

On Hamiltonian perturbations of hyperbolic PDEs

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The simplest nonlinear hyperbolic PDE with one spatial dimension

$$u_t + a(u)u_x = 0$$

can be considered as an infinite-dimensional analogue of a completely integrable Hamiltonian system. Adding terms with highest derivatives can be considered as a small perturbation while dealing with slow varying solutions. Surprisingly enough the complete integrability survives up to some high order of an arbitrary Hamiltonian perturbation. We will discuss the role of this “approximate integrability” in the study of the problem of critical behaviour of solutions to the perturbed equation near the point of gradient catastrophe of the unperturbed one.