Hyperbolic polynomials

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Hyperbolic polynomials are symbols of linear hyperbolic partial differential operators. They may come from scalar operators of arbitrary order, as well as from first order systems, through a characteristic polynomial. Roughly speaking, they are defined by the condition that $t \mapsto P(te - \xi)$ has only real roots $\lambda_j(\xi)$. The vector e is a direction of hyperbolicity.

The study of HPs began with L. Garding in 1952, who proved several convexity results, one of them having an important role in fully nonlinear elliptic equations !! Other inequalities were proved by Lax and Weinberger in 1958. Since then, HPs have invaded several mathematical areas, like real algebraic geometry, combinatorics and optimization.

P. Lax (1958) made a conjecture about the representation of HPs in three variables, recently proved by Helton and Vinnikov. A by-product is that every linear inequality valid between the eigenvalues of three Hermitian matrices A, B and $A+B$, is valid for the roots of a HP at points ξ, η and $\xi+\eta$. However this is a far from elementary way to prove even the simplest inequalities.

In this talk, I shall give an overview of the involvement of HPs in various topics. Then I shall provide a rather elementary proof that the roots at ξ, η and $\xi + \eta$ satisfy the analogue of Weyl and Lidskii–Wielandt inequalities, as well as a few other. However, the method does not provide all the relevant inequalities, which were described by A. Horn.