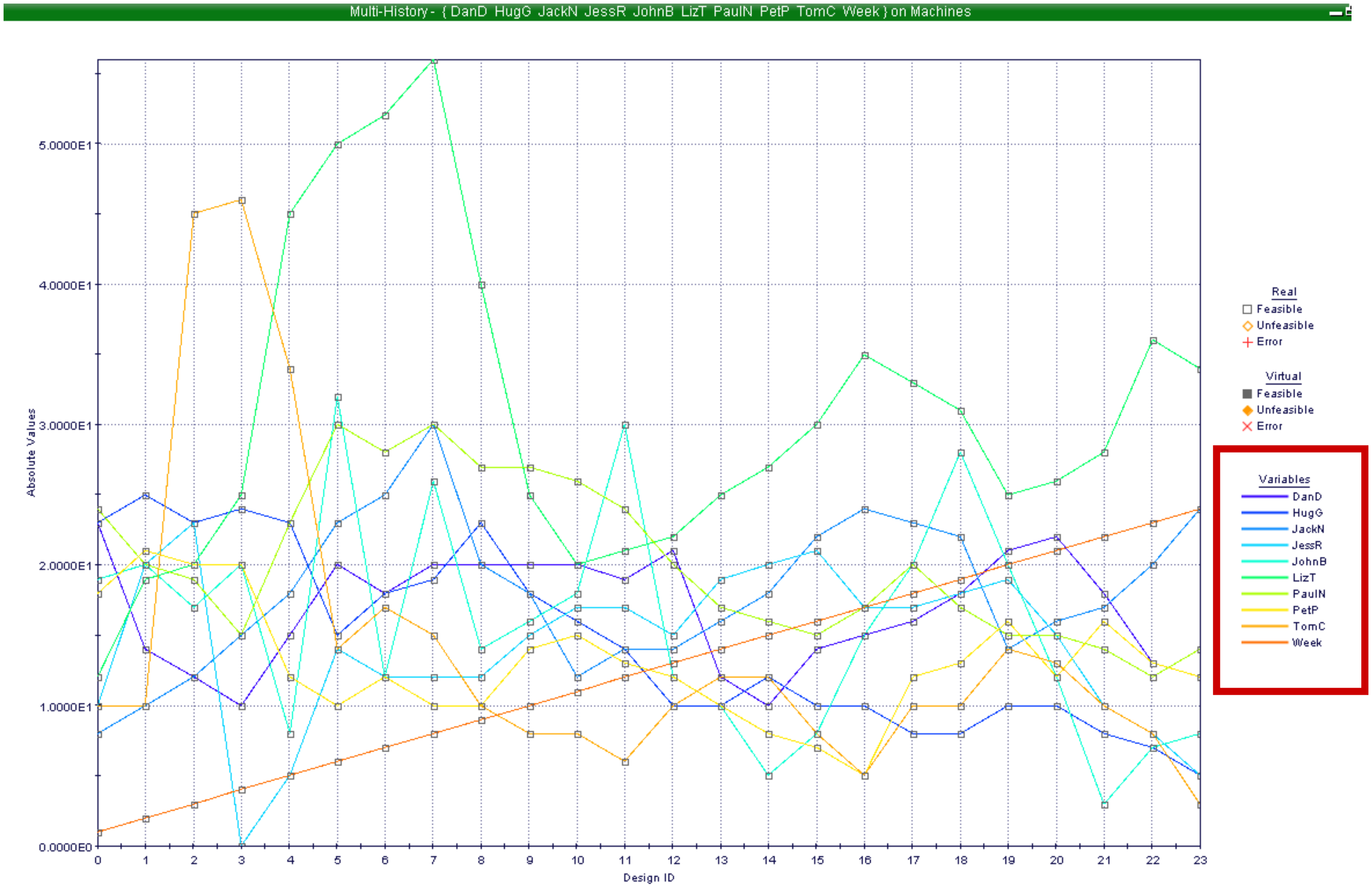
- import database (or use existing one in Design table)
- each column is a different distribution



ID	M	CATEGORY	DanD	HugG	JackN	JessR	JohnB	LizT	PaulN	PetP	TomC	Week	Total
0	☐		2.300E1	2.300E1	8.000E0	1.000E1	1.900E1	1.200E1	2.400E1	1.800E1	1.000E1	1.000E0	1.470E2
			1.400E1	2.500E1	1.000E1	2.000E1	2.000E1	1.900E1	2.000E1	2.100E1	1.000E1	2.000E0	1.590E2
2	☐		1.200E1	2.300E1	1.200E1	2.300E1	1.700E1	2.000E1	1.900E1	2.000E1	4.500E1	3.000E0	1.910E2
3	☐		1.000E1	2.400E1	1.500E1	0.000E0	2.000E1	2.500E1	1.500E1	2.000E1	4.600E1	4.000E0	1.750E2
4	☐		1.500E1	2.300E1	1.800E1	5.000E0	8.000E0	4.500E1	2.300E1	1.200E1	3.400E1	5.000E0	1.830E2
5	☐		2.000E1	1.500E1	2.300E1	1.400E1	3.200E1	5.000E1	3.000E1	1.000E1	1.400E1	6.000E0	2.080E2
6	☐		1.800E1	1.800E1	2.500E1	1.200E1	1.200E1	5.200E1	2.800E1	1.200E1	1.700E1	7.000E0	1.940E2
7	☐		2.000E1	1.900E1	3.000E1	1.200E1	2.600E1	5.600E1	3.000E1	1.000E1	1.500E1	8.000E0	2.180E2
8	☐		2.000E1	2.300E1	2.000E1	1.200E1	1.400E1	4.000E1	2.700E1	1.000E1	1.000E1	9.000E0	1.760E2
9	☐		2.000E1	1.800E1	1.800E1	1.500E1	1.600E1	2.500E1	2.700E1	1.400E1	8.000E0	1.000E1	1.610E2
10	☐		2.000E1	1.600E1	1.200E1	1.700E1	1.800E1	2.000E1	2.600E1	1.500E1	8.000E0	1.100E1	1.520E2
11	☐		1.900E1	1.400E1	1.400E1	1.700E1	3.000E1	2.100E1	2.400E1	1.300E1	6.000E0	1.200E1	1.580E2
12	☐		2.100E1	1.000E1	1.400E1	1.500E1	1.200E1	2.200E1	2.000E1	1.200E1	1.000E1	1.300E1	1.360E2
13	☐		1.200E1	1.000E1	1.600E1	1.900E1	1.000E1	2.500E1	1.700E1	1.000E1	1.200E1	1.400E1	1.310E2
14	☐		1.000E1	1.200E1	1.800E1	2.000E1	5.000E0	2.700E1	1.600E1	8.000E0	1.200E1	1.500E1	1.280E2
15	☐		1.400E1	1.000E1	2.200E1	2.100E1	8.000E0	3.000E1	1.500E1	7.000E0	8.000E0	1.600E1	1.350E2
16	☐		1.500E1	1.000E1	2.400E1	1.700E1	1.500E1	3.500E1	1.700E1	5.000E0	5.000E0	1.700E1	1.430E2
17	☑		1.600E1	8.000E0	2.300E1	1.700E1	2.000E1	3.300E1	2.000E1	1.200E1	1.000E1	1.800E1	1.590E2
18	☐		1.800E1	8.000E0	2.200E1	1.800E1	2.800E1	3.100E1	1.700E1	1.300E1	1.000E1	1.900E1	1.650E2
19	☐		2.100E1	1.000E1	1.400E1	1.900E1	2.000E1	2.500E1	1.500E1	1.600E1	1.400E1	2.000E1	1.540E2
20	☐		2.200E1	1.000E1	1.600E1	1.500E1	1.200E1	2.600E1	1.500E1	1.200E1	1.300E1	2.100E1	1.410E2
21	☐		1.800E1	8.000E0	1.700E1	1.000E1	3.000E0	2.800E1	1.400E1	1.600E1	1.000E1	2.200E1	1.240E2
22	☐		1.300E1	7.000E0	2.000E1	8.000E0	7.000E0	3.600E1	1.200E1	1.300E1	8.000E0	2.300E1	1.240E2
23	☐		1.200E1	5.000E0	2.400E1	5.000E0	8.000E0	3.400E1	1.400E1	1.200E1	3.000E0	2.400E1	1.170E2

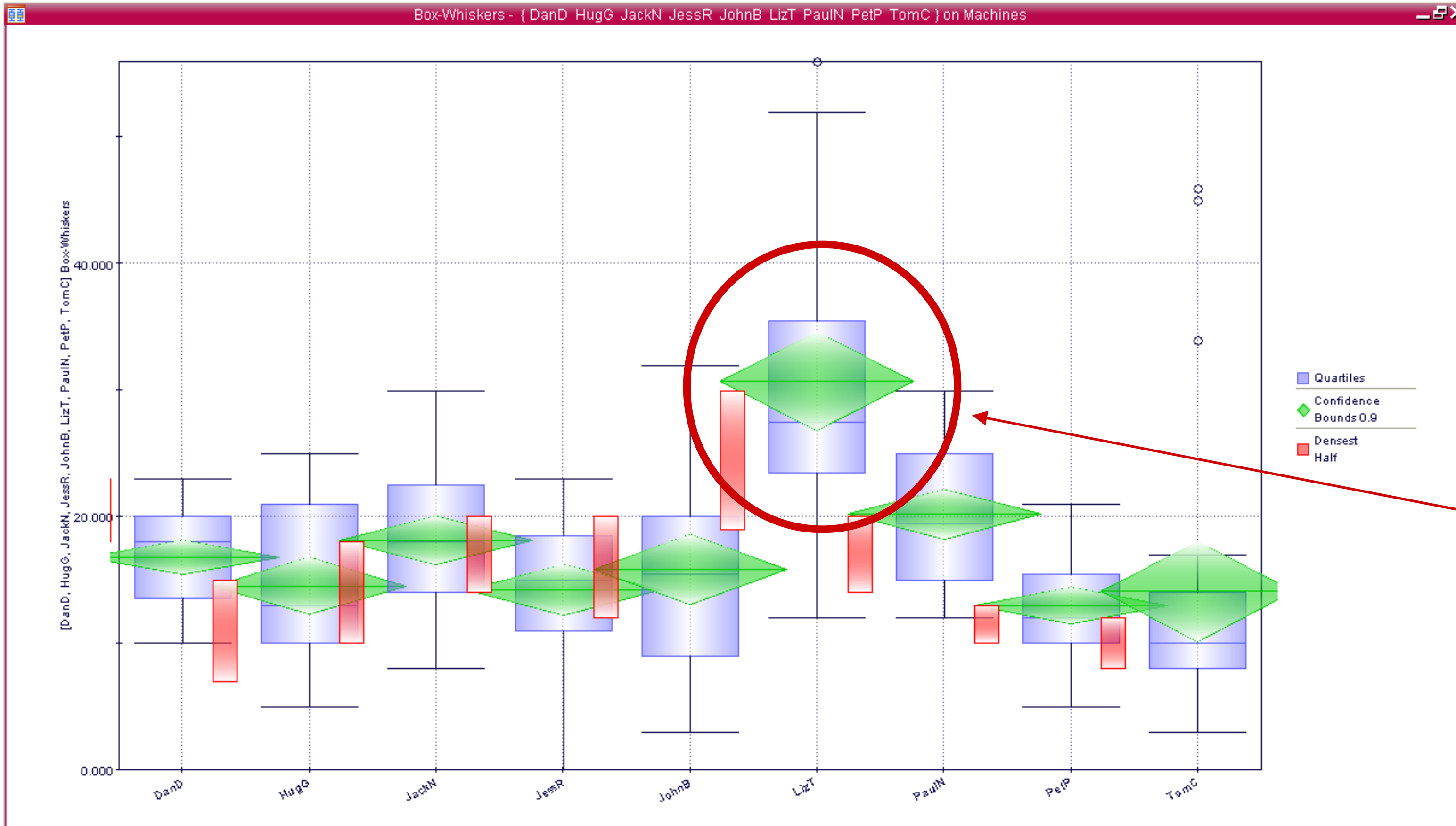
e.g., each column is a different **production line**, and shows the distribution of **defects** per day

Question: which line is most defective?



History chart for each line

Box Whiskers: Distributions Means and Variances



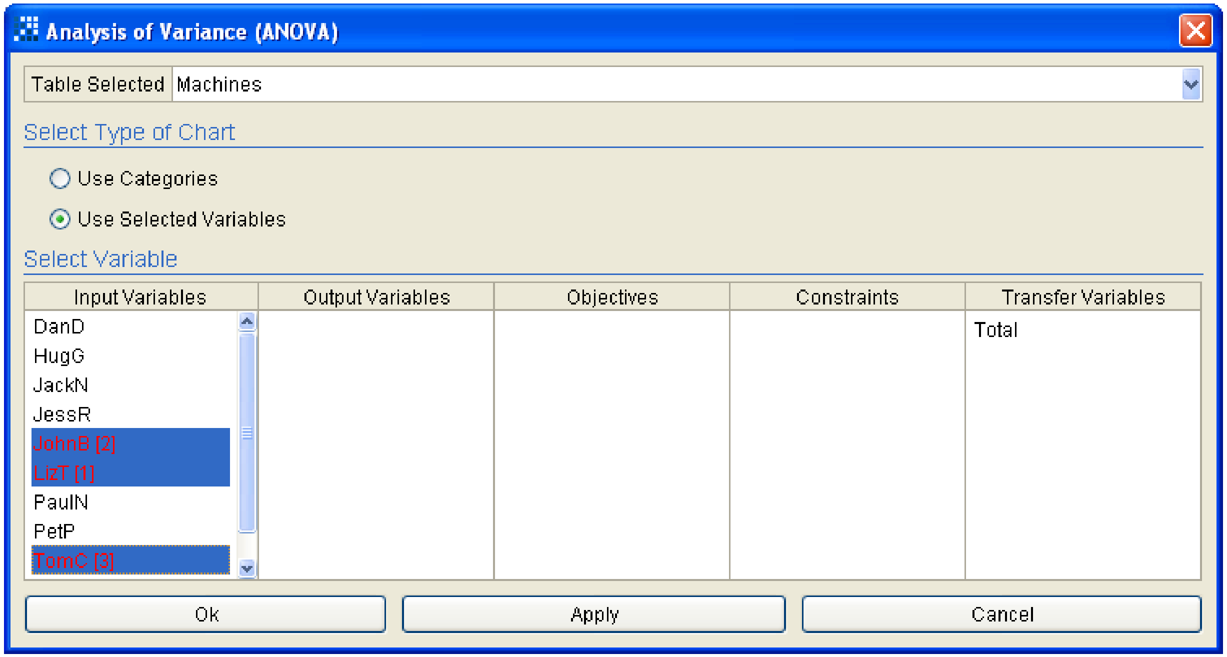
Value	N. of Designs	Min	Max	Arithmetic Mean	Std. Deviation	Std. Error	I Quartile	Median	III Quartile	Interquartile Range	Confidence Interval
DanD	24	10.000	23.000	16.792	3.956	0.808	13.500	18.000	20.000	6.500	(15.410,18.173)
HugG	24	5.000	25.000	14.542	6.386	1.304	10.000	13.000	21.000	11.000	(12.311,16.772)
JackN	24	8.000	30.000	18.125	5.343	1.091	14.000	18.000	22.500	8.500	(16.259,19.991)
JessR	24	0.000	23.000	14.208	5.687	1.161	11.000	15.000	18.500	7.500	(12.222,16.195)
JohnB	24	3.000	32.000	15.833	7.872	1.607	9.000	15.500	20.000	11.000	(13.084,18.583)
LizT	24	12.000	56.000	30.708	11.188	2.284	23.500	27.500	35.500	12.000	(26.801,34.616)

Box Whiskers can give a preliminary indication of which is most defective:

LizT seems to have highest mean, but **we need a tool to validate distribution analysis**: this is **ANOVA**

For more information visit: www.esteco.com or send an e-mail to: modeFRONTIER@esteco.com

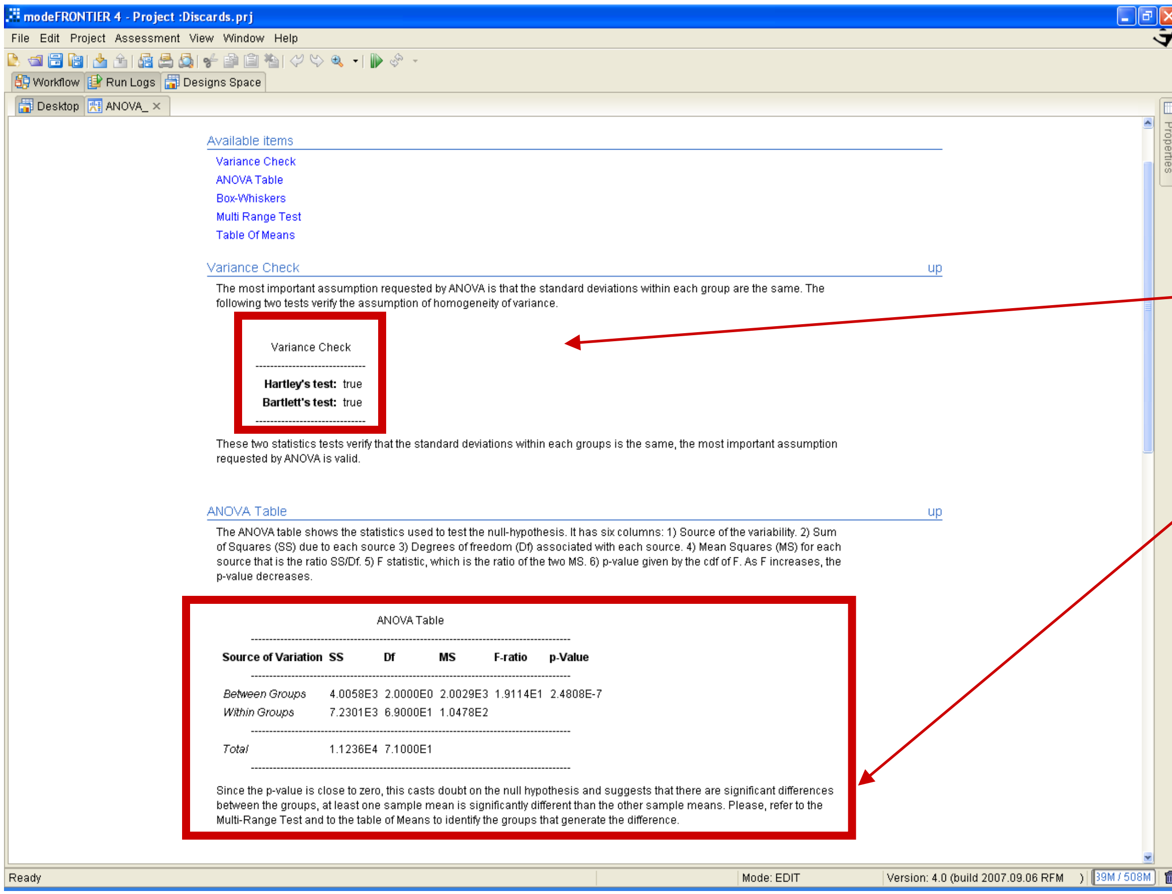
Statistical analysis for distributions (**ANOVA**)



- Select ANOVA tool
- Select table
- Select variables (distributions) to analyze

- in this case LizT is compared with other two production lines

Statistical analysis for distributions (**ANOVA**)



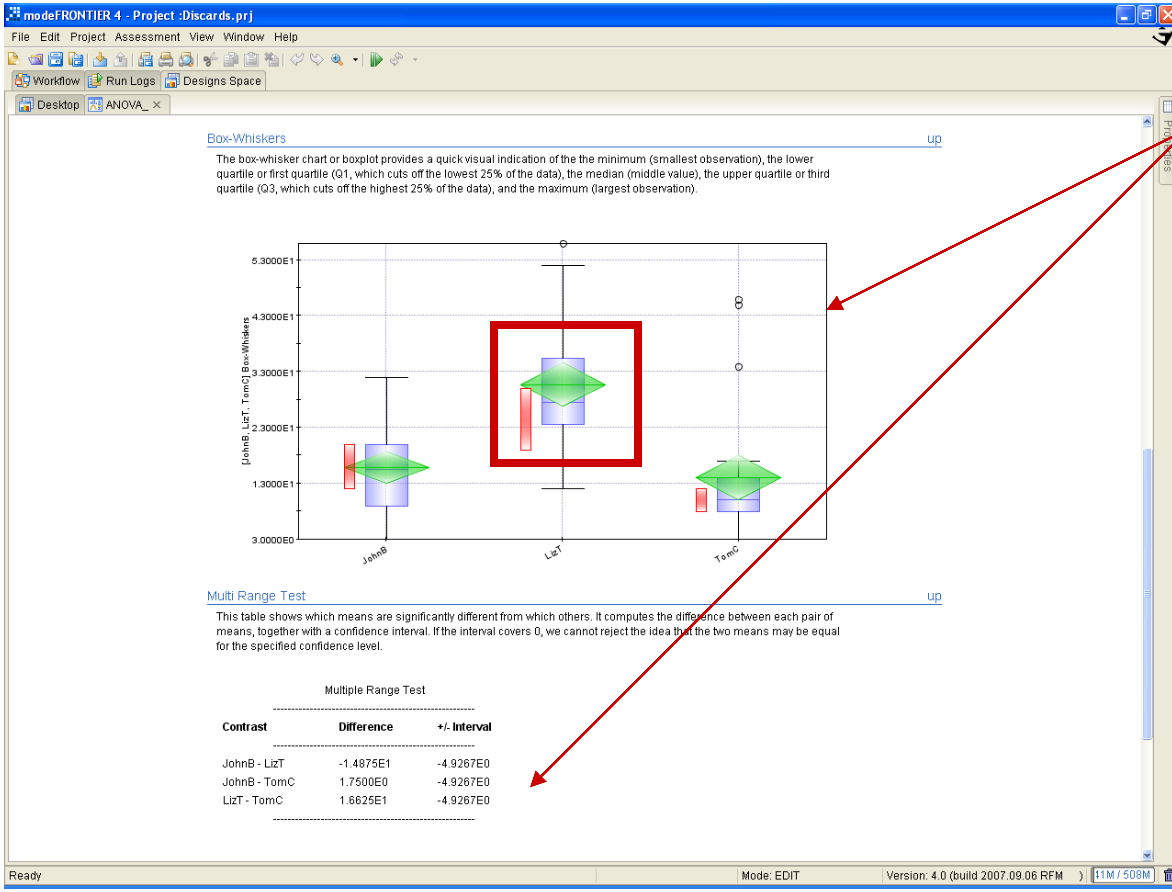
- **ANOVA analysis summary** is given to compare influence of different variables

-<u>Variance check</u>: only if variances of variables is similar, ANOVA can be performed

-**Hartley's test (**F_{max} test**) is performed** to verify that different groups have a similar variance, an assumption needed for ANOVA.

-<u>ANOVA table</u>: determines if variables have different significance

Statistical analysis for distributions (**ANOVA**)



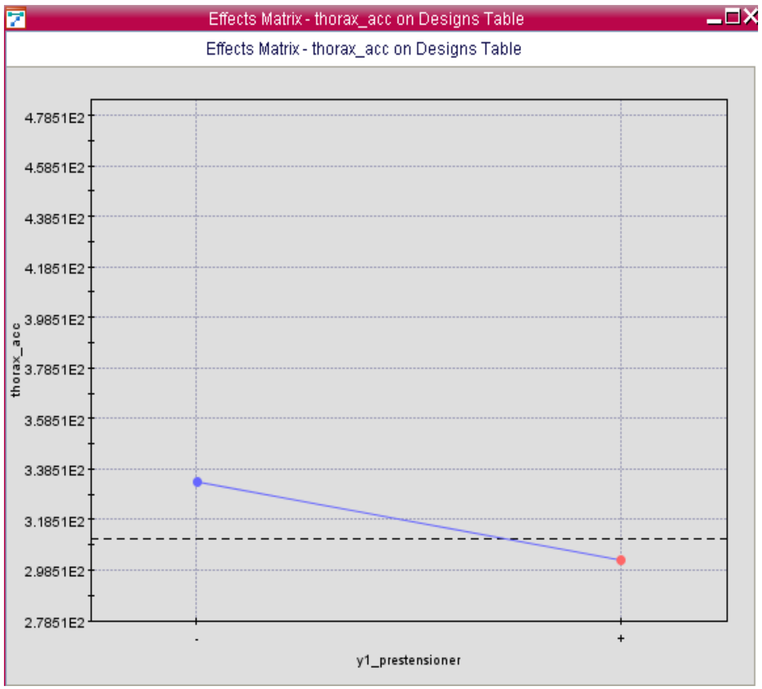
- Box-Whiskers, table of means and differences: if previous tests are valid, indicates the most significant variable

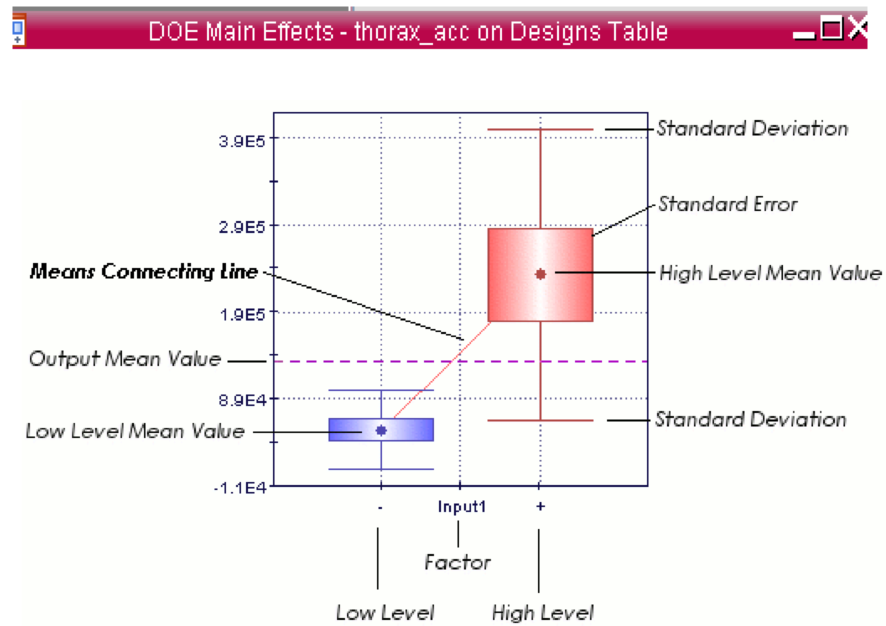
- **here we conclude that LizT is statistically the most defective line**

Tools for correlation analysis

- Main and interaction effects
- Student
- Correlation Matrix
- Scatter Matrix

Statistical analysis for correlations (**Effect matrix**)





- Effect of one input (*abscissa*: range is split in – and + half) to one output (*ordinate*: mean value)

- Same effect: output are in this case represented by bars (mean and STDEV)

Details on definition of Effect

	A	B	C	D	OBJ
1	-	-	-	-	65.6
2	-	-	+	+	79.3
3	-	+	-	+	51.3
4	-	+	+	-	69.6
5	+	-	-	+	59.8
6	+	-	+	-	77.7
7	+	+	-	-	74.2
8	+	+	+	+	87.9

Definition of Effect of input A for output Obj:

- Medium value of the function for every variable (computed for half range + or -):

A - ; A +

Effect = (Mean+) – (Mean-)

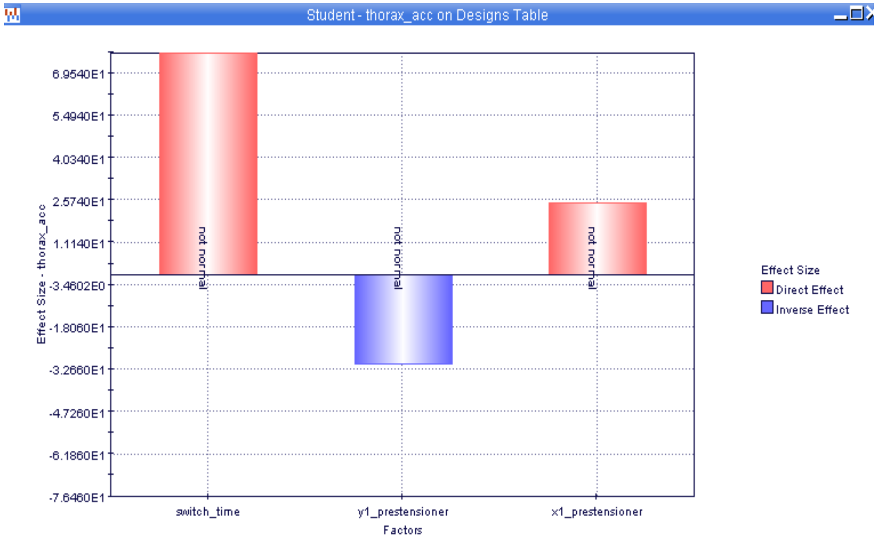
•The same for the interactions between the variables:

AB + +/ - - (concord); AB + -/ - + (discord)

	A	B	C	D
Mean +	75	70.8	78.8	69.5
Mean -	66.5	70.6	62.8	71.8
Effect	8.5	0.2	16	-2

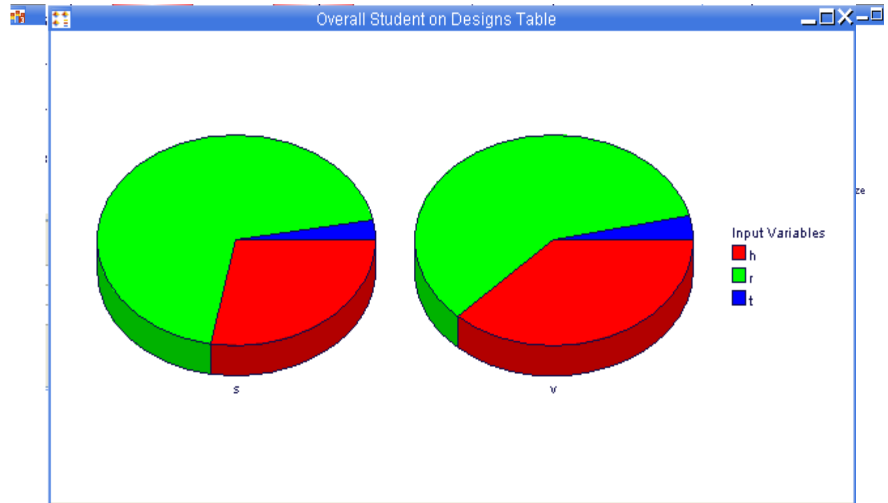
	AB
Mean AB+	76.7
Mean AB-	64.6
Effect	15.1

Statistical analysis for correlations (Student charts)



Factors	Effect Size	Significance	t-Student	critical t (α = 0.10)	critical t (α = 0.05)	critical t (α = 0.01)
switch_time	7.6460E1	0.000	9.4529E0	1.2864E0	1.6536E0	2.3478E0
y1_prestensioner	-3.0796E1	0.000	5.9703E0	1.2864E0	1.6536E0	2.3478E0
x1_prestensioner	2.4792E1	0.000	4.8951E0	1.2864E0	1.6536E0	2.3478E0

- t-Student chart shows the effect of each input variable accordingly to a selected output



Factors	s [Significance]	v [Significance]
h	269.8 [0.070]	1980.9 [0.017]
r	671.2 [0.000]	3168.2 [0.000]
t	29.4 [0.438]	191.3 [0.429]

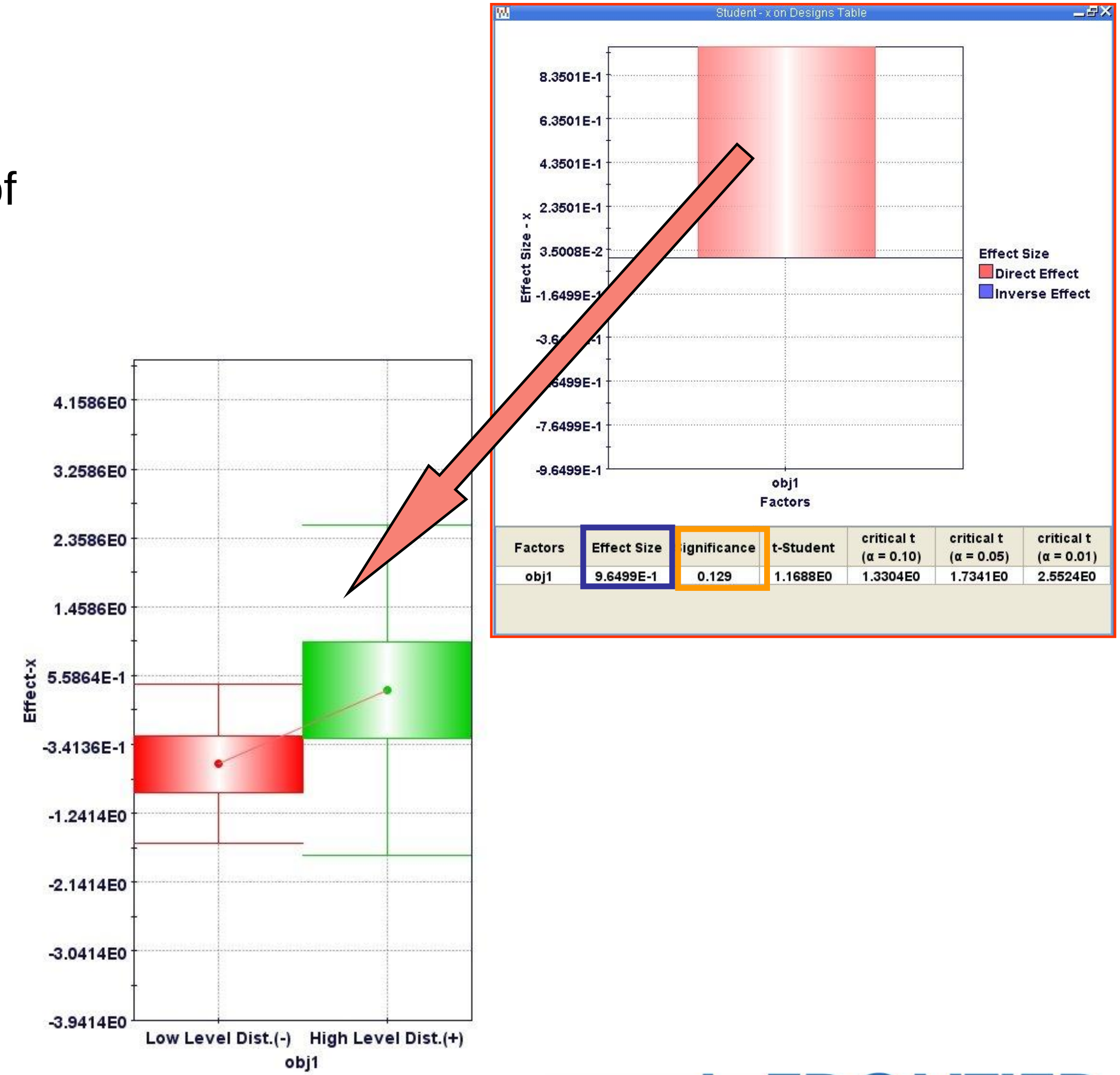
- Accordingly to t-Student parameter definition, overall 3D chart shows the normalised effect of each input variable accordingly to each output

Significance indicates the probability that response variable and the factor are not correlated (i.e. it is the probability that a difference in the response at factor variation is due to chance).

Details on Student test

- **Effect size** indicates the kind of relationship between the factor and the response variable: a value less than zero indicates that the relationship is inverse.

- An high value of **Significance** parameter (max value 0.5) indicates that there is a very high probability that the factor doesn't influence the response variable.



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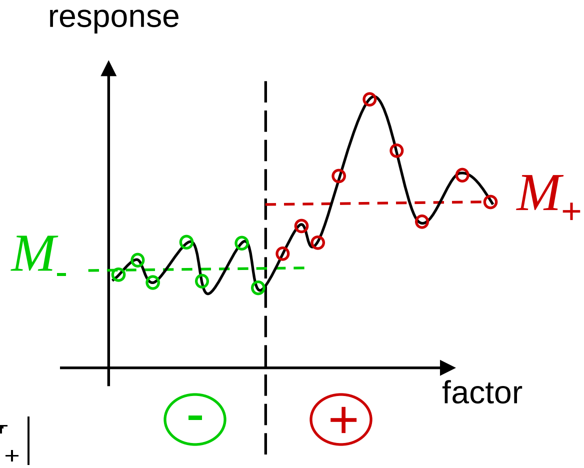
Details on Student test



$$t = \frac{|M_- - M_+|}{\sqrt{\frac{S_G^2}{n_-} + \frac{S_G^2}{n_+}}}$$

$$S_G^2 = \frac{(n_- - 1)S_-^2 + (n_+ - 1)S_+^2}{(n_+ + n_- - 2)}$$

$$S_+^2 = \frac{\sum (x_+ - M_+)^2}{n_+ - 1}$$

$$S_-^2 = \frac{\sum (x_- - M_-)^2}{n_- - 1}$$

- n_+ and n_- are the numbers of values in the upper and lower parts of domain of the input variable
- M_+ and M_- are the means of the values for the output variable x in the upper and lower parts of domain of the input variable
- $S_G{}^2$ is the general variance
- $S_+{}^2$ and $S_-{}^2$ are the variances of the population for the output variable x in the upper and lower parts of domain of the input variable

If *t* follows a well known distribution called Student distribution then M_- and M_+ are not statistically distinct i.e. probably there is no correlation between factor and response variable (significance close to 0.5)

Details on Student test

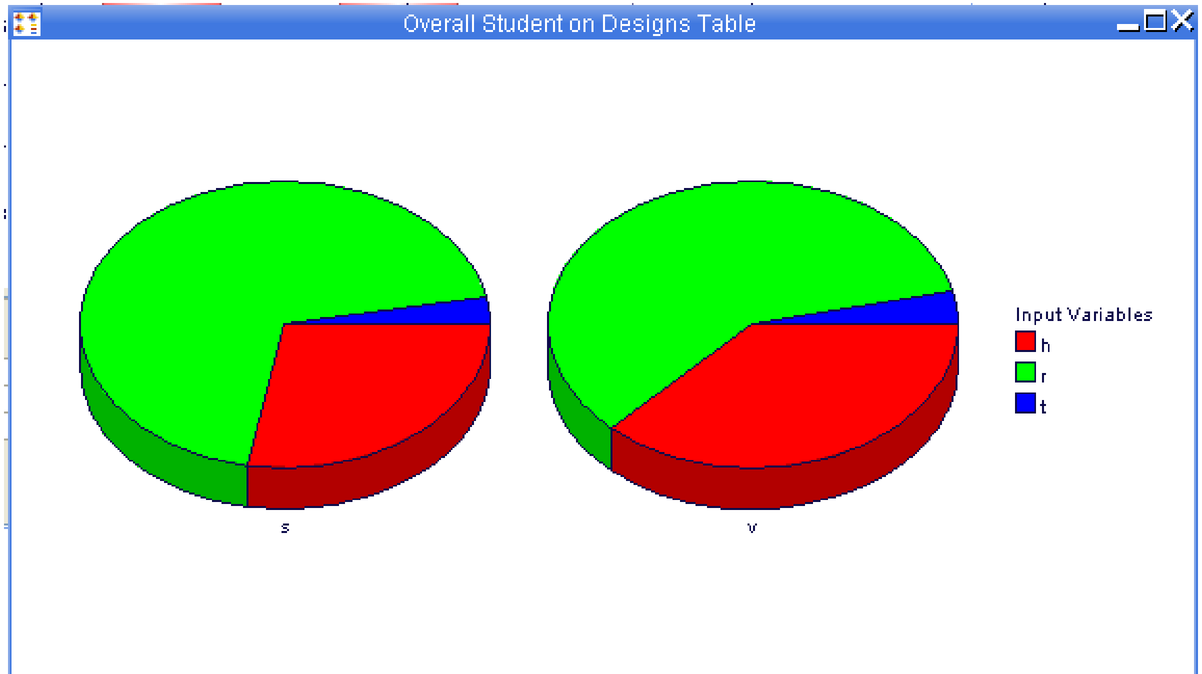
Factors	Effect Size	Significance	t-Student	critical t (α = 0.10)	critical t (α = 0.05)	critical t (α = 0.01)
obj1	9.6499E-1	0.129	1.1688E0	1.3304E0	1.7341E0	2.5524E0

- **critical t (α=0.1)=1.33** → *means that if $t \geq 1.33$ the response variable and the factor are correlated with a significance = 10% (i.e. there is 10% of probability that the difference between the range + and - of the response variable is due to chance).*
- **critical t (α=0.05)=1.73** → *means that if $t \geq 1.73$ the response variable and the factor are correlated with a significance = 5% (i.e. there is 5% of probability that the difference is due to chance).*
- **critical t (α=0.01)=2.55** → *means that if $t \geq 2.55$ the response variable and the factor are correlated with a significance = 1% (i.e. there is 1% of probability that the difference is due to chance).*

In the example t =1.688 and the significance is 0.129 → **means that the probability that the difference between the range + and - of the response variable is due to chance is 12.9%.**

The significance α is always between 0, i.e. correlation between factor and response variable, and 0.5, i.e. not correlation between factor and response variable (50% of probability that that the difference in the response variable ranges is due to chance).

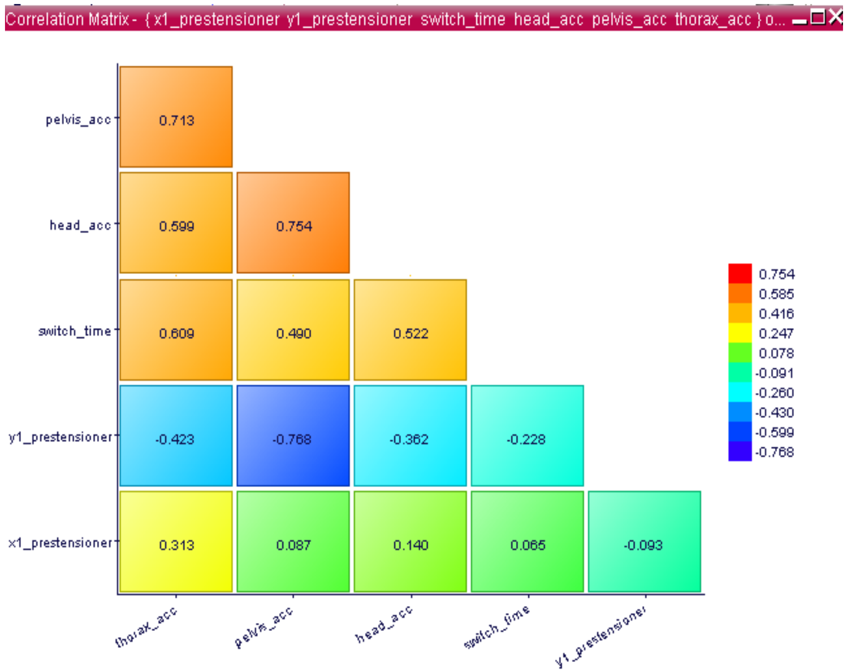
Overall Student chart

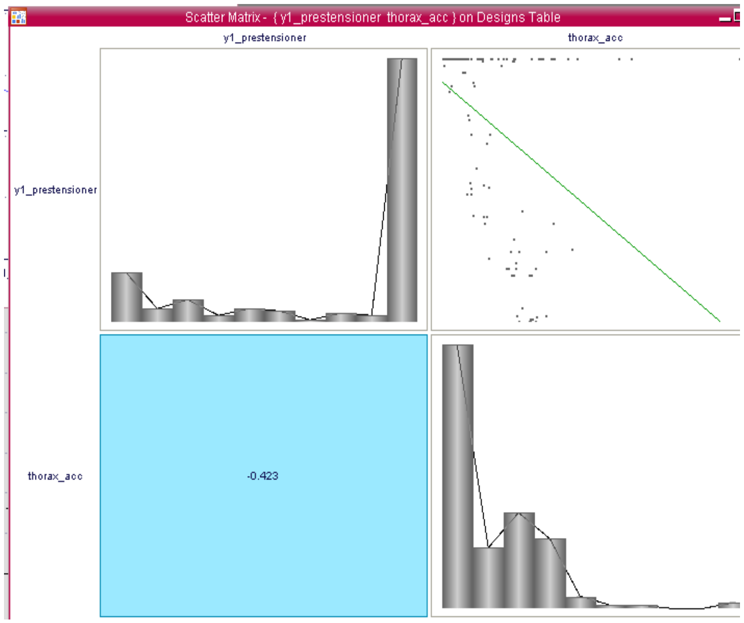


Factors	s [Significance]	v [Significance]
h	269.8 [0.070]	1980.9 [0.017]
r	671.2 [0.000]	3168.2 [0.000]
t	29.4 [0.438]	191.3 [0.429]

For each response (S and V), effect of inputs are reported in an overall chart on a common scale

Statistical analysis for correlations (Correlation chart and Scatter matrix)





Correlation chart:

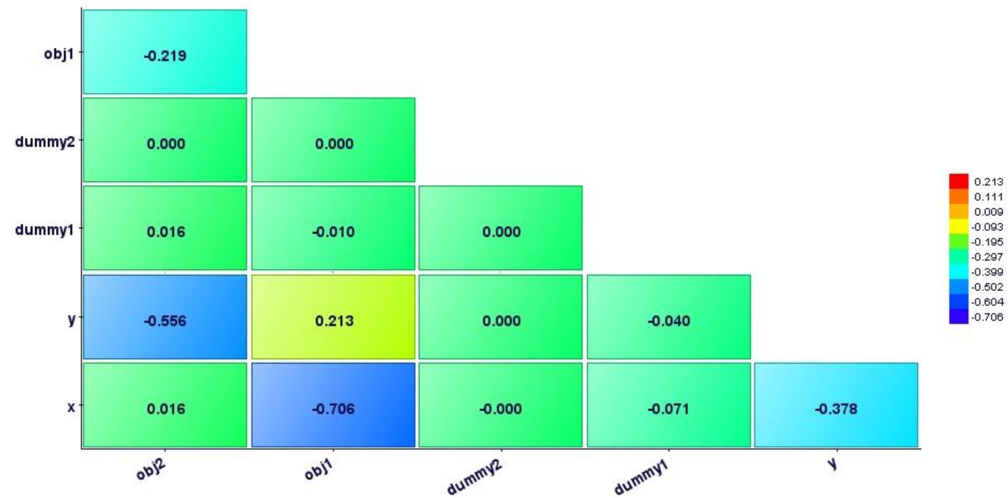
- +1 max direct correlation
- -1 max inverse correlation
- 0 no correlation

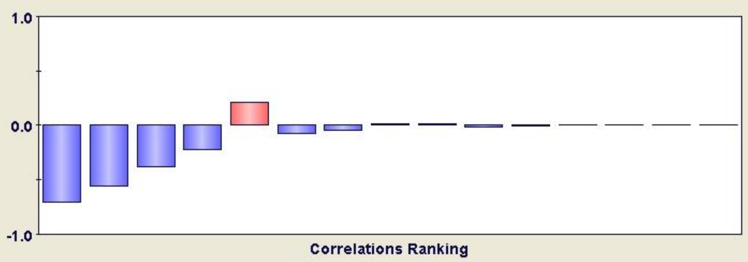
Scatter matrix:

- Report correlation, scatter plot and distribution charts for a pair of variables

Details on Correlation Matrix

- The correlation is a number (between -1 and 1) describing the degree of relationship between two variables
- The correlation is a measure of the linear association
- If it is exactly equal to 1, the two variables are perfectly positively correlated and the values all lie on a straight line with positive slope
- If it is equal to zero, the variables are uncorrelated, that is linearly unassociated
- If it is equal to -1, then the two variables are perfectly negatively correlated

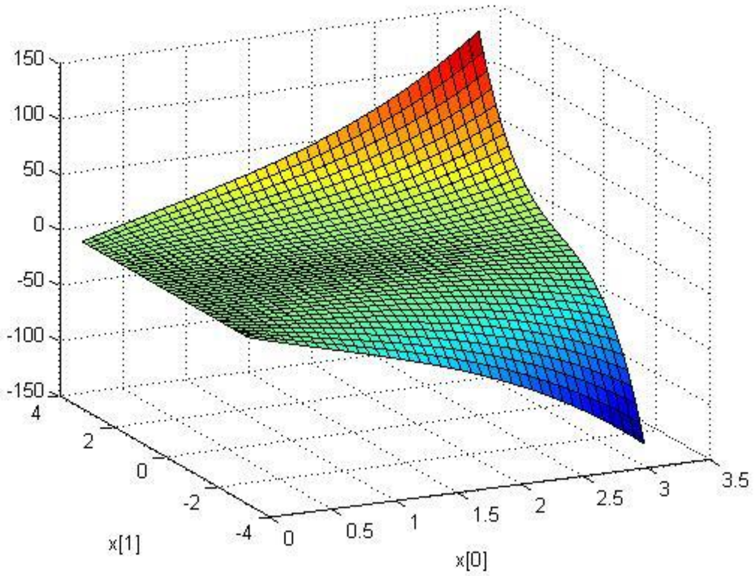


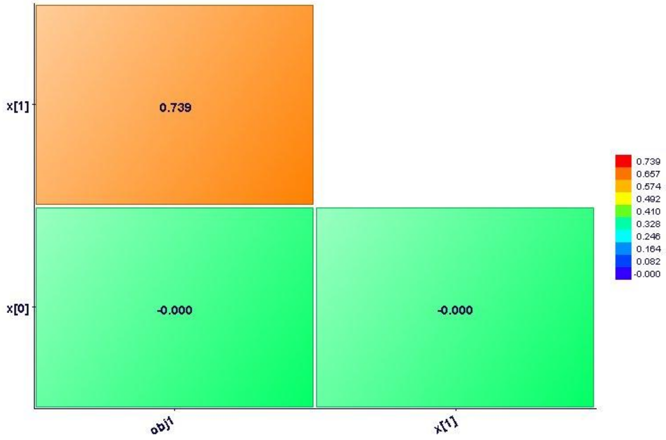


Statistics	
Absolute Max	0.706
Absolute Min	0.000
Arithmetic Mean	0.148

Statistical Analysis: **Interaction Effects**

Neither the t-Student test nor the correlation matrix are able to assess if interaction effects between factors exist.





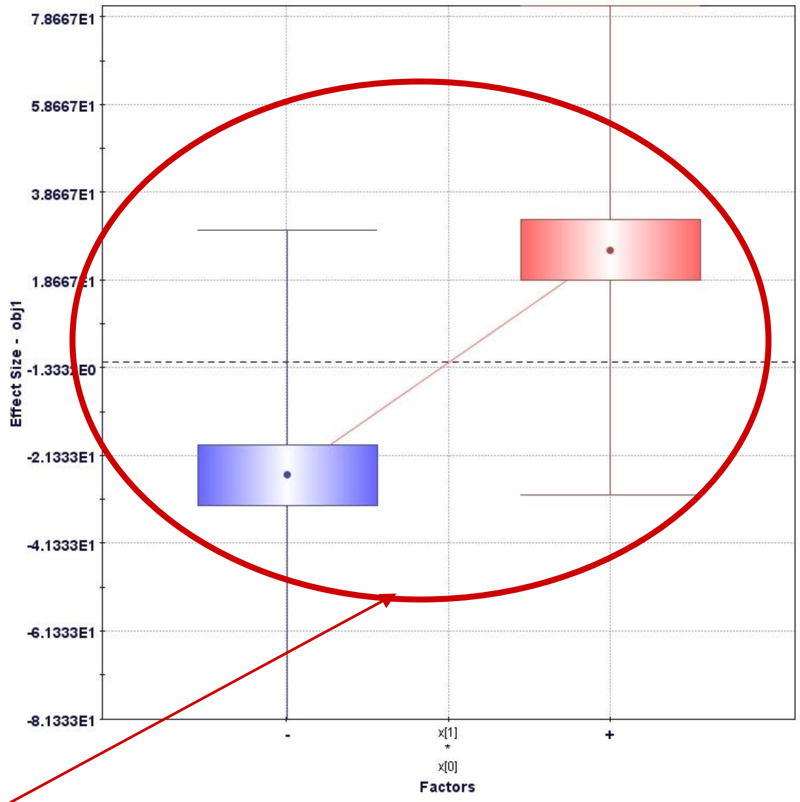
Factors	Effect Size	Significance	t-Student	critical t (α = 0.10)	critical t (α = 0.05)	critical t (α = 0.01)
x[1]	6.8079E1	0.009	2.7074E0	1.3450E0	1.7613E0	2.6245E0
x[0]	-3.1086E-15	0.500	1.0016E-16	1.4149E0	1.8946E0	2.9980E0

Statistical Analysis: Interaction Effects

By using DOE Interaction Effects and Effects Matrix is possible to assess if factors interactions play a role in the response variable.

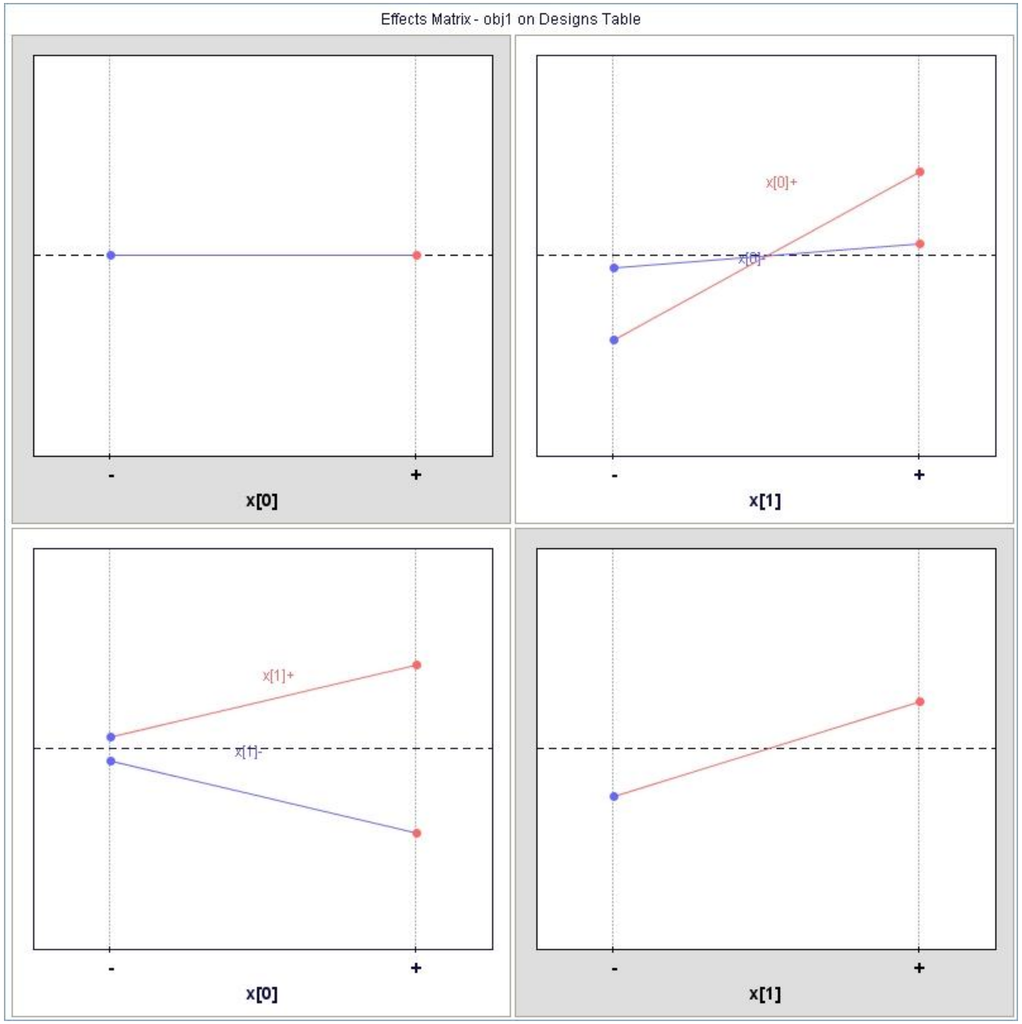
Statistical Analysis: Interaction Effects

Doe Interaction Effects



If x[0] and x[1] are both set at upper or lower values the response variable is higher, further information can be obtained by the effects matrix chart.

Statistical Analysis: Interaction Effects



It is possible to see that, though x[0] has not a main effect on the response variable, increasing both x[0] and x[1] an higher value of the response variable is obtained. On the other side if both variables are set to lower values there is no such an effect.

Example 1

How to use modeFRONTIER to get the most relevant qualitative information from a data-base of experiments

Statistical Analysis: Example 1

This experiment was conducted on a *catapult* – a table-top wooden device often used to teach design of experiments and statistical process control. The catapult has several controllable factors and a response easily measurable.

Statistical Analysis: Example 1

***Variables*:**

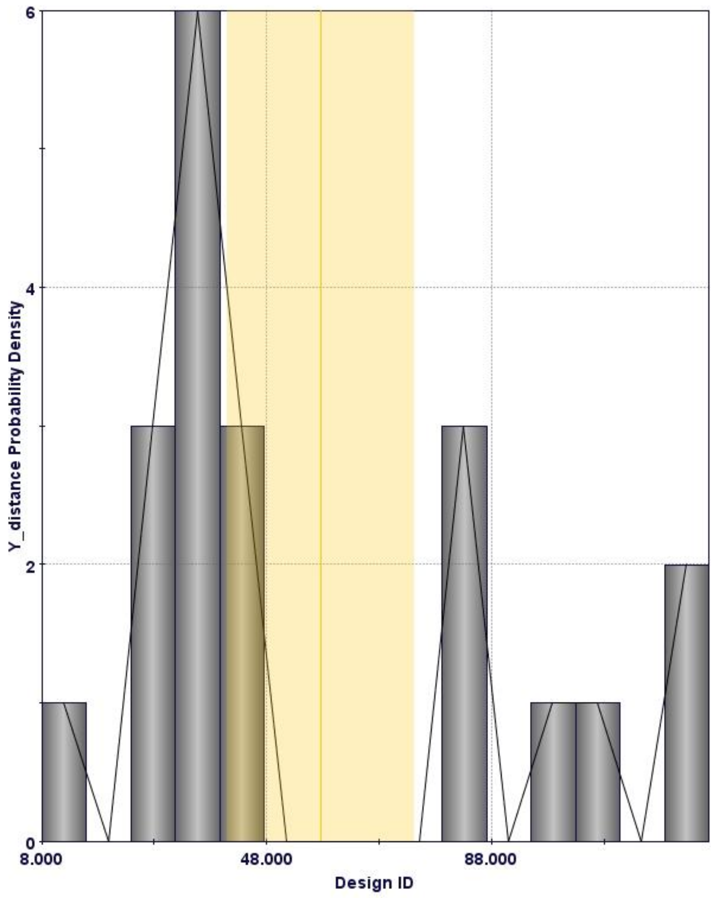
- **Response Variable *Y* = distance**
- **Factor 1 = band height** (height of the pivot point for the rubber bands – levels were 3.25 and 4.75 inches with a centerpoint level of 4) ;
- **Factor 2 = start angle** (location of the arm when the operator releases– starts the forward motion of the arm – levels were 0 and 20 degrees with a centerpoint level of 10 degrees)
- **Factor 3 = rubber bands** (number of rubber bands used on the catapult– levels were 1 and 2 bands)
- **Factor 4 = arm length** (distance the arm is extended – levels were 0 and 4 inches with a centerpoint level of 2 inches)
- **Factor 5 = stop angle** (location of the arm where the forward motion of the arm is stopped and the ball starts flying – levels were 45 and 80 degrees with a centerpoint level of 62 degrees)

A reduced factorial technique was used, the number of designs evaluated is $2^{5-1}=16$ +4 center points = 20 designs

Statistical Analysis: Example 1

Pattern	band height	start angle	rbands	arm length	stop angle	Y_distance
----+	3.25	0	1	0	80	28
00000	4	10	2	2	62	99
+++++	4.75	20	2	4	80	126.5
+-++-	4.75	0	2	4	45	126.5
-+++-	3.25	20	2	4	45	45
+----	4.75	0	1	0	45	35
00000	4	10	1	2	62	45
++--+	4.75	20	1	0	80	28.25
+--++	4.75	0	1	4	80	85
-+---	3.25	20	1	0	45	8
++-+-	4.75	20	1	4	45	36.5
---+-	3.25	0	1	4	45	33
00000	4	10	2	2	62	84.5
+++--	4.75	20	2	0	45	28.5
--+--	3.25	0	2	0	45	33.5
-++-+	3.25	20	2	0	80	36
+-+-+	4.75	0	2	0	80	84
-+-++	3.25	20	1	4	80	45
00000	4	10	1	2	62	37.5

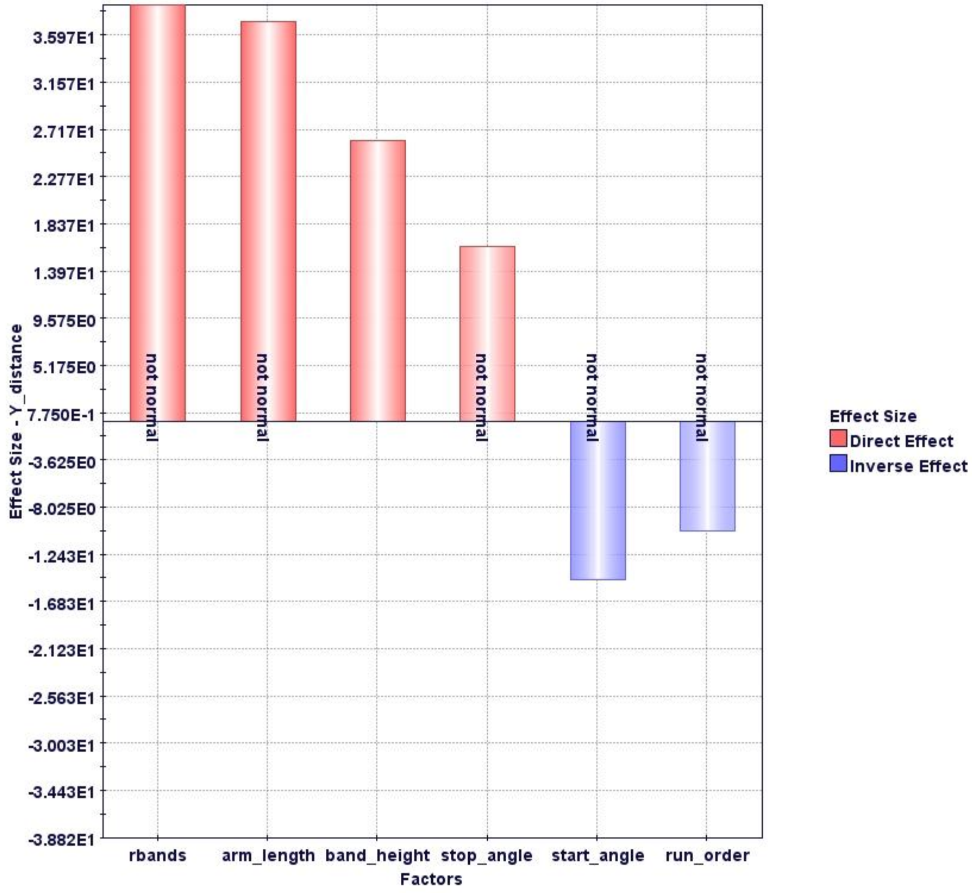
Statistical Analysis: Example 1

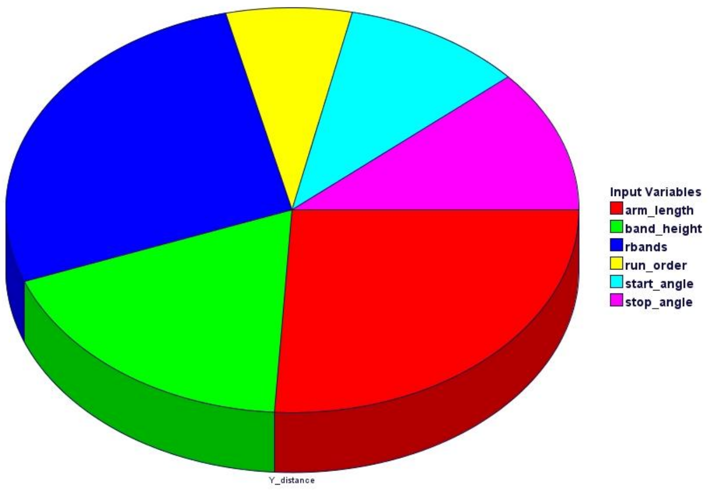


The probability density function and the box whiskers chart show the large spread of the data and a pattern that should be explained by the analysis.

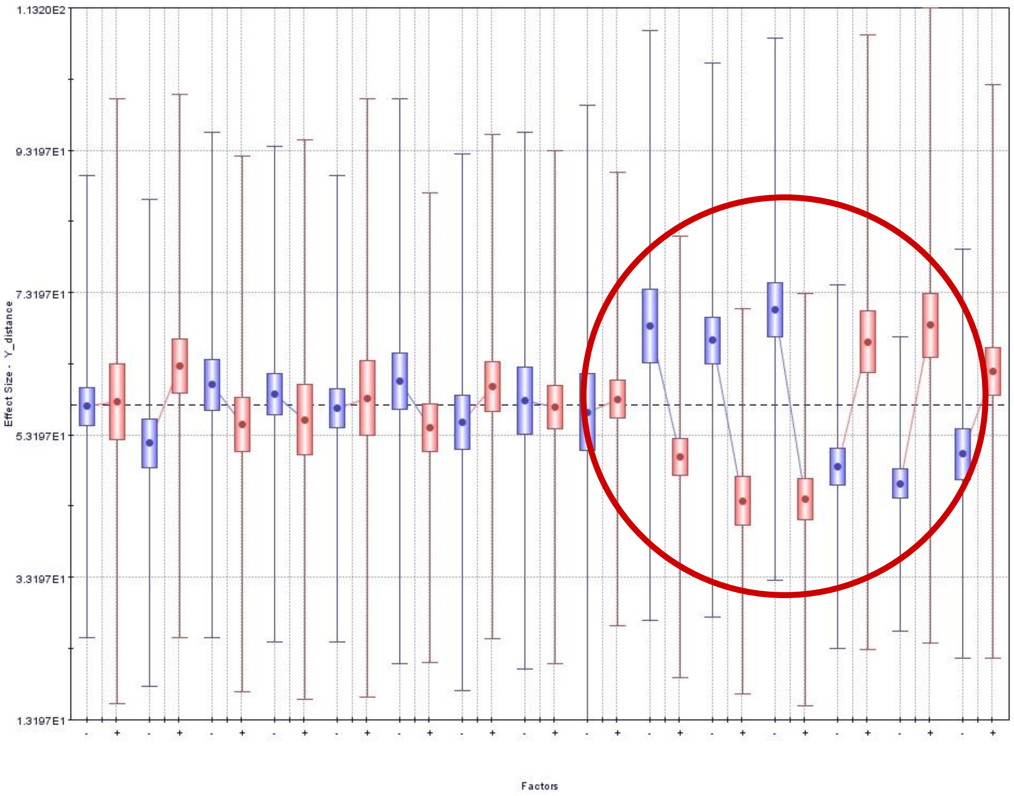
Statistical Analysis: Example 1

Student chart is useful to see the relevance and the effects of the different parameters.



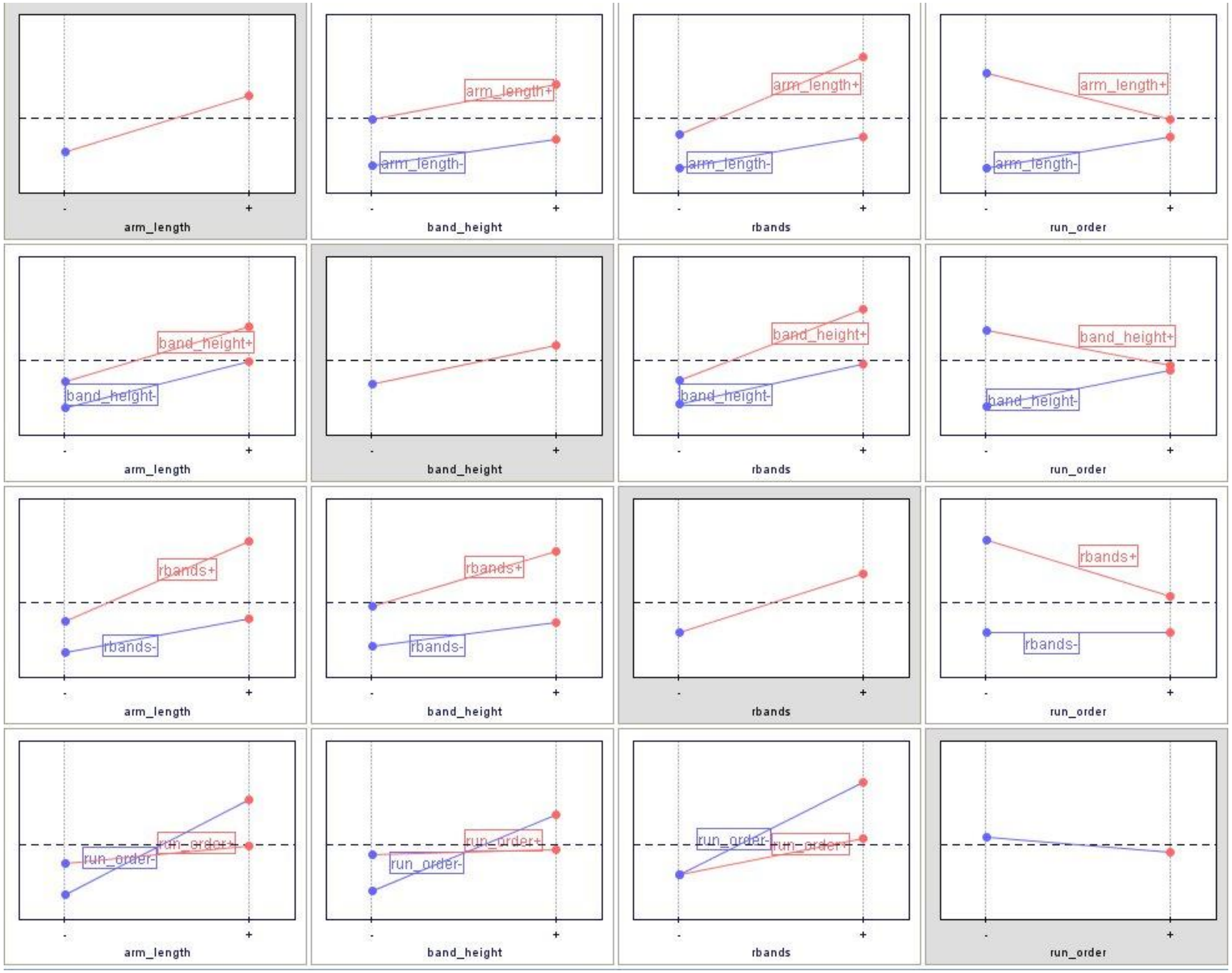


Statistical Analysis: Example 1



Interaction effects can play a role as well. This can be assessed by the DOE interaction effects chart: it is easy to understand if some parameters have a strong interaction. Further details can be gained by the matrix interaction charts.

Statistical Analysis: Example 1



- Run order interacts with arm length, band height and number of rubber bands.

- Bands height and number of rubber bands interact.

- Arm length and number of rubber bands interact.

Statistical Analysis

- An accurate assessment of the DOE is useful in any case: it gives a better insight of the problem and can reduce the complexity, limiting the number of variables and the variables definition range.

- Be aware: the statistical tools need DOE tables able to represent correctly all the design space.

Example 2

Choosing the proper DOE for statistical analysis

Statistical Analysis: Example 2

$$F_1(x,y) = -[1+(A_1+B_1)^2+(A_2+B_2)^2] \qquad F_2(x,y) = -[(x+3)^2+(y+1)^2]$$

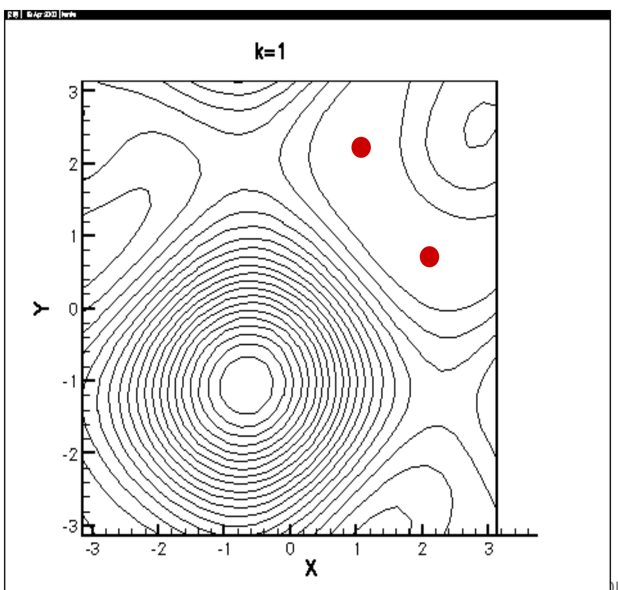
$$A_i = \sum_{j=1}^{2}(a_{i,j}\cdot\sin(\alpha_j)+b_{i,j}\cdot\cos(\alpha_j))$$

$$B_i = \sum_{j=1}^{2}(a_{i,j}\cdot\sin(\beta_j)+b_{i,j}\cdot\cos(\beta_j))$$

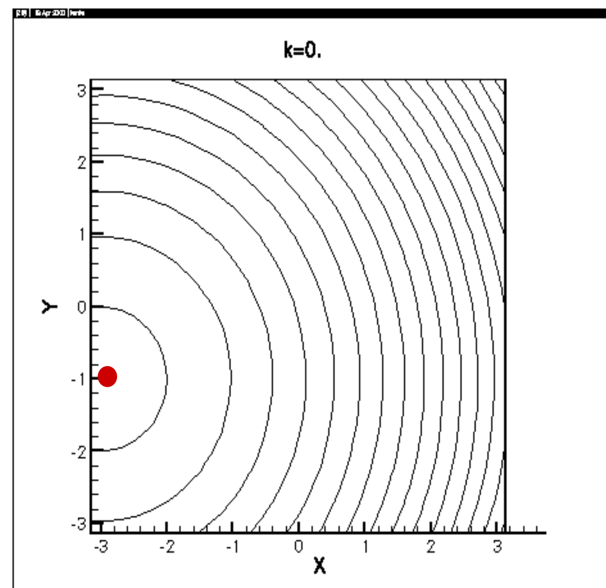
$$x,y \quad \in \quad [-\pi,\pi]$$

$$a = \begin{bmatrix} 0.5 & 1.0 \\ 1.5 & 2.0 \end{bmatrix} \qquad b = \begin{bmatrix} -2.0 & -1.5 \\ -1.0 & -0.5 \end{bmatrix} \qquad \alpha = [1.0 \quad 2.0]$$

$$\beta = (x,y) \quad \in \quad [-\pi,\pi]$$



Two different mathematical functions



Statistical Analysis: Example 2

16 Designs computed with Full Factorial

Id	x	y	dummy1	dummy2	o1	o2
0	-3.14	-3.14	-100	-100	-9.45804	-4.5992
1	-3.14	3.14	-100	-100	-9.45883	-17.1592
2	3.14	-3.14	-100	-100	-9.45444	-42.2792
3	3.14	3.14	-100	-100	-9.4553	-54.8392
4	-3.14	-3.14	-100	100	-9.45804	-4.5992
5	-3.14	3.14	-100	100	-9.45883	-17.1592
6	3.14	-3.14	-100	100	-9.45444	-42.2792
7	3.14	3.14	-100	100	-9.4553	-54.8392
8	-3.14	-3.14	100	-100	-9.45804	-4.5992
9	-3.14	3.14	100	-100	-9.45883	-17.1592
10	3.14	-3.14	100	-100	-9.45444	-42.2792
11	3.14	3.14	100	-100	-9.4553	-54.8392
12	-3.14	-3.14	100	100	-9.45804	-4.5992
13	-3.14	3.14	100	100	-9.45883	-17.1592

Initial input variables

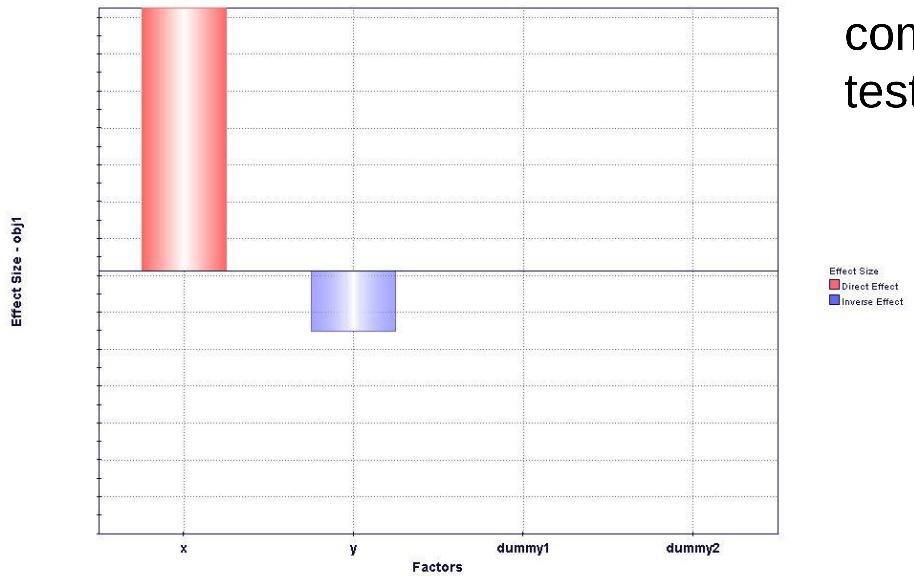
Added input variables

Results

Statistical Analysis: Example 2

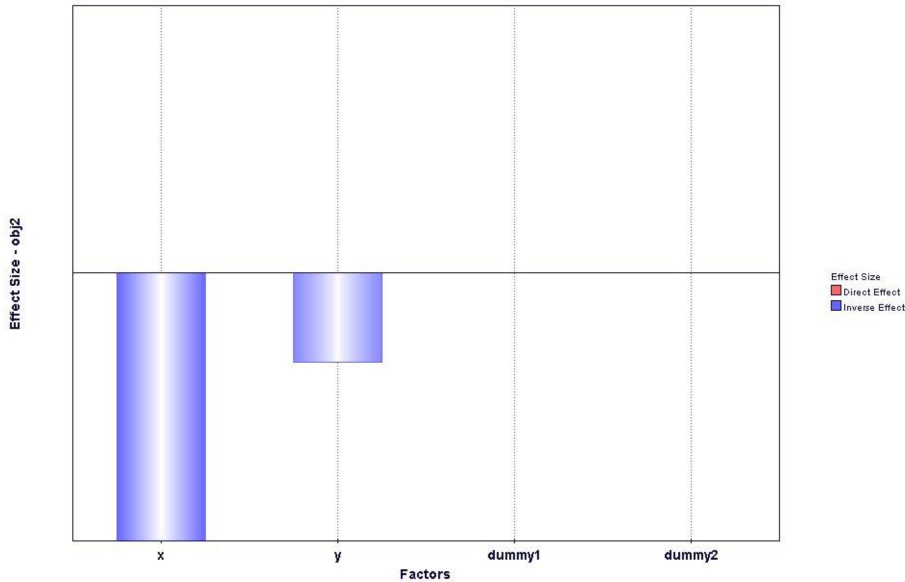
dummy1 and dummy2 have significance 0.5 in both functions.

Hint: *“The number of design variables can be reduced.”*



Factors	Effect Size	Significance	t-Student	critical t (α = 0.10)	critical t (α = 0.05)	critical t (α = 0.01)
x	3.5656E-3	0.000	1.6194E1	1.4149E0	1.8946E0	2.9980E0
y	-8.2307E-4	0.208	8.6368E-1	1.4149E0	1.8946E0	2.9980E0
dummy1	0.0000E0	0.500	0.0000E0	1.3450E0	1.7613E0	2.6245E0
dummy2	0.0000E0	0.500	0.0000E0	1.3450E0	1.7613E0	2.6245E0

Full factorial (or reduced factorial) gives a complete information on variables if t-Student test is used.



Factors	Effect Size	Significance	t-Student	critical t (α = 0.10)	critical t (α = 0.05)	critical t (α = 0.01)
x	-3.7680E1	0.000	1.1225E1	1.4149E0	1.8946E0	2.9980E0
y	-1.2560E1	0.116	1.2472E0	1.3450E0	1.7613E0	2.6245E0
dummy1	0.0000E0	0.500	0.0000E0	1.3450E0	1.7613E0	2.6245E0
dummy2	0.0000E0	0.500	0.0000E0	1.3450E0	1.7613E0	2.6245E0

Statistical Analysis: Example 2

16 Designs computed with Random DOE

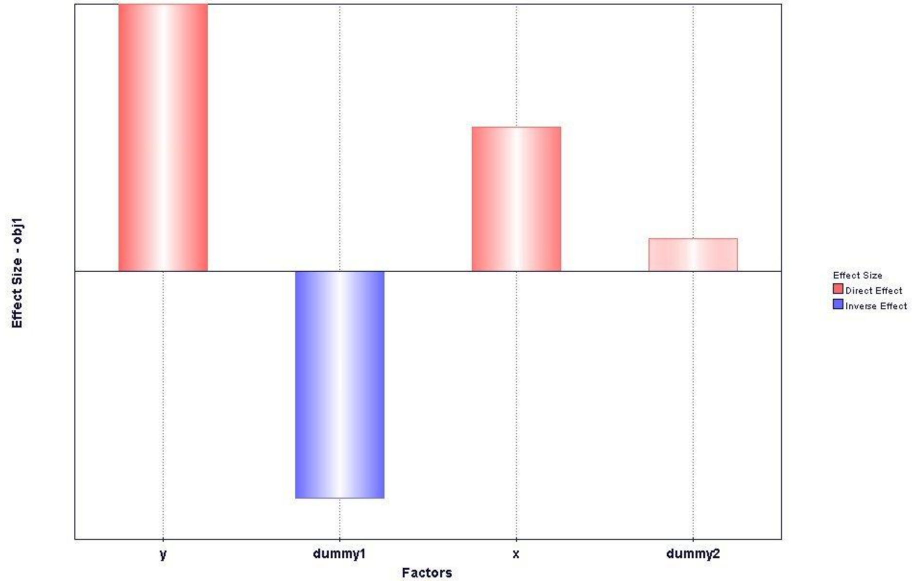
Id	x	y	dummy1	dummy2	o1	o2
0	-1.83551	-1.05044	46.17462	-17.9918	-42.0142	-1.35857
1	2.912641	2.762534	93.55936	-98.7799	-9.76235	-49.116
2	-0.64596	-0.95748	89.43894	87.41874	-61.4466	-5.5433
3	-2.41208	1.699224	-41.1941	1.290129	-4.06747	-7.63147
4	-0.76467	-2.2626	31.9732	-68.6569	-39.9631	-6.59087
5	-3.1086	0.145397	38.9939	61.05611	-5.06738	-1.32373
6	-0.11462	0.279802	48.79488	-71.5972	-32.432	-9.9633
7	0.774715	-1.97997	15.43154	-59.0159	-22.5719	-15.2088
8	-2.02205	0.253423	-97.8798	-67.7968	-17.0172	-2.52746
9	-0.66229	-1.77334	94.77948	-50.9151	-52.725	-6.06293
10	2.449131	-2.89945	-13.5914	-53.3753	-6.8771	-33.3009
11	-2.38758	0.957482	18.47185	31.0331	-3.9244	-4.20679
12	-0.78728	-0.23019	96.87969	-58.6559	-49.5447	-5.48876
13	0.026065	3.133719	-33.2733	-11.3511	-13.0261	-26.2447

Initial input variables (x, y)

Added input variables (dummy1, dummy2)

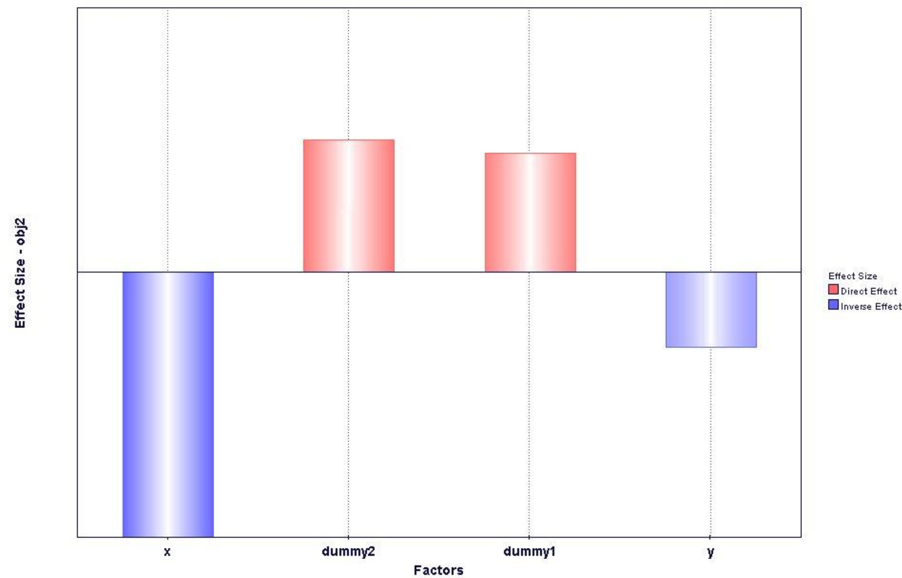
Results (o1, o2)

Statistical Analysis: Example 2



Factors	Effect Size	Significance	t-Student	critical t (α = 0.10)	critical t (α = 0.05)	critical t (α = 0.01)
y	2.8305E1	0.001	3.9826E0	1.3450E0	1.7613E0	2.6245E0
dummy1	-2.3942E1	0.010	2.6057E0	1.3450E0	1.7613E0	2.6245E0
x	1.5318E1	0.072	1.5461E0	1.3450E0	1.7613E0	2.6245E0
dummy2	3.5457E0	0.378	3.1781E-1	1.3450E0	1.7613E0	2.6245E0

Random DOE does not provide reasonable coverage of the experiments space unless the number of samples is large enough to cover uniformly the variables range.



The variable significances are not correct.

Factors	Effect Size	Significance	t-Student	critical t (α = 0.10)	critical t (α = 0.05)	critical t (α = 0.01)
x	-2.2990E1	0.005	3.9412E0	1.4737E0	2.0106E0	3.3508E0
dummy2	1.1509E1	0.070	1.5678E0	1.3450E0	1.7613E0	2.6245E0
dummy1	1.0342E1	0.094	1.3855E0	1.3450E0	1.7613E0	2.6245E0
y	-6.4465E0	0.192	8.9852E-1	1.3450E0	1.7613E0	2.6245E0