① Standard

② Standard

③ Usiamo i punti dati come riferimento:

$$P_0=\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\ P_1=\begin{pmatrix}0\\1\\0\\0\end{pmatrix},\ P_2=\begin{pmatrix}0\\0\\1\\0\end{pmatrix},\ P_3=\begin{pmatrix}0\\0\\0\\1\end{pmatrix},\ U=\begin{pmatrix}1\\1\\1\\1\end{pmatrix},$$

allora

$$P_0'=(P_0\vee U)\wedge(P_1\vee P_2\vee P_3)=\begin{pmatrix}0\\1\\1\\1\end{pmatrix}$$

$$\begin{pmatrix}1+\alpha\\1\\1\\1\end{pmatrix}\qquad x_0=0$$

$$P_1'=\begin{pmatrix}1\\0\\1\\1\end{pmatrix},\ P_2'=\begin{pmatrix}1\\1\\0\\1\end{pmatrix},\ P_3'=\begin{pmatrix}1\\1\\1\\0\end{pmatrix}$$

e sono punti in posizione generale ($\det(P_0'P_1'P_2'P_3')=0$)
come $P_0'+P_1'+P_2'+P_3'=3U$,
quindi anche P_0',P_1',P_2',P_3',U formano un riferimento.

$Q_0\in UvP_0$ $\quad Q_1=\begin{pmatrix}1\\1+\beta\\1\\1\end{pmatrix}$ $\quad Q_2=\begin{pmatrix}1\\1\\1+\gamma\\1\end{pmatrix}$ $\quad Q_3=\begin{pmatrix}1\\1\\1\\1+\delta\end{pmatrix}$

$=\begin{pmatrix}1+\alpha\\1\\1\\1\end{pmatrix}$

Calcolo i birapporti:

$$(U\ P_0\ P_0'\ Q_0)=\left(\begin{pmatrix}1\\1\\1\\1\end{pmatrix},\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\1\\1\end{pmatrix},\begin{pmatrix}1+\alpha\\1\\1\\1\end{pmatrix}\right)=(1\ \infty\ 0\ 1+\alpha)$$
$$=(0\ \infty\ -1\ \alpha)$$
$$=(\infty\ 0\ \alpha\ -1)=-1/\alpha$$

$(U\ P_1\ P_1'\ Q_1)=-1/\beta$
$(U\ P_2\ P_2'\ Q_2)=-1/\gamma$
$(U\ P_3\ P_3'\ Q_3)=-1/\delta$

e Q_0,Q_1,Q_2,Q_3 sono complanari sse $\begin{vmatrix}1+\alpha&1&1&1\\1&1+\beta&1&1\\1&1&1+\gamma&1\\1&1&1&1+\delta\end{vmatrix}=0$

$$\begin{array}{l}\text{I-IV}\\\text{II-IV}\\\text{III-IV}\\\ \end{array}\begin{vmatrix}\alpha&0&0&-\delta\\0&\beta&0&-\delta\\0&0&\gamma&-\delta\\1&1&1&1+\delta\end{vmatrix}=0$$

$$\alpha\begin{vmatrix}\beta&0&-\delta\\0&\gamma&-\delta\\1&1&1+\delta\end{vmatrix}+\beta\gamma\delta=0$$

$$\alpha\beta\gamma\delta+\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta=0$$
$$1+\frac{1}{\delta}+\frac{1}{\gamma}+\frac{1}{\beta}+\frac{1}{\alpha}=0$$

dunque: complanari sse $-\frac{1}{\alpha}-\frac{1}{\beta}-\frac{1}{\gamma}-\frac{1}{\delta}=1$

sse $\sum(UP_iP_i'Q_i)=1$ //

④ $\begin{pmatrix}\alpha&&&\\&\beta&&\\&&\gamma&1\\&&1&\gamma\end{pmatrix}$ oppure $\begin{pmatrix}\alpha&&&\\&\beta&&\\&&\gamma&1\\&&&\gamma\end{pmatrix}$

con $\alpha\neq\beta\neq\gamma\neq\alpha$ (tre blocchi di Jordan relativi ad autovalori diversi)

punti uniti: P_0,P_1,P_2 $\qquad$ P_0,P_1,P_3

rette unite: $P_0P_1, P_0P_2, P_1P_2, P_3P_3$ $\qquad$ $P_0P_1, P_0P_3, P_1P_2, P_1P_3, P_3P_4$

piani uniti:

iperpiani uniti: π_0,π_1,π_5 $\qquad$ π_0,π_2,π_5

Gli spazi uniti sono in numero finito,
perché ci sono solo 3 ipersp. uniti (per dualità)
e ciascuno con un numero finito di punti uniti ...