

Estimates for ordinary differential equations with Sobolev coefficients

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Inspired by a result of Ambrosio, Lecumberry and Maniglia, we show a simple derivation of integral logarithmic bounds for solutions of ordinary differential equations

$$\begin{cases} \frac{d\Phi(t, x)}{dt} = b(t, \Phi(t, x)) \\ \Phi(0, x) = x. \end{cases}$$

These bounds depend only on the L^∞ and $W^{1,p}$ ($p > 1$) norm of b , and on the compressibility constant of Φ (which in turn can be bound by $\|\operatorname{div} b\|_\infty$). These a-priori estimates allow to recover in a simple way many known old and new results about existence, uniqueness, stability, and differentiability properties of solutions of ODE's with Sobolev coefficients. In addition, as a new corollary, we conclude that the Cauchy problem for transport equations with Sobolev coefficients preserve a mild regularity property of the initial data.