11^{th} *Meeting on Nonlinear Hyperbolic PDEs and Applications*
On the occasion of 60^{th} *birthday of Alberto Bressan*

Book of Abstracts

June 13-17, 2016 - SISSA (Trieste, Italy)

Well-posedness of weak solutions to the kinetic Kuramoto model

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This talk is devoted to the kinetic Kuramoto model, arising in the study of collective synchronized phenomena in a mean-field limit, as the number of oscillators tends to infinity. This model is represented by a scalar conservation law for the distribution function having a non-local flux due to the mean-field interactions among the oscillators.

The aim of the talk is to present an analysis of bounded weak solutions for this equation: existence of BV solutions, their stability and their large time behavior. The analysis is based on a modified wave-front tracking algorithm which is suitable for scalar conservation laws with nonlocal terms. This study reveals the interesting phenomena of solutions whose sup-norm may grow to infinity as t goes to infinity.

Joint work with: Seung-Yeal Ha (SNU, Seoul, Korea) and Jinyeong Park (SNU, Seoul, Korea).

Joint works with: Seung-Yeal Ha (*SNU, Seoul, Korea*) and Jinyeong Park (*SNU, Seoul, Korea*).

Global existence for a model of multi-phase flow with one or two interfaces

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We consider a hyperbolic system of three conservation laws modeling the flow of an inviscid fluid in lagrangian variables that undergoes phase transitions ([1]). We study the global in time existence of weak solutions to the initial-value problem for large initial data, focusing on the case of data consisting of either two ([2]) or three ([3],[4],[5]) different phases separated, respectively, by one or two interfaces that are contact discontinuities. The latter case translates into the modeling of a tube divided into three regions, where the fluid lies in a specific phase, separated by two stationary phase transitions. This gives rise to the interesting cases in the applications, given by a drop of liquid in a gaseous environment ([5]) or a bubble in a liquid one ([3]). In particular, we find explicit bounds on the (possibly large) initial data in order that weak entropic solutions exist for all times. The proof exploits a carefully tailored version of front tracking scheme.

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Joint work with: Debora Amadori (*University of L'Aquila*), Edda Dal Santo (*University of Udine and Lublin University of Technology*), Andrea Corli (*University of Ferrara*).

A simple uniqueness result for the transport equation with Sobolev, continuous field

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Transport phenomena are ubiquitous in physics, starting from the dynamics of liquids and gasses. The transport / continuity equation

$$\partial_t u + b \cdot \nabla u = 0$$

is the linear case of a conservation law, and it is an essential building brick in modelling. A very rich theory was built and is being built up to now, among which the seminal works by Ambrosio-DiPerna-Lions provide well posedness in the context of vector fields which are either Sobolev or of bounded variation. Yet, the picture is not complete.

Important issues in the analysis arise from the fact that flows of the ODE associated to the transport equation are not smooth in the space variables, even when considering Regular Lagrangian Flows. This makes a challenge passing from the distributional PDE formulation (Eulerian) to the ODE one (Lagrangian). The talk is mostly based on an ongoing collaboration with Gianluca Crippa (Basel), where we address how to derive uniqueness of L^1 solutions for Sobolev vector fields which are also continuous by somehow reducing the Eulerian formulation to the Lagrangian one. This is an interesting situation where DiPerna-Lions theory does not apply, as it would require higher integrability of the solution.

Properties of solutions to variational problems

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We consider several related properties of solutions, in particular the problem of existence (for functionals of slow growth); the validity of the Euler-Lagrange equation and its main application, i.e., the regularity of solutions, presenting some new results and open problems.

A model for the morphogens evolution in a growing tissue

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Recently Averbukh, Ben-Zvi, Mishra, and Barkai proposed a model for the regulation of growth and patterning in developing tissues by diffusing morphogens. The model consists of a free boundary nonhomogeneous Neumann problem with two equations, a diffusion one and a transport one. We show that solutions of the underlying coupled system of nonlinear PDEs exist, are unique and are stable in a suitable sense. The key tool in the analysis is the transformation of the underlying system to a porous medium equation.

Joint work with: Mario Michele Coclite (University of Bari) and Siddartha Mishra (ETH-Zürich).

The maximum principle for the controlled Moreau's process

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Moreau's (or sweeping) process is a differential inclusion involving the external normal cone $N_{C(t)}$ to a (convex) moving set $C(t)$. More precisely we consider the problem

$$\dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)), \qquad x(0) = x_0 \in C(0), \tag{1}$$

where $u(t) \in U$ a.e. is the control and f is smooth and bounded and the state is in $\mathbb{R}^n$. For every measurable u, the above Cauchy problem admits one and only one solution on an interval $[0, T]$. The dynamics at the right hand side of (1) may be seen as a state constrained problem, where the constraint is build in the dynamics. In fact, the normal part acts by deleting the component of f which point outside the constraint, thus keeping the trajectory inside $C(t)$ for all t.

A characterization of the minimum time function to reach a given target subject to (1) through Hamilton-Jacobi inequalities was recently obtained in [1]. Instead, a formulation of Pontryagin's Maximum Principle, even for the simplest case of minimizing a smooth cost depending only on the state at time T - except from a particular case (see [2]) - is still missing. A joint work with Chems Eddine Arroud gives a formulation of the maximum principle in this simple case, by requiring the moving set to be smooth enough and an outward pointing condition on f. The main challenges are due to the discontinuity of the right hand side of (1) with respect to the state. We adopt the method developed in [2] which is based on passing to the limit along the adjoint equation for controlled Moreau-Yosida approximation, which involves the Hessian of the squared distance from $C(t)$. Its discontinuity along the boundary of $C(t)$ provides serious difficulties.

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Recent results about nonlocal conservation laws

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Various situations lead to conservation or balance laws where the flux is a nonlocal functional of the unknown variable. This talk will overview recent analytic results motivated by specific applications where nonlocal fluxes have a prominent role.

Sharp semi-wavefront profiles for scalar conservation laws with degenerate diffusivity

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We consider the scalar parabolic equation

$$\rho_t + f(\rho)_x = \left(D(\rho)\rho_x\right)_x + g(\rho) \tag{2}$$

for $(x,t) \in \mathbb{R} \times [0,+\infty)$, where $f \in C^1([0,\overline{\rho}])$, $f(0) = 0$, $g \in C([0,\overline{\rho}])$ and $D \in C^1([0,\overline{\rho}])$, for some $\overline{\rho} > 0$.

On the diffusion coefficient (or diffusivity) D we assume $D(\rho) > 0$ for $\rho \in (0, \overline{\rho})$; however, D can vanish at 0 or $\overline{\rho}$, and even at both points simultaneously, at any (even infinite) order. About the forcing term g we require, for instance, that $g(\rho) > 0$ for $\rho \in [0, \overline{\rho})$ and $g(\overline{\rho}) = 0$; we consider as well the case when g is negative or it changes sign in the interval $(0, \overline{\rho})$.

The reaction-diffusion-convection equation (2), with D vanishing (as a power function) at some points, models several physical and biological phenomena. However, there are no results about source terms g as above. Our main source of inspiration has been the appearance of (2) with $g = 0$ in the framework of collective movements [2], while the source term was included analogously to [1].

We first prove the existence of semi-wavefront solutions for every wave speed; their properties are investigated. Then, a family of traveling wave solutions is constructed by a suitable combination of the previous semi-wavefront solutions. Proofs exploit comparison-type techniques; they can be extended to the case of several space variables in a straightforward way.

More information is contained in [3].

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Joint work with: L. di Ruvo, (*University of Modena and Reggio Emilia*) and Luisa Malaguti (*University of Modena and Reggio Emilia*)

On the structure of solutions of multidimensional conservation laws with discontinuous flux

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We study the structure of solutions of conservation laws with discontinuous flux of the form

$$\mathrm{div}_z \boldsymbol{A}(z, u) = 0, \tag{3}$$

in order to establish a general framework for studying uniqueness of solutions of the Cauchy problem associated to the evolutionary equation

$$u_t + \mathrm{div}_x \boldsymbol{F}(t, x, u) = 0, \qquad \text{in } (0, +\infty) \times \mathbb{R}^N. \tag{4}$$

Here $\boldsymbol{A}$ (respectively $\boldsymbol{F}$) is discontinuous in its first variable z (respectively (t, x)). More precisely we will assume that $\boldsymbol{A}(z, \cdot) \in C^1(\mathbb{R}, \mathbb{R}^n)$, $\boldsymbol{A}(\cdot, v) \in$

$SBV(\mathbb{R}^n, \mathbb{R}^n)$ where SBV is the space of special function of bounded variation and that $\boldsymbol{A}$ satisfies some mild structural assumptions.

These structural assumptions on $\boldsymbol{A}$ and the results in [1] guarantee the existence of a $\mathcal{H}^{n-1}$–rectifiable set $\mathcal{N}$ that represents a universal jump set of $\boldsymbol{A}(\cdot, v)$, independent of v.

Our structure results are valid for any *weak entropy solution* (WES) of (3), that is, a distributional solution $u \in L^\infty(\mathbb{R}^n)$ of (3) for which there exists a non-negative Radon measure μ such that $\mu(\mathbb{R}^n \setminus \mathcal{N}) = 0$ and, for every $k \in \mathbb{R}$,

$$\mathrm{div}_z\Big(\mathrm{sign}(u-k)[\boldsymbol{A}(z,u) - \boldsymbol{A}(z,k)]\Big) + \mathrm{sign}(u-k)\mathrm{div}_z^a \boldsymbol{A}(z,k) \leq \mu. \quad (5)$$

First, we prove that every WES admits traces on $\mathcal{N}$ (in a generalized sense). Then we prove a generalized Kato inequality that implies the *quasi*–contractivity of the L^1 norm $\|u(t) - v(t)\|_1$ of the difference of solutions of (4). More precisely, the contraction property in time is valid up to the integral of a quantity $W(u^\pm, v^\pm)$ which is concentrated on the jump set $\mathcal{N}$ and that admits an explicit representation in terms of the generalized traces $u^\pm$, $v^\pm$.

In order to get uniqueness it is enough to require an additional admissibility condition on $\mathcal{N}$ in the spirit of [2]. We stress that a general existence result for such admissible solutions is a major open problem.

The results presented here are published in [3].

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Joint work with: Virginia De Cicco (*Univ. di Roma "La Sapienza"*), Guido De Philippis (*SISSA Trieste*) and Francesco Ghiraldin (*Max Planck Institut, Leipzig*)

Around Bressan's compactness and mixing conjectures

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In 2003 Alberto Bressan proposed two conjectures involving ODE flows of non smooth velocity fields. The first conjecture concerns the strong compactness of almost measure preserving flows associated to velocity fields equi-bounded in BV. The second conjecture states that, if an incompressible flow mixes a given set at scale ε in unit time (i.e., the set becomes "homogeneous" if observed with "resolution" ε), then the BV norm of the associated velocity field is lower bounded by a logarithm of ε.

Both conjectures are still open, but the attempts towards their solution stimulated a large amount of important mathematical advances.

In the talk I will offer my perspective on this area. I will first explain an approach to the conjectures in the Sobolev context together with some more recent extensions. Then I will describe how the simple and elegant "chessboard" example motivating the logarithmic lower bound in the mixing conjecture can be made more regular (Sobolev, or even Lipschitz). I will finally indicate the implications of this on the loss of regularity for solutions of transport equations with non Lipschitz velocity field.

Joint work with: G. Alberti (*University of Pisa*), F. Bouchut (*University of Paris Est*), A. Bohun (*formerly University of Basel*), C. De Lellis (*University of Zurich*), A. L. Mazzucato (*Penn State University*)

Redistribution of Damping in Hyperbolic Conservation Laws

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The lecture will survey a program for constructing BV solutions to the Cauchy problem for hyperbolic systems of balance laws with partially dissipative source.

Well-posedness for a monotone solver for traffic junctions

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In this talk we present well-posedness results for solutions obtained as vanishing viscosity limits for the Cauchy problem on a traffic junction where m incoming and n outgoing roads meet. We assume that the traffic on each road is governed by a scalar conservation law like in the classical LWR model.

Our proof relies on the introduction of a family of Kruzhkov-type adapted entropies at the junction and a suitable definition of admissible solution.

The key step in our construction consists in the description and analysis of the set of stationary solutions for the inviscid problem from the point of view developed by Andreianov, Karlsen, Risebro and Cancès to deal with scalar conservation laws with discontinuous flux.

Joint work with: Boris Andreianov (*Université de Tours*) and Giuseppe M. Coclite (*Università di Bari*).

On a low Mach number limit for supernovae

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Fluid dynamic equations are used to model various phenomena arising from physics, engineering, astrophysics.

In particular these type of equations are useful to model some phenomena taking place at the level of supernovae, where the modeling equations are given by the coupling of the compressible Navier Stokes equations with equations that take into account of the chemical reactions and heat effects.

One feature of these flows is that they take place under a low Mach number and high Reynolds number regime and so they are affected by the presence of high oscillating acoustic waves. In order to understand this type of dynamic one has to derive a model for low speed flows (low Mach number) in a hydrostatically balanced, radially stratified background that removes acoustic waves and allows for the development of finite amplitude temperature and density variation.

Here, we analyze a simplified model for supernovae and we identify the asymptotic limit in the regime of low Mach, low Froude and high Reynolds number. The system is driven by a long range gravitational potential. We show convergence to an anelastic system for ill-prepared initial data. The proof is based on frequency localized Strichartz estimates for the acoustic equation based on the recent work of Metcalfe and Tataru.

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Joint work with: Eduard Feireisl (*Institute of Mathematics of the Academy of Sciences of the Czech Republic, Czech Republic.*).

Control problems for structured population dynamics

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We introduce a model of structured population dynamics on a simple network. When the population under consideration consist of two stages, say juveniles developing into adults as in [1], it is natural to write a renewal equation for each of the two stages and couple them through a "nodal condition" analytically similar to those arising in road traffic or fluid dynamics in pipes. Models with similar structures arise when dealing with the demography of sexual reproduction or with the exploitation of biological resources. We are interested in control problems for similar structures. Here the control functions act as distribution matrices at the nodes. A well posedness result applicable to these situations is presented, see [2, 3]. Moreover we prove that the solution is differentiable with respect to the control.

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Joint work with: R.M. Colombo (*Università di Brescia*)

Scalar conservation laws with moving constraints arising in traffic flow modeling

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We consider the Cauchy problem for a strongly coupled PDE-ODE system introduced in [2, 4, 5] to model the influence of a slow and large vehicle on the surrounding road traffic. The model consists of a conservation law describing the main traffic evolution and an ODE accounting for the trajectory of the slower vehicle that depends on the downstream traffic density. The moving constraint is operated by an inequality on the flux, which accounts for the bottleneck created on the road by the presence of the slower vehicle.

We present the proof of existence of weak entropy solutions obtained via the wave-front tracking method [2], and two finite volume schemes able to capture exactly the non-classical discontinuities that may arise at the constraint position [1, 3, 5].

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Joint work with: Maria Laura Delle Monache (*Rutgers University in Camden*), Christophe Chalons (*Université de Versailles St. Quentin*), Stefano Villa (*Università di Milano - Bicocca*)

Uniqueness for a non–linear 1D compressible to incompressible limit in the non smooth case

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In the first part of the talk we introduce some local existence and uniqueness results for some kind of systems of partial and ordinary differential equations coupled through boundary conditions.

Then we introduce the problem of the compressible to incompressible limit for a $1D$ fluid (which we call "liquid") with finite mass m surrounded by a $1D$ fluid (which we call "gas") and we show the existence of the limit, defined for all times, through a compactness argument.

Finally, the fact that this limit satisfies a system of partial and ordinary differential equations, coupled through suitable boundary conditions, implies the uniqueness of the compressible to incompressible limit. Moreover, since the limit is defined for all times, we also have the existence for all times of the solution to this particular coupled system ODE-PDE.

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Joint work with: Rinaldo M. Colombo (*University of Brescia*),

Relative entropy for Euler–Korteweg system and relaxation to diffusion theories

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For an Euler system with dynamics generated by a potential energy functional, we present a functional format for the relative energy identity, firstly introduced in the works by Dafermos [1,2] and DiPerna [3]. Depending on the selection of the functional, several models of interest fit under the proposed framework, as the equations of isentropic gas dynamics, the Euler–Poisson system, the system of quantum hydrodynamics, the Euler–Korteweg system [4]. The resulting formula is then used to establish stability results between weak and strong solutions for the system under consideration.

In the presence of a friction term, the relative energy calculation is also used to connect the Euler flow to the gradient flow generated by the energy functional in the diffusive (high friction) limit regime. We apply this approach to prove the convergence from the Euler–Poisson system with friction to the Keller-Segel system, and the convergence from the Euler–Korteweg theory with friction to the Cahn–Hilliard equation when the limit has smooth solutions [5], generalizing the results of [6] on convergence from the compressible Euler system with friction to the porous media equation.

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Joint works with: Jan Giesselmann (*University of Stuttgart*), Athanasios E. Tzavaras (*King Abdullah University of Science and Technology — KAUST*)

From Shape Formation to Monge-Ampère Anomalies

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This lecture is concerned with the analysis of thin elastic films exhibiting residual stress at free equilibria. Examples of such structures and their actuations include: plastically strained sheets; specifically engineered swelling or shrinking gels; growing tissues; atomically thin graphene layers, etc.

These and other phenomena can be studied through a variational model, pertaining to the non-Euclidean version of nonlinear elasticity, which postulates formation of a target Riemannian metric, resulting in the morphogenesis of the tissue which attains an orientation-preserving configuration closest to being the metric's isometric immersion. In this context, analysis of scaling of the energy minimizers in terms of the film's thickness leads to the rigorous derivation of a hierarchy of limiting theories, differentiated by the embeddability properties of the target metrics and, a-posteriori, by the emergence of isometry constraints with low regularity.

Conversely, rigidity and flexibility of solutions to the weak formulations of the related PDEs, result in the bounds on the energy scaling. These equations include the Monge-Ampère equation. In particular, we observe that the set of $C^{1,\alpha}$ solutions to the Monge-Ampère equation in 2 dimensions is dense in C^0 provided that $\alpha < 1/7$, whereas rigidity holds when $\alpha > 2/3$.

Shock Waves in Conservation Laws with Physical Viscosity

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Consider a $n \times n$ system of conservation laws

$$u_t + f(u)_x = (B(u)u_x)_x$$

where $B(u)$ is physical viscosity matrix. Prime examples include the compressible Navier-Stokes equations, MHD, and nonlinear viscous elasticity equations. Due to the off-diagonal elements and the degeneracy of the viscosity matrix, and the nonlinearity of the flux function $f(u)$, there are rich wave interactions. With Yanni Zeng, [1], we describe the wave propagation for a perturbation of a shock profile. We use the Green's function approach for our analysis. The construction of the Green's function around the shock profile, [2], is based on the fundamental solutions, [3], around the end states of the profile. Waves of various decaying rates are described explicitly in pointwise sense.

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Joint works with: Yanni Zeng (*Department of Mathematics, University of Alabama at Birmingham*)

Splash singularities for a free-boundary incompressible viscoelastic fluid model in 2-D

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We study the existence of splash singularities (namely where the free-boundary remain smooth but self-intersects at a point) for a nonlinear viscoelastic fluid system of Oldroyd tepee (for high Weissenberg number). The boundary conditions are given by the equilibrium of the force fields at the interface. Similar results have been obtained in 2-D for the free boundary Navier-Stokes equations by Castro, Córdoba, Fefferman,Gancedo, Gomez-Serrano in 2015 [2] The idea consists in mapping the plane by a conformal transformation, then by an iteration scheme to show the local existence in time and? the? stability under small perturbations of the initial domain. These properties allow to design a suitable initial setting for the formation of a splash singularity?. The iteration method used here takes inspiration from the paper of T.Beale [1]

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Joint work with Elena Di Iorio and Stefano Spirito.

Metastability for conservation laws without and with reaction term

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Metastability is a broad term describing the existence of a very sensitive equilibrium possessing a weak form of stability. Such behavior is usually related to the presence of a small first eigenvalue for the linearized operator at the given equilibrium state, revealed at dynamical level by the appearance of slowly moving structures. A fascinating feature is the presence of a finite dimensional manifold describing the large time behavior of the solutions, indicating a drastic dimensional reduction of the infinite-dimensional dynamics.

In the field of conservation laws, metastability has been analyzed since the pioneering paper authored by Kreiss and Kreiss [1] on the Burgers equation

$$u_t + \left(\tfrac{1}{2}u^2\right)_x = \varepsilon u_{xx}.$$

Any constant value is solution to the equation and transitions between such states evolves following the rule dictated by Rankine–Hugoniot condition and entropy-like admissibility criteria, at least in the regime of small viscosity ε. When the equation is considered in a bounded interval, stationary shocks and boundary conditions exhibit a very weak long-range interaction which determines an exponentially slow motion of the shock. In the first part, the aim of the talk is to present metastability behavior for such equation and some of its possible modifications: general scalar conservation laws with viscosity, regularizations of relaxation type, prototypical systems such as the isentropic Navier–Stokes equations (mainly taken from [2,3,4]).

Another field of appearance of metastable behavior is phase transition phenomena. Here, a key role is played by the presence of an appropriate zero-order reaction term, which is usually assumed to be of cubic type, so that two distinct stable constant states are identified. The simplest prototypes are the Allen–Cahn equation (see [5, 6]) and the Cahn–Hilliard equation, (see [7] and descendants) both of which concerns with first-order in time dynamical systems. Assuming a retarded relation between flux and gradient of the unknown (corresponding to the substitution of the standard Fourier law with a Cattaneo-type one) give raise to second order in time evolutive PDEs of hyperbolic type, as

$$\tau u_{tt} + g(u)u_t = \varepsilon^2 u_{xx} + f(u).$$

The language for describing presence of slow motions has then to be adapted, taking into account this inherent "inertial" effect. After recalling the main properties of metastability analysis for the classical Allen–Cahn equation, the second part of the talk will be devoted to discuss the modification in the techniques and in the dynamical properties arising from this second-order dynamical structure (mainly taken from [8,9]).

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Joint works with: Raffaele Folino (*Università dell'Aquila*), Corrado Lattanzio (*Università dell'Aquila*), Marta Strani (*Université Paris–Diderot*).

Hybrid mathematical model of cell movement

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In this talk we present some discrete in continuous mathematical model for the growth of stem cells aggregates. Our model follows closely the results obtained in recent biological experiments concerning the growth of neuromastes in the lateral line of zebrfish and in the development of the so called cardiosphere, composed by human cardiac stem cells. In both cases, we rely on a hybrid description: discrete for the cellular level and continuous for the molecular level. We prove the existence of steady solutions consistent with the formation of particular biological structure, the neuromasts. Dynamical numerical simulations are performed to show the behavior of the model and its qualitative and quantitative accuracy to describe the evolution of the cell aggregate.

Conservation laws on networks

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We consider systems of conservation laws defined on a topological graph. This problem appears naturally in a number of applications, including vehicular traffic on road networks, irrigation channels, blood flow, supply chains and other. The dynamics needs to be supplemented by coupling conditions at nodes.
The theory for the single conservation law on a general network (graph) is fairly well developed for a number of different coupling conditions at nodes, which reflect the physical properties of the modeled systems. On the other side, the theory for systems is still in its infancy. We provide a general presentation of the main results, including numerical methods and application.
Specific problems, such as Dynamic Traffic Assignment and moving bottlenecks for vehicular traffic or supply chains, require to pair the conservation laws with other partial differential equations and or ordinary differential equations. We show some examples and results for these extended systems, on which Alberto gave recently important contributions.

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Joint work with: Ke Han (*Imperial Colleg*), Mauro Garavello (*University of Milan*).

Lyapunov-like functions and Lie brackets

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Under some controllability assumptions,the optimal time function is a particular Lyapunov function, a very efficient one indeed, for it also minimizes a cost, namely, the time to reach the target. In general, the optimal time function is not smooth, and this is somehow the price one has to pay for the high performance it guarantees. The situation is similar when the integral of a nonnegative current cost l replaces the time cost (which is the integral of $l = 1$). To pave the way towards an augmented regularity, we embed the standard dissipative relation in a differential inequality (DI) involving Hamiltonians built from the iterated Lie brackets of the dynamical vector fields. Actually, the solutions of (DI), besides yielding reachability of the target (in finite or infinite time), provide upper estimates for the minimum value function. Furthermore, because of the explicitly displayed controllability, solutions of (DI) can likely be expected more regular than the value function.

On the approximate current-vortex sheets near the onset of instability

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We consider the free boundary problem for 2D current-vortex sheets in ideal incompressible magneto-hydrodynamics near the transition point between the linearized stability and instability. In order to study the dynamics of the discontinuity near the onset of the instability, Hunter and Thoo have introduced an asymptotic quadratically nonlinear integro-differential equation for the amplitude of small perturbations of the planar discontinuity.

In this talk we present our results about the well-posedness of the problem in the sense of Hadamard, under a suitable stability condition, that is the local-in-time existence in Sobolev spaces and uniqueness of smooth solutions to the Cauchy problem, and the strong continuous dependence on the data in the same topology.

Joint works with: Alessandro Morando and Paola Trebeschi (*DICATAM, University of Brescia*)

From Relaxation to Slow Erosion

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In this talk I will present some works I did jointly with Alberto throughout the years. In particular, I will present an early work on a relaxation model for multi-component chromatography and a recent work on slow erosion of granular flows. I will explain the main ideas for establishing the BV bound for the chromatography model, and for the slow erosion model, a backward Euler approximation together with a projection operator for pointwise constraint.

On the local limit of continuity equations with nonlocal fluxes

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Consider a family of continuity equations where the velocity field is given by the convolution of the solution with a regular kernel. In the singular limit where the convolution kernel is replaced by a Dirac delta, one formally recovers a conservation law. My talk will aim at discussing the behavior of the solutions of the nonlocal continuity equation in the singular limit. I will recall some positive results and discuss some recent counter-examples. I will also discuss the nonlocal to local limit for a related family of continuity equations with a viscous second order term. The talk will be based on an ongoing collaboration with Stefano Bianchini, Maria Colombo and Gianluca Crippa.