Relaxed schemes for non linear convection-diffusion problems

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Several relaxation approximations to partial differential equations have been recently proposed, including conservation laws, non-linear degenerate diffusion problems [1, 3, 4, 5, 6]. The present work focuses onto suitable relaxation schemes for the numerical approximation of non-linear convection-diffusion equations. These schemes are based on an appropriate semilinear hyperbolic system with relaxation terms. In particular the convective and diffusive terms are relaxed separately. Moreover the numerical scheme is considered in the relaxed form, i.e. when the relaxation parameter $\varepsilon \to 0$. High order methods are obtained by coupling ENO and WENO schemes [7] for space discretization with IMEX schemes for time integration [2]. Linear stability analysis is provided together with various numerical results.

References

[1] D. Aregba-Driollet, R. Natalini, S. Tang. Explicit diffusive kinetic schemes for nonlinear degenerate parabolic systems. *Math. Comp.*, **73** (2004), pp. 63-94 (electronic).

[2] U. Asher, S. Ruuth, R.J. Spiteri. Implicit-explicit Runge-Kutta methods for time dependent Partial Differential Equations. *Appl. Numer. Math.*, **25** (1997), pp. 151-167

[3] F. Cavalli, G. Naldi, G. Puppo, M. Semplice. High order relaxation schemes for non linear diffusion problems. *Technical report*, `http://arxiv.org/abs/math?papernum=0604572`, (2006).

[4] S. Jin, Z. Xin. The relaxation schemes for systems of conservation laws in arbitrary space dimension. *Comm. Pure and Appl. Math.*, **48** (1995), pp. 235-276.

[5] P.L. Lions, G. Toscani. Diffusive limit for two-velocity Boltzmann kinetic models. *Rev. Mat. Iberoamericana*, **13** (1997), pp. 473-513.

[6] G. Naldi, L. Pareschi, G. Toscani. Relaxation schemes for partial differential equations and applications to degenerate diffusion problems. *Surveys Math. Indust.*, **10** (2002), pp. 315-343.

[7] C.W. Shu, S. Osher. Efficient implementation of essentially nonoscillatory shock-capturing schemes. II. *J. Comput. Phys.*, **83** (1989), pp. 32-78.