<u>Esercizio</u> Calcolare $\log\left(\frac{3}{2}\right)$ con un errore minore di $\frac{1}{100}$.

Soluzione. Per $x>-1$ si ha lo sviluppo

$$\log(1+x) = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k} x^k + \frac{(-1)^n (1+\xi)^{-n-1}}{n+1} x^{n+1}$$

dove $\xi \in (0,x)$. Con $x = \frac{1}{2}$ e $0<\xi<\frac{1}{2}$ si trova

$$\log\left(\frac{3}{2}\right) = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k} x^k + \underbrace{\frac{(-1)^n (1+\xi)^{-n-1}}{(n+1)\, 2^{n+1}}}_{= R_n}$$

dove il resto verifica

$$|R_n| \le \frac{1}{(n+1)\, 2^{n+1}} \cdot \frac{1}{(1+\xi)^{n+1}} \le \frac{1}{(n+1)\, 2^{n+1}}.$$

Cerco $n \in \mathbb{N}$ tale che

$$\frac{1}{(n+1)\, 2^{n+1}} < \frac{1}{100} \iff 100 < (n+1)\, 2^{n+1}.$$

Con $n=4$ si trova

$$(n+1)\, 2^{n+1} = 5 \cdot 32 = 160.$$

Quindi

$$\log\left(\frac{3}{2}\right) = \frac{1}{2} - \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{3}\left(\frac{1}{2}\right)^3 - \frac{1}{4}\left(\frac{1}{2}\right)^4 + \text{Errore}$$

con $|\text{Errore}| \leq \frac{1}{100}$.

$\square$