Well posedness of balance laws with non–characteristic boundary

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We study the well posedness of the following initial-boundary value problem for a nonlinear system of balance laws

$$\left\{\begin{array}{rcll} \partial_t u + \partial_x f(u) & = & g(t,x,u) & (t,x)\in\Omega \\ u(t_o,x) & = & \bar{u}(x) & x \le \Psi(t_o) \\ b\left(u\left(t,\Psi(t)\right)\right) & = & h(t) & t \ge t_o \end{array}\right. \tag{1}$$

in the non-characteristic case. Here $\bar{u}$, h are L^1 functions with small total variation, b is smooth, $t_o \in \mathbf{R}$ is the initial time, $\Omega = \{(t,x) \in \mathbf{R}^2 \colon t \ge t_o,\ x \le \Psi(t)\}$ for a suitable Lipschitz map $\Psi \colon [t_o, +\infty[\mapsto \mathbf{R}$, and $u \in \mathcal{U}$ denotes the unknown vector of the conserved quantities.

We consider separately the convective part

$$\left\{\begin{array}{rcll} \partial_t u + \partial_x f(u) & = & 0 & (t,x)\in\Omega \\ u(t_o,x) & = & \bar{u}(x) & x \le \Psi(t_o) \\ b\left(u\left(t,\Psi(t)\right)\right) & = & h(t) & t \ge t_o \end{array}\right. \tag{2}$$

and the source part

$$\left\{\begin{array}{rcll} \partial_t u & = & g(t,x,u) & (t,x)\in\Omega \\ u(t_o,x) & = & \bar{u}(x) & x \le \Psi(t_o) \\ b\left(u\left(t,\Psi(t)\right)\right) & = & h(t) & t \ge t_o \end{array}\right. \tag{3}$$

of (1). We require those assumptions on f and g that make (2) and (3) well posed, and ask that there exists a domain (not necessarily compact) which is invariant for both (2) and (3). Finally, to obtain the well posedness globally in time, we assume that (2) is a Temple system. In fact, we need to require on (2) hypotheses that ensure the well posedness for large data because the total variation and the L^∞ norm of the solution may well grow exponentially with time.

We follow the definition [1, Definition NC] of solution to the boundary value problem (2). This framework is suitable, for instance, in applications to traffic modeling, where no physical viscosity is present.

The operator splitting scheme gives an approximate solution to (1) starting from an approximate solution to (2), given by the wave front tracking procedure [2, 3, 4], and an approximate solution to (3).

The results achived, related to [1, 5, 6], extend those in [3, 4].

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