ES 2 $x \in \mathbb{R}$ tali che $\displaystyle\sum_{n=1}^{\infty} \frac{\lg(n+1)}{n+2} (x-1)^n$

Converge semplice e assolut.

Sol. CA: $\displaystyle\sum_{n=1}^{\infty} \frac{\lg(n+1)}{n+2} |x-1|^n$

Crit. Radice

$$L = \lim_{n\to\infty} \sqrt[n]{\frac{\lg(n+1)}{n+2} |x-1|^n}$$

$$= |x-1| \lim_{n\to\infty} \frac{\sqrt[n]{\lg(n+1)}}{\sqrt[n]{n+2}} = |x-1|$$

Lim. noti

1° caso $|x-1| < 1 \Rightarrow$ CA $\Rightarrow$ CS

2° caso $|x-1| > 0 \Rightarrow$ NO CA e termine generale non infinitesimo

$\Rightarrow$ NO CS

$|x-1| = 1 \iff x^2 - 2x + 1 = 1 \iff x(x-2) = 0$

$\iff x = 0$ opp $x = 2$

- $x = 2 \to \displaystyle\sum_{n=1}^{\infty} \frac{\lg(n+1)}{n+2} \geq \sum_{n=1}^{\infty} \frac{\log 2}{n+2} = \infty$

- $x = 0 \to \displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\lg(n+1)}{n+2}$. NO CA

CS: Leibniz

□ $\displaystyle\lim_{n\to\infty} \frac{\lg(n+1)}{n+2} = 0$ NOTO $f(x) = \dfrac{\lg(x+1)}{x+2}$

□ $f'(x) = \dfrac{1}{(x+1)(x+2)} + \lg(x+1)\left(-\dfrac{1}{(x+2)^2}\right) \xrightarrow[x\to\infty]{} 0$

Quindi: $f' = \dfrac{1}{(x+2)^2}\left(\dfrac{x+2}{x+1} - \lg(x+1)\right) < 0$ definit. □

ES. 3 Integrale:

$$I = \int_{-\pi/2}^{\pi/2} (2 - \cos^2 x)\, e^{-2|\sin x|} \cos x \, dx$$

Sostituzione: $\sin x = t$, $dt = \cos x\, dx$

$x = -\pi/2 \to t = -1$

$x = \pi/2 \to t = +1$

Inoltre $\cos^2 x = 1 - \sin^2 x = 1 - t^2$.

$$I = \int_{-1}^{1} (2 - 1 + t^2)\, e^{-2|t|} \, dt$$

$$= \int_{-1}^{1} (1+t^2)\, e^{-2|t|} \, dt \qquad \text{simmetria}$$

Ricordare

$$= 2 \int_0^1 (1+t^2)\, e^{-2t} \, dt$$

Calcolo primitiva per parti con integrali indefiniti:

- $\int e^{-2t} \, dt = -\frac{1}{2} e^{-2t}$

- $\int t^2 e^{-2t} dt = t^2 \left(-\frac{1}{2}\right) e^{-2t} - \int 2t \left(-\frac{1}{2}\right) e^{-2t} \, dt =$

$$= -\frac{t^2}{2} e^{-2t} + \int t\, e^{-2t} \, dt$$

$$= -\frac{t^2}{2} e^{-2t} + \left(t \frac{e^{-2t}}{-2} - \int -\frac{1}{2} e^{-2t} \, dt \right)$$

$$= -\frac{t^2}{2} e^{-2t} - \frac{t}{2} e^{-2t} + \frac{1}{2} \frac{e^{-2t}}{-2}$$

Dmopue

$$I = 2\left[e^{-2t}\left(-\frac{1}{2} - \frac{t^2}{2} - \frac{t}{2} - \frac{1}{4}\right)\right]_{t=0}^{t=1}$$

$$= \left[e^{-2t}\left(-\frac{3}{2} - t^2 - t\right)\right]_{t=0}^{t=1}$$

$$= e^{-2}\left(-\frac{3}{2} - 1 - 1\right) - \left(-\frac{3}{2}\right)$$

$$= -\frac{7}{2}e^{-2} + \frac{3}{2}.$$

$$I = \frac{3}{2} - \frac{7}{2}\frac{1}{e^2}$$

ES. 4 $\qquad f(z) = 3i + 2 - i\,\bar{z}^3$

$$A = \{ f(z) \in \mathbb{C} : z \in \mathbb{C} \text{ con } \operatorname{Im} z = 0 \}$$

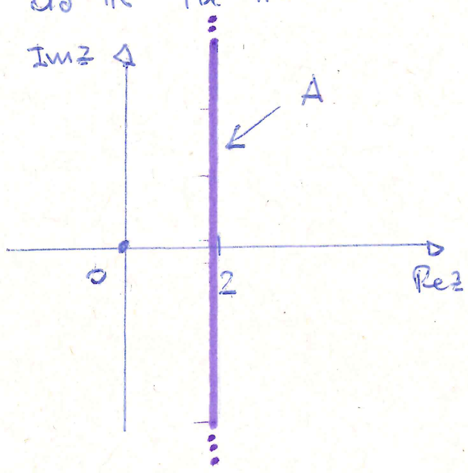
$$B = \{ z \in \mathbb{C} : f(z) = 29 + 3i \}$$

Disegnare A e B.

(A) $\quad z = x + iy$ con $\operatorname{Im} z = y = 0 \iff z = x \in \mathbb{R}$

$$f(z) = f(x) = 3i + 2 - i x^3 = 2 + i(3 - x^3)$$

Osservo che $x \mapsto 3 - x^3$ è 1-1 e su da $\mathbb{R}$ su $\mathbb{R}$



(B) $\quad 3i + 2 - i\,\bar{z}^3 = 29 + 3i$

$$\Updownarrow$$

$$-i\,\bar{z}^3 = 27$$

$$\Updownarrow$$

$$\bar{z}^3 = 27i$$

$$\Updownarrow$$

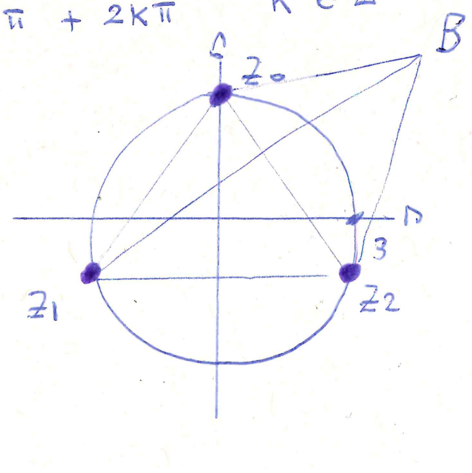
$$z^3 = \overline{27i} = -27i = 27\, e^{\frac{3}{2}\pi i}$$

Con $z = re^{i\vartheta}$ $\quad r \geq 0$ e $\vartheta \in [0, 2\pi)$ trovo

$$r^3 e^{3i\vartheta} = (re^{i\vartheta})^3 = z^3 = 27\, e^{\frac{3}{2}\pi i}$$

$r^3 = 27 \iff r = 3$ e poi $\quad 3\vartheta = \frac{3}{2}\pi + 2k\pi \quad k \in \mathbb{Z}$

e quindi



$$\vartheta_k = \frac{\pi}{2} + \frac{2k\pi}{3} \qquad k = 0, 1, 2$$

$z_0 = 3e^{\pi/2\, i} = 3i \qquad z_1 = 3e^{\frac{7\pi}{6} i}$

$z_2 = 3e^{\frac{11\pi}{6} i}$