Discontinuous solutions for the Degasperis-Procesi equation

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Consider the *Degasperis-Procesi equation*

$$u_t - u_{txx} + 4uu_x = 3u_x u_{xx} + uu_{xxx}, \qquad (t,x) \in (0,\infty) \times \mathbb{R}\,,$$

which can be viewed as a shallow water approximation to the Euler equations. We are interested in the Cauchy problem for this equation, so we augment it with an initial condition $u_0 \in L^1(\mathbb{R}) \cap BV(\mathbb{R})$.

Formally, the problem is equivalent to an hyperbolic-elliptic system or to a conservation law with a nonlocal flux function. We prove the exstistence and uniqueness of the entropy weak solution, that is a distributional solution satisfying some additional entropy conditions.

In addition the unique entropy weak solution satisfies the Oleinik type estimate

$$u_x(t,x) \leq K_T \left(1 + \frac{1}{t}\right), \qquad 0 < t \leq T,\ \ x \in \mathbb{R}\,,$$

for some positive constant K_T depending on T and on the total variation of u_0. An implication is that a shock wave in an entropy weak solution to the Degasperis-Procesi equation is admissible only if it jumps down in value (like the inviscid Burgers equation).