First and higher-order necessary conditions
for an impulsive optimal control problem with final state constraints

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We consider control systems governed by nonlinear O.D.E.s of the form:

$$\dot{x}(t) = f(x(t), u(t), v(t)) + \sum_{\alpha=1}^{m} g_\alpha(x(t), u(t))\dot{u}_\alpha(t), \quad \text{for } t \in [0, T],$$

where $x : [0,T] \to \mathbb{R}^n$ is the *state variable,* $u : [0,T] \to \mathbb{R}^m$ is the *impulsive control* and $v : [0,T] \to \mathbb{R}^l$ is the *ordinary control.* Since we assume the commutativity hypothesis

$$[g_\alpha, g_\beta] = 0, \quad \forall\ \alpha, \beta,$$

we can allow u to be of class L^1, which of course gives the system an impulsive character. For these control systems, we consider optimization problems in the Mayer form and with final state constraints.

We prove (see [2]) that the problem with integrable u provides a "proper extension" of the standard problem with absolutely continuous controls u and, furthermore, we show that the impulsive problem can be regarded as a variational limit of problems corresponding to controls u with bounded variation.

Finally, we establish (see [1]) a maximum principle and higher-order necessary conditions in terms of the adjoint state and some Lie brackets of the involved vector fields, the latter including an impulsive version of the generalized Legendre-Clebsch second order condition.

References

[1] M.S. Aronna and F. Rampazzo. Necessary conditions involving Lie brackets for impulsive optimal control problems; the commutative case. 2012. [Published online as arXiv:1210.4532].

[2] M.S. Aronna and F. Rampazzo. On optimal control problems with impulsive commutative dynamics. 2013. [submitted].