The p-System at a junction

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We consider the evolution of a non viscous isentropic or isothermal fluid, described by the p-system in Eulerian coordinates, in n tubes exiting a single junction. The simplest examples are elbows and junctions with 3 tubes; see Figure 1. Each tube is modeled by a copy of the

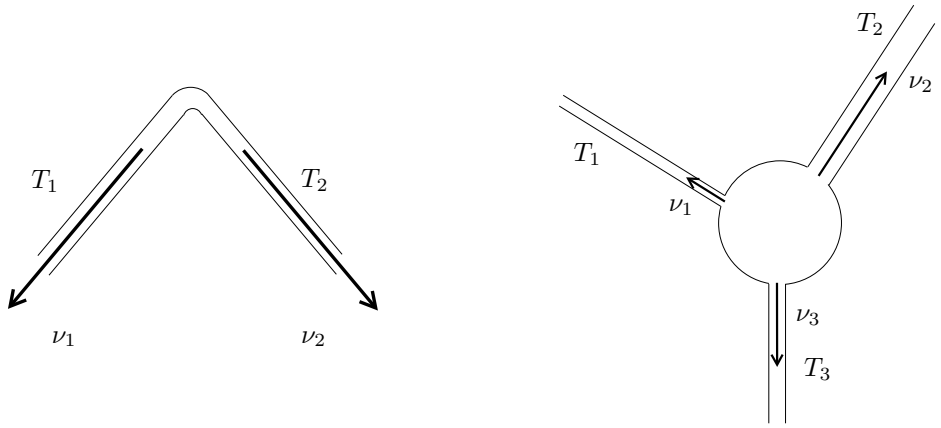

Figure 1: Left: an elbow. Right: a junction connecting 3 tubes.

real half line, the relative positions and cross sections of the ducts also have an essential role in the model. Our goal is the description of fluid flow through 1D conservation laws (along the tubes) equipped with suitable coupling conditions at the junction. Usually, in the engineering literature, these descriptions are obtained through 2D or 3D models, computationally far more expensive.

First, we state the Riemann problem at the junction. Different approaches to the definition of its solution are available. We present two different frameworks, the former motivated by the search for the well posedness of the Cauchy problem, the latter by physical considerations. The two approaches yield the same solutions and allow to extend to the case of the junction several properties of Lax solutions to standard Riemann problems.

Second, we consider the Cauchy problem, obtaining the existence and well posedness of solutions to Cauchy problems on the junction by means of the wave-front tracking technique.

Finally, we compare the above constructions with the model [1] proposed by Banda, Herty and Klar.

References

[1] M. K. Banda, M. Herty, A. Klar. Gas flow in pipeline networks. *Networks and Heterogeneous Media (NHM)*, **1** (2006), pp. 41-56.

[2] R. M. Colombo, M. Garavello. A well posed Riemann problem for the p-system at a junction. *Networks and Heterogeneous Media (NHM)*, to appear.

[3] R. M. Colombo, M. Garavello. On the p-system at a junction. *Preprint - Quaderno del Seminario Matematico di Brescia*, n°8, (2006).