Artificial compressibility approximation for the incompressible Navier - Stokes equations in $3-D$

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We study the approximation of the Leray weak solution of the $3D$ Navier Stokes equations

$$\begin{cases} u_t + (u \cdot \nabla)\, u - \nu \Delta u + \nabla p = 0 \\ \operatorname{div} u = 0 \\ u(x,0) = u_0(x), \end{cases} \tag{1}$$

where $(x,t) \in \mathbf{R}^3 \times [0,T]$, $u = u(x,t) \in \mathbf{R}^3$ is the particle fluid velocity, $p = p(x,t) \in \mathbf{R}$ is the fluid pressure and ν is the viscosity.
We introduce the following approximating system

$$\begin{cases} u_t^\varepsilon + \nabla p^\varepsilon = \nu \Delta u^\varepsilon - (u^\varepsilon \cdot \nabla)\, u^\varepsilon - \dfrac{1}{2}(\operatorname{div} u^\varepsilon) u^\varepsilon \\ \varepsilon p_t^\varepsilon + \operatorname{div} u^\varepsilon = 0. \end{cases} \tag{2}$$

By exploiting the wave equation structure of the pressure in the system (2), we achieve the convergence of the approximating sequences by means of dispersive estimates of Strichartz type combined with nonlinear waves interactions estimates for bilinear terms. We prove that the projection of the approximating velocity fields on the divergence free vectors is relatively compact and converges to a Leray weak solution of the incompressible Navier Stokes equations.

References

[1] D. Donatelli. P. Marcati. A Dispersive approach to the artificial compressibility approximations of the Navier Stokes equations in $3-D$. *Journal of Hyperbolic Differential Equations*, **3** (2006), pp. 1-14.