
**ERRATA and ADDITIONS: Outline of the lectures on
Distributions and Integral Operators
Academic Year 2018/19, updated March 16, 2019**

p. 21, line 5 of Theorem 2.2.2: Please replace

The map $\mathcal{J}f$

by

The map \mathcal{J}

p. 22, end of section 2.2: Please add:

We also state the following, which we do not prove. For a proof, we refer to Treves [?].

Proposizione 0.0.1 *Let Ω be an open subset of \mathbb{R}^n . Let $\{u_j\}_{j \in \mathbb{N}}$ be a sequence in $\mathcal{D}'(\Omega)$ which converges to $u \in \mathbb{K}^{\mathcal{D}'(\Omega)}$ pointwise, i.e.,*

$$\lim_{j \rightarrow +\infty} \langle u_j, \varphi \rangle = \langle u, \varphi \rangle \quad \forall \varphi \in \mathcal{D}(\Omega).$$

Then $u \in \mathcal{D}'(\Omega)$ and $\{u_j\}_{j \in \mathbb{N}}$ converges to u in $\mathcal{D}'_s(\Omega)$.

p. 33, line 6 to bottom: replace

$\text{supp } \varphi \subseteq \Omega_1$

by

$\overline{\mathbb{B}_n(x_0, \epsilon_0)} \cup \text{supp } \varphi \subseteq \Omega_1$

p. 43, statement (ii) of Proposition 2.11.3: Please replace

If $m \in \mathbb{N}$ and if $\beta \in \mathbb{N}^n$ and $|\beta| \leq m$

by

If $m \in \mathbb{N}$, and if $\alpha \in C_c^m(\mathbb{R}^n)$ and if $\beta \in \mathbb{N}^n$ and $|\beta| \leq m$

p. 55, formula in display line 3 to bottom: Please replace

$\forall x \in \mathbb{R}^n$

by

$\forall x \in \mathbb{R}^n \setminus \{0\}$

p. 54, line 2 of Theorem 2.14.1: Please replace

, then

by

which is not identically equal to 0, then

p. 60, line 2: Please replace
and that
by
and that

$$D^{e_j} S_n(x) = \frac{1}{s_n} \frac{x_j}{|x|^n} \quad \forall x \in \mathbb{R}^n \setminus \{0\},$$

defines a locally integrable function in \mathbb{R}^n and that

p. 100, end of the page, please add:

Exercise 0.0.2 Let $m \in \mathbb{N}$. Let $c_\alpha \in \mathbb{C}$ for all $\alpha \in \mathbb{N}^n$ such that $|\alpha| \leq m$. Then the distribution $\sum_{|\alpha| \leq m} c_\alpha D^\alpha \delta_0$ is positively homogeneous of degree $h \in]-n, +\infty[$ if and only if $c_\alpha = 0$ for all $\alpha \in \mathbb{N}^n$ such that $|\alpha| \leq m$.

Solution. The sufficiency is obvious. We now prove the necessity of the condition and thus we assume that

$$t^h \left[\sum_{|\alpha| \leq m} c_\alpha D^\alpha \delta_0 \right] = h_t \left[\sum_{|\alpha| \leq m} c_\alpha D^\alpha \delta_0 \right] \quad \forall t \in]0, +\infty[.$$

Since $D^\alpha \delta_0$ is positively homogeneous of degree $-n - |\alpha|$, we have

$$t^h \left[\sum_{|\alpha| \leq m} c_\alpha D^\alpha \delta_0 \right] = \sum_{|\alpha| \leq m} c_\alpha h_t [D^\alpha \delta_0] = \sum_{|\alpha| \leq m} c_\alpha t^{-n-|\alpha|} D^\alpha \delta_0 \quad \forall t \in]0, +\infty[,$$

and thus

$$\sum_{|\alpha| \leq m} c_\alpha (t^h - t^{-n-|\alpha|}) D^\alpha \delta_0 = 0 \quad \forall t \in]0, +\infty[.$$

Hence,

$$c_\alpha (t^h - t^{-n-|\alpha|}) = 0 \quad \forall t \in]0, +\infty[,$$

for all $\alpha \in \mathbb{N}^n$ such that $|\alpha| \leq m$ and accordingly

$$c_\alpha (t^{h+n+|\alpha|} - 1) = 0 \quad \forall t \in]0, +\infty[,$$

for all $\alpha \in \mathbb{N}^n$ such that $|\alpha| \leq m$. Since $h > -n$, we can take the limit as t tends to 0 conclude that $c_\alpha = 0$ for all $\alpha \in \mathbb{N}^n$ such that $|\alpha| \leq m$. \square