The Riemann problem for Euler equations at a junction

Cristina Mauri - *University of Brescia*
Joint work with Rinaldo M. Colombo (University of Brescia)

We pose the Riemann problem for the full Euler 3×3 system of gas dynamics at a junction and estabilish its well posedness in a number of cases.

More precisely, each duct exiting the junction is described through a copy of the positive half real axis and the evolution of the fluid in the duct is described by the Euler system

$$\left\{ \begin{array}{cccl} \partial_t \rho & + & \partial_x q & = 0 \\ \partial_t q & + & \partial_x P(\rho, q, E) & = 0 \\ \partial_t E & + & \partial_x F(\rho, q, E) & = 0 \end{array} \right. \qquad \begin{array}{l} t \geq 0 \\ x \geq 0 \end{array} \tag{1}$$

where
rho is the fluid density, q is its linear momentum, p the pressure, e the internal energy and

$$P(\rho, q, E) = \frac{q^2}{\rho} + p(\rho, e), \quad F(\rho, q, E) = \frac{q}{\rho}\left(E + p(\rho, e)\right), \quad E = \rho\, e + \frac{1}{2}\frac{q^2}{\rho}.$$

By Riemann problem at a junction with n tubes we mean the problem consisting of n copies of (1), each equipped with constant initial data. This talk presents two different definitions of solution for this problem that extend the constructions obtained in [2, 3] for the p-system. In particular, we state under which conditions the two definitions lead to the same solution. We provide examples in which the two constructions yield over- or underdetermined problems. We also show that whenever the two definitions coincide, the resulting solution keeps the main properties of Lax solutions to standard Riemann problems.

Finally, we show some preliminary results towards the well posedness theory for the Cauchy problem at a junction.

References

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