Variable unilateral constraints and non classical shocks

Paola Goatin - *University of Sud Toulon - Var*
Joint work with Rinaldo M. Colombo (University of Brescia)

We consider the following constrained Cauchy problem:

$$\text{(CCP)} \qquad \begin{cases} \partial_t u + \partial_x f(u) = 0 & u \in \mathbf{R} \\ u(0,x) = \bar{u}(x) & x \in \mathbf{R} \\ f\left(u(t,0)\right) \leq \tilde{f}(t) & t \in [0,+\infty[\end{cases}$$

for a scalar conservation law. This problem may serve as a model for a toll gate along a road, u being the (mean) traffic density, f the (mean) traffic flow and $\tilde{f}$ the throughput of the toll gate.

A first result is obtained through the wave front tracking technique. The key point in this construction is the choice of a suitable non classical Riemann solver at $x = 0$ and the availability of a notion of entropy solutions.

Second, we consider a class of conservation laws with a discontinuous space-time dependent flux. A suitable limiting procedure on these problems yields weak entropy solutions to (CCP). This allows to strengthen the first result.

References

[1] R. M. Colombo, and P. Goatin. A Well Posed Conservation law with a Variable Unilateral Constraint *Preprint*, 2006.