The boundary Riemann solver coming from the real vanishing viscosity approximation

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It is considered the hyperbolic-parabolic approximation

$$\begin{cases} v^\varepsilon_t + \tilde{A}\big(v^\varepsilon,\, \varepsilon v^\varepsilon_x\big) v^\varepsilon_x = \varepsilon \tilde{B}(v^\varepsilon) v^\varepsilon_{xx} & v^\varepsilon \in \mathbf{R}^N \\ v^\varepsilon(t,\, 0) \equiv \bar{v}_b & \\ & \\ v^\varepsilon(0,\, x) \equiv \bar{v}_0 & \end{cases}$$

of an hyperbolic boundary Riemann problem. The conservative case is, in particular, included in the previous formulation.

The hypotheses imposed on the matrices $\tilde{A}$ and $\tilde{B}$ are essentially those introduced by Kawashima and most of them are implied, in the conservative case, by the presence of a dissipative entropy. In particular, the boundary characteristic case is allowed, i.e. one eigenvalue of $\tilde{A}$ is allowed to be close to zero. Moreover, no hypothesis of invertibility is made on the viscosity matrix $\tilde{B}$.

It is also introduced on the structure of matrix $\tilde{A}$ a new condition of block linear degeneracy. It is shown that this condition is essential, even in the boundary free case, in order to prevent the appearance of pathological behaviors.

It is assumed that the approximating solutions v^ε converge in L^1_{loc} to a unique limit, which is supposed to depend continuously in L^1 with respect to the initial and the boundary data. Moreover, it is assumed that in the hyperbolic limit the disturbances travel with finite propagation speed.

Under these hypotheses, it is given a complete characterization of the boundary Riemann solver induced in the hyperbolic limit when the difference between the boundary and the initial datum is small.

The most interesting behavior is observed when the viscosity matrix is not invertible. Indeed, if one interprets the boundary datum $\bar{v}_b$ as the trace of the solution on the axis $x = 0$, it turns out that when $\tilde{B}$ is singular the initial boundary value problem associated to the parabolic approximation is, in general, ill posed for a boundary datum $\bar{v}_b \in \mathbf{R}^N$. Thanks to a perturbation technique, it is obtained a precise characterization of the admissible boundary conditions. Finally, it is described a way of assigning the boundary datum which is coherent with the admissibility criterium obtained.