Long time behavior of particles systems in the mean-field limit

A system of N particles, interacting via a two-body potential, in the mean field limit is described by the Vlasov equation. The approximation is proven to be good for times of the order of log N.

What happens for longer times? While it is reasonable (and numerically verified) that asymptotically the particle system will reach the Maxwellian equilibrium it is not clear how and in how long times.

An interesting case is when initially the particles are extracted from a distribution which is a stable stationary solution of the Vlasov equation. In fact, the Vlasov equations admits stable stationary solution which are not equilibrium states for the particle system (i.e. not maxwellian). We prove that in this case the particles system will stay close to this state at least for times of the order of $N^{1/8}$, so that these states can be interpreted as quasi-stationary states of the particle system.

The results presented here are obtained in collaboration with F. Rousset.