Strichartz estimates for Dirac and Schroedinger equations with a magnetic potential

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In this joint work with P. D'Ancona, we consider the Dirac and Schrödinger equations with magnetic potentials:

$$\begin{cases} iu_t + \mathcal{H}_0 u + V(x)u = 0 \qquad \text{in } \mathbb{R}^{1+3} \\ u(0,x) = f(x), \end{cases}$$

$$\begin{cases} iv_t + \Delta v + W(x,D)v = 0 \qquad \text{in } \mathbb{R}^{1+n} \\ v(0,x) = g(x), \end{cases}$$

where $\mathcal{H}_0$ is the Dirac operator, the potential V is matrix valued and $W(x,D) = A(x)\cdot\nabla + b(x)$, with $A : \mathbf{R}^n \to \mathbf{R}^n$, $b : \mathbf{R}^n \to \mathbf{R}$. We prove that the full set of Strichartz estimates hold as in the free case, if the potentials are assumed to satisfy the following assumptions:

$$|V(x)| \leq \frac{C}{|x^{1-}| + |x|^2}, \qquad C - \text{small},$$

$$|A(x)| \leq \frac{C}{|x^{1-}| + |x|^2}, \qquad C - \text{small},$$

$$|b(x)| \leq \frac{1}{|x^{1-}| + |x|^2};$$

we assume, moreover, that 0 is not a resonance for b and $\langle x \rangle^{3/2+} A \in \dot{C}^{1/2+}$. On the other hand, no regularity assumptions for V are required.

The techniques adopted are a suitable mix of free Strichartz estimates, smoothing effect (for Schrödinger) and resolvent estimates involving a theorem by Kato, all based on a TT^*-like argument.