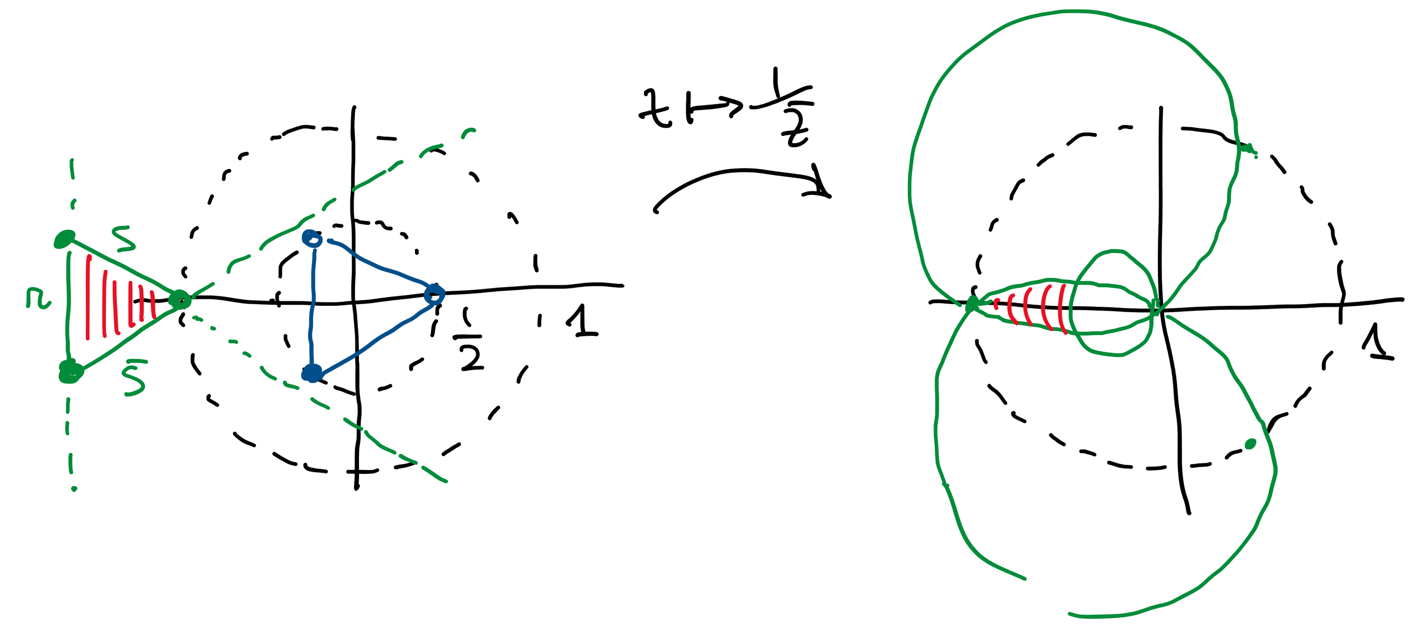
A1-G1A-234

giovedì 15 febbraio 2024 09:20

① $\left(z+\frac{3}{2}\right)^3 = \frac{1}{8}$

$z+\frac{3}{2}$ = radici cubiche di $\frac{1}{8}$ = $\begin{cases} \frac{1}{2} & = \frac{1}{2} \\ \frac{1}{2}\,\mathrm{cis}(2\pi/3) & = \frac{1}{2}\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) \\ \frac{1}{2}\,\mathrm{cis}(4\pi/3) & = \frac{1}{2}\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right) \end{cases}$

$z = \begin{cases} -1 \\ -\frac{7}{4}+i\frac{\sqrt{3}}{4} \\ -\frac{7}{4}-i\frac{\sqrt{3}}{4} \end{cases}$



② Esercizio standard.

③ $F: \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \longmapsto \begin{pmatrix} c \\ b \\ d \\ a \end{pmatrix} = \begin{pmatrix} 0&0&1&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 1&0&0&0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$

e_1, e_2, e_3, e_4

è funzione associata alla permutazione della base canonica con

e_2 fisso — autovalore 1, fattore $(x-1)$

$(e_1 e_4 e_3)$ ciclo — fattore $(x^3-1) = (x-1)(x^2+x+1)$
$= (x-1)(x-\alpha)(x-\bar{\alpha})$

$\alpha = -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$

e $\underset{\mathbb{C}}{\sim} \begin{pmatrix} 1 &&& \\ & 1 && \\ && \alpha & \\ &&& \bar{\alpha} \end{pmatrix}$

$\underset{\mathbb{R}}{\sim} \begin{pmatrix} 1 && \\ & 1 & \\ && \square \end{pmatrix}$, $\square = \boxed{\begin{matrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{matrix}}$

④ Standard: $\ker A \subsetneq \ker A^2 \subsetneq \ker A^3 \subsetneq \ker A^4 = K^5$

$\langle e_1, e_3\rangle \quad \langle e_1, e_3, e_4\rangle \quad \langle e_1, e_3, e_4, e_2\rangle$

Young:

$v = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \\ \varepsilon \end{pmatrix}$ con $\varepsilon \neq 0$

$Av = \begin{pmatrix} 2\delta+\varepsilon \\ 2\varepsilon \\ 0 \\ 3\beta \\ 0 \end{pmatrix}$ $\quad A^2v = \begin{pmatrix} 6\beta \\ 0 \\ 0 \\ 6\varepsilon \\ 0 \end{pmatrix}$ $\quad A^3v = \begin{pmatrix} 12\varepsilon \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} \alpha' \\ 0 \\ \beta' \\ 0 \\ 0 \end{pmatrix}$ con $\beta' \neq 0$

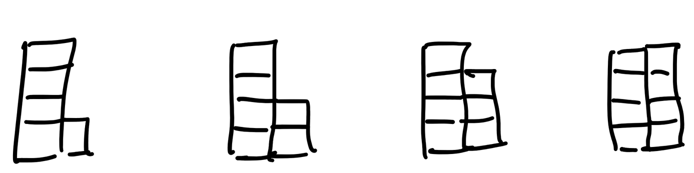
$\begin{pmatrix} 0&0&0&2&1 \\ 0&0&0&0&2 \\ 0&0&0&0&0 \\ 0&3&0&0&0 \\ 0&0&0&0&0 \end{pmatrix} = A\,H = H\left(\begin{array}{cccc|c} 0&1&&& \\ &0&1&& \\ &&0&1& \\ &&&0& \\ \hline &&&&0 \end{array}\right)$

$H = \begin{pmatrix} 12\varepsilon & 6\beta & 2\delta+\varepsilon & \alpha' \\ 0 & 0 & 2\varepsilon & 0 \\ 0 & 0 & 0 & \beta' \\ 0 & 6\varepsilon & 3\beta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\varepsilon \neq 0 \neq \beta'$.

dim ker $= 2$ (2 blocchi Jordan)

$m(x) = x^4$ (un blocco Jordan d'ordine 4, massimo):



$\begin{pmatrix} N_4 & \\ & N_1 \end{pmatrix}$ $\begin{pmatrix} N_4 & \\ & N_2 \end{pmatrix}$ $\begin{pmatrix} N_4 & \\ & N_3 \end{pmatrix}$ $\begin{pmatrix} N_4 & \\ & N_4 \end{pmatrix}$

(ordini 5, 6, 7, 8)

- $A \sim J = \begin{pmatrix} N_4 & \\ & N_1 \end{pmatrix}$

$A^2 \sim J^2 = \left(\begin{array}{c|c} N_4^2 & \\ \hline & N_1^2 \end{array}\right) \sim \left(\begin{array}{cc|c} N_2 && \\ & N_2 & \\ \hline && N_1 \end{array}\right)$

$A^3 \sim J^3 = \left(\begin{array}{c|c} N_4^3 & \\ \hline & N_1^3 \end{array}\right) = \left(\begin{array}{ccc|c} N_2 &&& \\ & N_1 && \\ && N_1 & \\ \hline &&& N_1 \end{array}\right)$

$A^4 \sim J^4 = \mathbb{0}_5$
$\overset{\shortparallel}{A^n}$ $(n \geq 4)$

- $(A - \mathbb{1}_5) \sim -\mathbb{1}_5 + J$

$(A-\mathbb{1}_5)^n \sim (-1)^n \mathbb{1}_5 + J \quad \forall n \geq 1$

stesso J, non è sbaglio!

$\begin{pmatrix} \alpha & 1 && \\ & \alpha & 1 & \\ && \alpha & 1 \\ &&& \alpha \end{pmatrix}^n = \begin{pmatrix} \alpha^n & * & * & * \\ & \alpha^n & * & * \\ && \alpha^n & * \\ &&& \alpha^n \end{pmatrix} \sim \begin{pmatrix} \alpha^n & 1 && \\ & \alpha^n & 1 & \\ && \alpha^n & 1 \\ &&& \alpha^n \end{pmatrix}$

se $\alpha \neq 0$.