Equazioni Differenziali 2 — Foglio 2

Nome: Consegna: 30 Aprile 2010

Esercizio 1 Compute the solution to the Cauchy Problem

$$\begin{cases} y' = \sin(x+y+3) \\ y(0) = -3. \end{cases}$$

Esercizio 2 Consider the Cauchy Problem

$$\begin{cases} y' = \sqrt{y^2+x^2+1} \\ y(0) = 0. \end{cases}$$

i) Prove that the problem has a unique local solution $y \in C^1(-\delta,\delta)$ for some $0 < \delta \leq +\infty$;

ii) Prove that the solution is even, i.e., $y(-x) = -y(x)$ for all $x \in (-\delta,\delta)$;

iii) Prove that the solution is convex for $x \geq 0$;

iv) Prove that the solution is defined for all $x \in \mathbb{R}$;

v) Show that $y(x) \geq \sinh(x)$ for all $x \geq 0$.

Esercizio 3 Let $f \in C(\mathbb{R})$ be a continuous function such that $tf(t) \geq 0$ for all $t \in \mathbb{R}$. Show that the Cauchy Problem

$$\begin{cases} y'' + e^{-x} f(y) = 0 \\ y(0) = y'(0) = 0 \end{cases}$$

has the unique solution $y = 0$.

Esercizio 4 Prove that the solution of the Cauchy Problem

$$\begin{cases} y' = -(x+1)y^2 + x \\ y(-1) = 1 \end{cases}$$

is globally defined on $\mathbb{R}$.