# Workshop "Analysis and Geometry in Control Theory and its Applications"

With a special tribute to Hélène Frankowska and Héctor J. Sussmann

## June 9-13, 2014 – INDAM, Rome, Italy

## CONFIRMED SPEAKERS

| | |
|---|---|
| F. ANCONA | (Padova, Italy) |
| M.S. ARONNA | (Rio De Janeiro, Brazil) |
| Z. ARTSTEIN | (Rehovot, Israel) |
| J. P. AUBIN | (Paris, France) |
| D. BARILARI | (Paris, France) |
| M. BARDI | (Padova, Italy) |
| A. BLOCH | (Ann Arbor, Michigan) |
| B. BONNARD | (Dijon, France) |
| U. BOSCAIN | (Paris, France) |
| P. CANNARSA | (Roma, Italy) |
| P. CARDALIAGUET | (Paris, France) |
| L. CHEN | (Paris, France) |
| J. M. CORON | (Paris, France) |
| A. L. DONTCHEV | (Ann Arbor, Michigan) |
| M. FALCONE | (Roma, Italy) |
| E. FELEQI | (Padova, Italy) |
| F. FORNI | (Liège, Belgium) |
| H. FRANKOWSKA | (Paris, France) |
| D. HOEHENER | (Cambridge, Massachusetts) |
| A. LAI | (Roma, Italy) |
| U. LEDZEWICZ | (Edwardsville, Illinois) |
| E. MARCHINI | (Milano, Italy) |
| C. MARICONDA | (Padova, Italy) |
| A. MARIGONDA | (Verona, Italy) |
| P. MASON | (Paris, France) |
| M. MAZZOLA | (Paris, France) |
| M. NERURKAR | (Camden, New Jersey) |
| T. K. NGUYEN | (State College, Pennsylvania) |
| L. POGGIOLINI | (Firenze, Italy) |
| F. PRIULI | (Roma, Italy) |
| M. QUINCAMPOIX | (Brest, France) |
| L. RIFFORD | (Nice, France) |
| A. SARYCHEV | (Firenze, Italy) |
| H. SCHAETTLER | (St. Louis, Missouri) |
| A. SICONOLFI | (Roma, Italy) |
| M. SIGALOTTI | (Paris, France) |
| C. SINESTRARI | (Roma, Italy) |
| H. J. SUSSMANN | (New Brunswick, New Jersey) |
| D. TONON | (Paris, France) |
| A. VAN DER SCHAFT | (Groningen, The Netherlands) |
| R. B. VINTER | (London, UK) |
| D. VITTONE | (Padova, Italy) |
| P. R. WOLENSKI | (Baton Rouge, Louisiana) |
| H. ZIDANI | (Paris, France) |

## ORGANIZING COMMITTEE

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## INFO

For further information see the conference WEB site

www.controltheoryworkshop.it

INDAM — Università degli Studi di Padova — ITN SADCO Initial Training Network Sensitivity Analysis for Deterministic Controller Design — GDRE ConEDP — Fondazione Cassa di Risparmio di Padova e Rovigo

# Complete program

**( Room V, 3rd floor )**

## Monday, 9/06/2014

- 9.00-9.30 **Registration (Indam room)**
- 9.30-9.50 **Opening (Room V, 3rd floor)**
- 9.50-10.20 **Zvi Artstein** - *Applications of Young Measures to Control Systems*
- 10.20-10.50 **Richard B. Vinter** - *Regularity Properties of the Hamiltonian for State Constrained Optimal Control Problems*

coffe break

- 11.20-11.50 **Hasnaa Zidani** – *HJB approach for stratified state-constrained control problems*
- 11.50-12.20 **Laura Poggiolini** - *State-local optimality in the minimum time problem*
- 12.20-12.50 **Antonio Siconolfi** - *Homogenization and singular limits for Hamilton--Jacobi Bellman equations*

Lunch

- 15.30-16.00 **Daniela Tonon**- *Some new results on Pointwise second order necessary conditions in optimal control under closed control constraints*
- 16.00-16.30 **Antonio Marigonda** - *Controllability of some nonlinear systems with drift via generalized curvature properties*

coffe break

- 17.00-17.30 **Anna Chiara Lai** - *Costly asymptotic controllability for control-polynomial systems*
- 17.30-18.00 **Andrey Sarychev** - *Fréchet Generalized Trajectories and Minimizers for Optimal Control Problems of Low Coercivity*

## Tuesday, 10/06/2014

- 9.00-9.20 **Opening**
- 9.20-9.50 **Ludovic Rifford** - *Control and Dynamics*
- 9.50-10.20 **Heinz Schaettler** - *On the Local Structure of Optimal Controls in the Context of Biomedical Applications*
- 10.20-10.50 **Bernard Bonnard** - *Energy minimization problem vs time minimization in the averaged orbital transfer*

coffe break

- 11.20-11.50 **Davide Barilari** - *Comparison theorems for conjugate points in sub-Riemannian geometry*
- 11.50-12.20 **T. Khai Nguyen** - *Quantitative compactness estimates for Hamilton-Jacobi equations*
- 12.20-13.05 **Héctor J. Sussmann** - *A very weak regularity result for minimizers of real analytic subriemannian metrics*

lunch

- 15.00-15.30 **Ermal Feleqi** - *Towards a useful set-valued iterated Lie bracket*

- 15.30-16.00 **Urszula Ledzewicz** - *Control of the Tumor Microenvironment: Geometric Insights*

coffe break

- 16.30-17.00 **Soledad Aronna** - *Proper extensions of optimal control problems with final constraints*
- 17.00-17.30 **Arjan Van Der Schaft** - *Maximal monotone relations and incrementally port-Hamiltonian systems*
- 17.30-18.00 **Davide Vittone** - *Some new results on sub-Riemannian geodesics*

**Social Dinner (**Trattoria degli Amici, 8.15 p.m.)

## Wednesday, 11/06/2014

- 9.20-9.50 **Ugo Boscain**- *On the relation between the heat diffusion and the value function for problems of Riemannian and sub-Riemannian geometry*
- 9.50-10.20 **Anthony Bloch**- *Dynamics and Geometry of Discrete and Smooth Optimal Control Problems on Manifolds*
- 10.20-10.50 **Mario Sigalotti** - *Control of the multi-input Schroedinger equation*

coffe break

- 11.20-11.50 **Fabio Ancona** - *The role of resonances in the homogeneous feedback stabilization: a case study*
- 11.50-12.20 **Martino Bardi** - *Portfolio optimisation with Gaussian and non-Gaussian stochastic volatility*
- 12.20-12.50 **Carlo Mariconda** - *The non occurrence of the Lavrentiev gap for scalar multi-dimensional variational problems*

lunch

free afternoon

**Speakers' Dinner** (Trattoria Pommidoro, 8.15 p.m.)

## Thursday, 12/06/2014

- 9.00-9.20 **Opening**
- 9.20-9.50 **Asen Donchev** - *Around the inverse function theorem*
- 9.50-10.20 **Jean-Pierre Aubin** - *How Long and How Much Money is Needed to Regulate a Viable Economy*
- 10.20-10.50 **Marc Quincampoix** - *Representation formulas for limit values in optimal control*

coffe break

- 11.20-11.50 **Pierre Cardaliaguet** – *An optimal control approach to Mean Field Games*
- 11.50-12.20 **Elsa Marchini** - *Optimal control for evolution equations with memory*
- 12.20-13.05 **Hélène Frankowska** - *Jets of Set-Valued Maps and Necessary Optimality Conditions in Optimal Control*

lunch

- 15.00-15.30 **Daniel Hoehener** - *Variational Approach to Second-Order Optimality*

*Conditions in Optimal Control*

- 15.30-16.00 **Luxi Chen** - *The Generalized Lax-Hopf Formula. Applications to Investment Problem*
- 16.00-16.30 **Jean-Michel Coron** - *On the control of the Korteweg de Vries equation*

coffe break

- 17.00-17.30 **Marco Mazzola** - *A relaxation result for state constrained inclusions in infinite dimension*
- 17.30-18.00 **Piermarco Cannarsa** -*The Mayer problem with differential inclusions: regularity, sensitivity, and optimality conditions*

## Friday, 13/06/2014

- 9.20-9.50 **Maurizio Falcone** - *A Geometric Control Approach to Domain Decomposition for Hamilton-Jacobi-Bellman equations*
- 9.50-10.20 **Paolo Mason** - *Approximate controllability of the Schroedinger equation via*
- *adiabatic techniques*
- 10.20-10.50 **Fulvio Forni** - *Differentially positive systems*

coffe break

- 11.20-11.50 **Fabio Priuli** - *Modeling different degrees of rationality in pedestrian crowds motion*
- 11.50-12.20 **Peter R. Wolenski** - *Another look at the strong invariance property*
- 12.20-12.50 **Carlo Sinestrari** - *Generalized characteristic flow for eikonal equation*

**Conclusion**

# The role of resonances in the homogeneous feedback stabilization: a case study

Fabio Ancona, Henry Hermes and Matthias Kawski

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**Keywords** : homogeneous system, state feedback stabilization, resonance relations.

We will survey existing necessary conditions for a homogeneous control system (w.r.t. to a given dilation) to admit a homogeneous (of correct order), continuous, asymptotically stabilizing state feedback control. For a class of three dimensional, homogeneous, locally controllable, systems, we shall provide new necessary conditions for their homogeneous (continuous) stabilization based on resonance relations among the eigenvalues of the linear part. The existence of such stabilizing feedback laws is basic in utilizing high order, homogeneous, approximations of nonlinear systems to construct local asymptotically stabilizing feedback controls when linear approximations of the nonlinear system are not locally controllable.

# Proper extensions of optimal control problems with final constraints

M.-Soledad Aronna, Monica Motta and Franco Rampazzo

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**Keywords** : impulsive systems, unbounded control, proper extension

For a control system

$$\dot{x} = f(x, u, v) + \sum_{\alpha=1}^{m} g_\alpha(x)\dot{u}_\alpha, \quad x(a) = \bar{x}, \tag{0.1}$$

on an interval $[a, b]$, we shall rely on the notion of *limit solution* $x$, which is defined for $\mathcal{L}^1$ inputs $u$ and for standard, bounded measurable, controls $v$. Here $\mathcal{L}^1$ denotes the space of *everywhere defined* Lebesgue integrable functions. A limit solution $x$ corresponding to a control $u$ in $\mathcal{L}^1$ is a function belonging to $\mathcal{L}^1$: actually, $x$ is the limit (in a suitable, non metric topology) of standard solutions corresponding to absolutely continuous controls that approximate $u$.

We shall consider an optimal control problem with a final constraint of the type $(x(b), u(b)) \in \mathcal{S}$. We shall investigate the question whether the concept of limit solution provides a *proper extension* of the standard problem with absolutely controls $u$. In particular, we shall provide a sufficient condition guaranteeing that no gaps phenomena arise as a consequence of the extension of the solution's concept.

# Applications of Young Measures to Control Systems

Zvi Artstein

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We shall display the role of Young Measures in constructing variational limits for some control and optimal control systems that depend on highly oscillatory parameters.

# How Long and How Much Money is Needed to Regulate a Viable Economy

Jean-Pierre Aubin

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**Keywords** : Viability, Hamilton-Jacobi Equations, Regulation, Optimal Duration, Optimal Endowment

Duration of temporal windows and endowment in units of numéraire are needed for regulating the evolution of economically viable commodities and prices by financing transactions of commodity allocations during a propitious temporal window. Examples of temporal windows range from the centuries required for building cathedrals to the milliseconds of the high frequency markets.

Regulation means that we compute also the transactions of commodities (velocities of commodity evolutions) and the price fluctuations (velocities of prices) for allowing economic agents to satisfy dynamically (at each instant) their needs while obeying the scarcity constraints.

Viability Theorems imply that a sufficient condition requires that the sum of the transactions values and the "impact of price fluctuations should be non positive. The implicit current orthodox belief requires that budgetary constraint on (virtual) units of numéraire should be respected at the cost of violating (real) economic viability.

The explicit proposition we raise is that the economic viability must be maintained, and, whenever it is at stakes, the budgetary rule should be violated in the following sense:

*a strictly positive endowment in monetary units should be injected by an "endowment provider" in the economy for providing means of payment to the economic agents until economic viability is restored.* See [1]

[1] Aubin J.-P. (2013) *Time and Money. How Long and How Much Money is Needed to Regulate a Viable Economy*, Lecture Notes in Economics and Mathematical Systems, 670, Springer, DOI. 10.1007/978-3-319-00005-3.

# Portfolio optimisation with Gaussian and non-Gaussian stochastic volatility

Martino Bardi , Annalisa Cesaroni and Andrea Scotti

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**Keywords** : financial models, stochastic volatility, singular perturbations, jump Lévy processes

We consider the classical Merton portfolio optimisation problem with volatility depending on a stochastic process $Y_t$. Motivated by a large literature on financial models we assume that $Y_t$ evolves at a faster time scale $t/\epsilon$ than the assets, and we study the asymptotics as $\epsilon \to 0$. This is a singular perturbation for a Hamilton-Jacobi-Bellman equation. Moreover we assume that $Y_t$ is a mean-reverting ergodic process, either driven by a Brownian motion or by a pure jump Lévy process. The HJB equation is parabolic in the former case and integro-differential in the latter. By viscosity solutions methods under suitable conditions we find an effective averaged Hamiltonian $\bar{H}$ and prove convergence of the optimal wealth to the solution of a limit HJB equation governed by $\bar{H}$. This turns out to be associated to a portfolio optimisation problem with constant volatility. However, such effective volatility is not the usual mean historical volatility as in uncontrolled systems, but it is instead a suitable nonlinear harmonic mean.

# Comparison theorems for conjugate points in sub-Riemannian geometry

Davide Barilari and Luca Rizzi

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**Keywords** : Conjugate points, Linear quadratic optimal control problems, Curvature, sub-Riemannian geometry.

In this talk I will present some recent results concerning the sub-Riemannian generalizations of sectional and Ricci-type comparison theorems for the existence of conjugate points along geodesics in Riemannian geometry.

On a Riemannian manifold, it is well known that after the first conjugate point a geodesic is no longer a local minimizer (i.e., here local means with respect to small variations of the initial parameter). Conjugate points can be characterized as the solution of the so-called "Jacobi equation", a second order ODE involving the sectional curvature computed along the geodesic.

The generalization of comparison results to the sub-Riemannian setting is a priori not straightforward, the main difficulty being the lack of a proper theory of Jacobi fields, an analytic definition of curvature and, a fortiori, constant curvature models.

The geodesic problem on a sub-Riemannian manifold is a nonlinear optimal control problem that is linear in the control and with quadratic cost. In this spirit, we regard sub-Riemannian geometry as a special class of optimal control problem. We propose a new approach to the theory of "Jacobi equation" for conjugate points in which the Riemannian sectional curvature is replaced by a suitable concept of curvature defined for affine control system, and discussed in [1, 2]. In this framework, Linear-Quadratic optimal control problems with constant potential appear naturally, playing the role of model spaces (i.e., spaces with constant curvature).

As an application, we prove a version of sub-Riemannian Bonnet-Myers theorem and we obtain some new results on conjugate points for three dimensional left-invariant sub-Riemannian structures.

**References:**

1. *The curvature: a variational approach* (with A. Agrachev and L. Rizzi). Submitted. Preprint ArXiv.

2. *Comparison theorems for conjugate points in sub-Riemannian geometry* (with L. Rizzi). Submitted. Preprint ArXiv.

# Dynamics and Geometry of Discrete and Smooth Optimal Control Problems on Manifolds

Anthony Bloch and P. Crouch, N. Nordkvist

*University of Michigan* abloch@umich.edu

In this talk I will discuss the dynamics and geometry that arises in the optimal control of various nonlinear systems on manifolds. In addition I will discuss their discretization.

I will discuss various systems on Riemannian manifolds and in particular on Lie groups, and show how certain cases give rise to integrable systems. I will discuss the case of Clebsch systems which occur when one has an infinitesimal group action. I will also discuss the more general notion of embedded optimal control problem where one does not necessarily have controllability.

# Energy minimization problem vs time minimization in the averaged orbital transfer

Bernard Bonnard, Institut Mathématiques de Bourgogne

April 17, 2014

**Key words**

Geometric optimal control, orbital transfer, averaging

**Abstract**

In this talk we compare the averaged orbital transfer using low thrust propulsion in the energy minimization case vs the time minimal case. In the energy case the averaged system leads to a Riemannian problem which can be completly analysed in the coplanar case (integration of the extremals trajectories, conjugate and cut loci computations). In the time minimal case the associated metric is Finslerian and we obtain only a qualitative description of the solutions in the case of a transfer towards a circular orbit.

# On the relation between the heat diffusion and the value function for problems of Riemannian and sub-Riemannian geometry

Ugo Boscain

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In this talk I will review some recent results relating the value function for an optimal control problem linear in the control and with quadratic cost (the so called sub-Riemannian problem) and the corresponding heat diffusion. In particular I will discuss what happen at the cut locus (the point where the value function is not smooth) in two relevant cases: generic low-dimensional Riemannian manifolds, generic 3D contact sub-Riemannian manifolds (close to the starting point). As a byproduct, I show that, for generic low-dimensional Riemannian manifolds, the only singularities of the exponential map, as a Lagragian map, that can arise along a minimizing geodesic are A3 and A5 (in classification of Arnol school).

# The Mayer problem with differential inclusions: regularity, sensitivity, and optimality conditions

Piermarco Cannarsa

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**Keywords** : Mayer problem, differential inclusions, sensitivity relations, optimality conditions

In this talk, we will survey recent results for the Mayer problem associated with the differential inclusion $\dot{x} \in F(x)$, aiming to recover sensitivity relations and optimality conditions. One important aspect of our approach is the fact that we do not assume the set-valued map $F$ to possess a parameterization of the form $F(x) = f(x, U)$ with $f$ smooth in $x$. Therefore, our regularity assumptions on the data will be stated in terms of the Hamiltonian $H(x, p) = \sup_{v \in F(x)} \langle v, p \rangle$. Such assumptions will require $H$ to be semiconvex in $x$ and differentiable in $p$ for $p \neq 0$.

Our first step will be to recall the method developed in [5] to prove the semiconcavity of the value function without assuming $F$ to have a smooth parameterization, as was the case in the earlier works [1] and [4].

Then, we will turn to the study of sensitivity relations, that is, inclusions that identify the pair formed by the dual arc and the Hamiltonian, evaluated along the associated minimizing trajectory, as a suitable generalized gradient of the value function. We shall discuss two kinds of relations obtained in recent joint papers with H. Frankowska and T. Scarinci [2, 3]. First, we will derive costate inclusions for the Fréchet and proximal *superdifferential* of the value function along an optimal trajectory. Such inclusions can, in turn, be applied to improve the information contained in the maximum principle. They can also be used to associate, with any nonzero limiting gradient of the value function at some point $(t, x)$, a family of optimal trajectories at $(t, x)$ with the property that families corresponding to distinct limiting gradients have empty intersection. Finally, we will investigate how proximal *sudbifferentiability* propagates along optimal trajectories yielding local regularity results for the value function.

# Bibliography


[1] P. Cannarsa and H. Frankowska. Some characterizations of optimal trajectories in control theory. *SIAM Journal on Control and Optimization*, 29(6):1322–1347, 1991.

[2] P. Cannarsa, H. Frankowska. and T. Scarinci. Sensitivity relations for the Mayer problem with differential inclusions.

[3] P. Cannarsa, H. Frankowska. and T. Scarinci. Regularity and second-order sensitivity relations for Mayer's problem in optimal control.

[4] P. Cannarsa and C. Sinestrari. *Semiconcave functions, Hamilton-Jacobi equations, and optimal control*. Progress in Nonlinear Differential Equations and their Applications, 58. Birkhäuser Boston Inc., Boston, MA, 2004.

[5] P. Cannarsa and P.R. Wolenski. Semiconcavity of the value function for a class of differential inclusions. *Discrete Contin. Dyn. Syst. Ser. B*, 29(2):453–466, 2011.

# An optimal control approach to Mean Field Games

Pierre Cardaliaguet

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The Mean Field Game system formalizes differential games with infinitely many players. We show that techniques typical of optimal control can be used to build solutions for this system and analyze their long-time average.

# The Generalized Lax-Hopf Formula Applications to Investment Problem

Luxi CHEN

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**Keywords** : Lax-Hopf Formula, Transaction of commodities, transaction-cost function, value-function of an intertemporal optimization problem, generalized Lax-Hopf formula, moderation of a transaction cost-function,terminal conditions

The *Lax-Hopf formula* simplifies the value function of an intertemporal optimization (infinite dimensional) problem associated with a convex *transaction-cost* function which depends only on the transactions (velocities) of a commodity evolution: it states that *the value function is equal to the marginal fonction of a finite dimensional problem with respect to durations and average transactions*, much simpler to solve. The average velocity of the value function on a investment temporal window is regarded as an *enrichment*, proportional to the profit and inversely proportional to the investment duration.
At optimum, the Lax-Hopf formula implies that *the enrichment is equal to the cost of the average transaction on the investment temporal window.*
In this study, we generalize the Lax-Hopf formula when the transaction-cost function *depends also on time and commodity*, for reducing the infinite dimensional problem to a finite dimensional problem. For that purpose, we introduce the *moderated transaction-cost* function which depends only on the duration and on a commodity.
Here again, the generalized Lax-Hopf formula reduces the computation of the value function to the marginal fonction of an optimization problem on durations and commodities involving the moderated transaction cost function. At optimum, the enrichment of the value function is still equal to the moderated transition cost-function of average transaction.

# On the control of the Korteweg de Vries equation

Jean-Michel Coron

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**Keywords** : Controllability, stabilization, Korteweg-de Vries equations

The Korteweg de Vries equation allows to describe approximately long waves in water of relatively shallow depth. We study this equation on a finite interval. The control is acting on the boundary of this interval. Depending on these boundary conditions we get different control systems. We present some results and methods to study the controllability and the stabilization of these control systems.

# Around the Inverse Function Theorem

Asen L. Dontchev

*Mathematical Reviews (AMS) and the University of Michigan*

**Keywords** : Inverse function theorem, Lipschitz localization, metric regularity, generalized differentiation, convex best interpolation

The classical inverse/implicit function theorems revolves around solving an equation in terms of a parameter and tell us when the solution mapping associated with this equation is a differentiable function with respect to the parameter. Already in 1927 Hildebrandt and Graves observed that one can put aside differentiability obtaining that the solution mapping is only Lipschitz continuous. In the last decades it has become clear that in order to derive estimates for the solution mapping, it is sufficient to differentiate what you can and leave the rest as is, hoping that the resulting problem is easier to handle. More sophisticated results may be obtained by employing various concepts of generalized differentiability. As an illustration I will present an unconventional implicit function theorem for an optimal control problem.

# A Geometric Control Approach to Domain Decomposition for Hamilton-Jacobi-Bellman equations

M. Falcone

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**Abstract**

In this talk we present a new algorithm for the solution of Hamilton-Jacobi-Bellman equations related to optimal control problems. The key idea is to divide the computational domain into subdomains which are shaped by the optimal dynamics of the underlying control problem, following some ideas introduced by Ancona and Bressan. This can result in a rather complex geometrical subdivision, but it has the advantage that every subdomain is invariant with respect to the optimal dynamics, and then the solution can be computed independently in each subdomain. The features of this dynamics-dependent domain decomposition can be exploited to speed up the computation and for an efficient parallelization, since the classical transmission conditions at the boundaries of the subdomains can be avoided. Although the method has been designed for hyperbolic equations, we can extend some of its features to semi-linear equations of second order. We present and discuss some preliminary results related to this class of equations. Several examples in two and three dimensions illustrate the properties of this dynamic domain decomposition method.

Joint works with S. Cacace, E. Cristiani, M. Falcone, A. Picarelli.

# Towards a useful set-valued iterated Lie bracket

Ermal Feleqi and Franco Rampazzo

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**Keywords** : asymptotic and integral formulas; generalized differential quotients; set-valued iterated Lie brackets; Chow-Rashevski's theorem

I will show how classical asymptotic formulas for iterated Lie bracket generating multiflows, such as

$$xe^{t_1f_1}e^{t_2f_2}e^{-t_1f_1}e^{-t_2f_2}e^{t_3f_3}e^{t_2f_2}e^{t_1f_1}e^{-t_2f_2}e^{-t_1f_1}e^{-t_3f_3} = x + t_1t_2t_3\,(x[[f_1,\,f_2],\,f_3]) + o(t_1t_2t_3)$$

as $(t_1, t_2, t_3) \to (0, 0, 0)$, in the Agrachev-Gamkrelidze formalism, for vector fields $f_1, f_2 \in C^2$, $f_3 \in C^1$, can be improved by integral formulas like

$$xe^{t_1f_1}e^{t_2f_2}e^{-t_1f_1}e^{-t_2f_2}e^{t_3f_3}e^{t_2f_2}e^{t_1f_1}e^{-t_2f_2}e^{-t_1f_1}e^{-t_3f_3} - x = \int_0^{t_1}\int_0^{t_2}\int_0^{t_3} xe^{t_1f_1}e^{t_2f_2}e^{-t_1f_1}e^{-t_2f_2}e^{s_3f_3}e^{t_2f_2}e^{t_1f_1}e^{(s_2-t_2)f_2}e^{-t_1f_1}e^{-s_2f_2}$$
$$\big[e^{s_2f_2}e^{s_1f_1}[f_1,f_2]e^{-s_1f_1}e^{-s_2f_2},\,f_3\big]e^{s_2f_2}e^{t_1f_1}e^{-s_2f_2}e^{-t_1f_1}e^{-s_3f_3}ds_1ds_2ds_3.$$

These formulas combined with H. Sussmann's GDQs–generalized differential quotients–suggest a generalization of the notion of "iterated bracket" to vector fields with low regularity. For instance, if $f_1, f_2 \in C^{1,1}$, $f_3 \in C^{0,1}$, we set

$$[[f_1,f_2],\,f_3]_{set}(x) = \overline{co}\Big\{\lim_{k\to\infty}(Df_3(y_k)[f_1,f_2](x_k) - D[f_1,f_2](x_k)f(y_k)) \;:$$
$$\forall\{x_k\}\subset DIFF([f_1,f_2]),\ \forall\{y_k\}\subset DIFF(f_3),\ x_k \overset{k}{\to} x,\ y_k \overset{k}{\to} x\Big\},$$

where $DIFF(g)$ denotes the set of differentiability points of $g$. The set-valued vector field $x \mapsto [[f_1, f_2],\, f_3]_{set}(x)$ turns out to be upper semi-continuous with compact, convex, nonempty values, and reduces to the singleton $\{[[f_1, f_2],\, f_3](x)\}$ at differentiability points $x \in DIFF([f_1, f_2]) \cap DIFF(f_3)$.

Then I will conclude by showing how set-valued iterated brackets of this kind can be used to obtain a nonsmooth version of Chow-Rashevski's theorem.

# Differentially positive systems

Fulvio Forni and Rodolphe Sepulchre

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**Keywords** : Positive systems, Differentially positive systems, Horizontal contraction, multi stability, limit cycles analysis.

The trajectories of a positive system leave a cone invariant. Positive systems have a rich history both because of the relevance in applications (e.g., when modeling a behavior defined by positive variables) and because positivity significantly restricts the behavior, leading to the Perron-Frobenius theory. Motivated by the importance of positivity in linear system analysis, the talk introduces and explores the concept of differential positivity for nonlinear behaviors. Differential positivity means positivity of the linearized system along any trajectory. We derive the analog of Perron-Frobenius theory in this generalized setup and explore its consequences for the asymptotic behavior of the system. We show that, under suitable conditions, differential positivity strongly restricts the asymptotic behavior and provides a novel analysis and design tool for multistable or limit cycle behaviors. The analysis of a nonlinear pendulum will illustrate the potential of the approach.

# Jets of Set-Valued Maps and Necessary Optimality Conditions in Optimal Control

Helene Frankowska

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**Keywords** : Derivatives of set-valued maps, optimal control, state constraints.

This talk concerns an approximation of set-valued maps that leads to a second order variational differential inclusion. In particular, it allows to obtain a new pointwise second order condition verified by the adjoint state of the maximum principle. Results were obtained in collaboration with D. Hoehener and D. Tonon.

# Variational Approach to Second-Order Optimality Conditions in Optimal Control

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**Keywords** : Second-order optimality conditions, state constraints

Using set-valued analysis techniques, we derive second-order variational equations. These results motivate the definition of sets of second-order feasible variations for control systems with pure state constraints. Having feasible variations, second-order optimality conditions for optimal control problems with pure state constraints in integral form can be derived quite naturally and under weak regularity assumptions on the data. A relationship between the nonemptiness of the set of feasible second-order variations and the normality of the maximum principle can also be shown.

# Costly asymptotic controllability for control-polynomial systems.

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**Keywords** : control-polynomial systems, unbounded controls, controllability, cost estimate.

We present some results on control systems characterized by polynomial growth in the control variable and by the unboundedness of the control set.

This class of systems generalizes control-quadratic systems, that are suitable to model mechanical systems controlled via time-varying, frictionless, holonomic constrains.

We show a general result on sufficient conditions for asymptotic controllability and a cost estimate.

We finally discuss the case of control-polynomial systems, whose algebraic properties play a special role in order to establish less demanding conditions for asymptotic controllability.

This is a joint work with Franco Rampazzo.

# Control of the Tumor Microenvironment: Geometric Insights

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**Keywords** : optimal control, bang-bang and singular controls, models for cancer treatment

Mathematical modeling of cancer growth and treatments is a rapidly developing field with methods and techniques from various areas being utilized. Optimal control has its relevant role when it comes to analyze the structure of theoretically optimal treatment protocols. It gives rise to mathematical problems with nontrivial solutions that are both interesting for their intrinsic merit and their implications on the underlying medical problem. For example, for an optimal control problem that models a homogeneous tumor under chemotherapy, optimal controls are bang-bang reaffirming the medical paradigm of maximum tolerated doses (MTD). However, as more aspects of the tumor microenvironment are taken into account—such as tumor heterogeneity, tumor vasculature or tumor-immune system interactions—alternatives to this paradigm become a viable option.

In this talk, we will present some models that account for both heterogeneity and elements of the tumor microenvironment and incorporate the effects of treatment, the controls of the system, on all these compartments. We will analyze the dynamics of the systems and geometric properties of the regions of attraction of uncontrolled stable equilibrium points. This analysis naturally leads to the formulation of an optimal control problem. The occurrence of singular controls as an integral piece of the solutions will be discussed and its connections with current medical research on alternative treatment protocols to MTD like metronomic chemotherapy, chemo-switch protocols or adaptive therapy will be addressed.

# Optimal control for evolution equations with memory

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**Keywords** : integrodifferential equations, optimal control

In this talk, we investigate the existence and regularity of solutions for Bolza optimal control problems in infinite dimension governed by a class of semilinear evolution equations. Our results apply to systems exhibiting hereditary properties, as heat propagation in real conductors and isothermal viscoelasticity, described by equations with memory terms which account for the past history of the variables in play.

**The non occurrence of the Lavrentiev gap for scalar multi-dimensional variational problems**

Carlo Mariconda - Università degli Studi di Padova

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function, $\Omega$ be an open and bounded subset of $\mathbb{R}^n$. We consider the functional

$$I(u) := \int_\Omega f(\nabla u(x))\, dx \quad u \in W^{1,1}(\Omega).$$

It is known [3] that if $\Omega$ is star-shaped then the Lavrentiev phenomenon does not occur if one does not consider a fixed boundary datum, i. e.

$$\inf\{I(u) : u \in W^{1,1}(\Omega)\} = \inf\{I(u) : u \in W^{1,\infty}(\Omega)\}.$$

The importance of the non occurrence of the Lavrentiev phenomenon is due to the fact that only in that case, the methods of numeric analysis allow to approximate the infimum value of the operator (finite elements method). When the boundary datum is taken into account, in spite of the paradigm saying that the Lavrentiev phenomenon should not occur, there are just a few results corroborating the statement, apart the obvious case where some "natural growth conditions" are assumed or when it is known a priori that the minima of $I$ are Lipschitz: we mention the recent paper [1] where it is proved that, if the lagrangian is radial and both the boundary datum and the domain are of class $\mathcal{C}^2$, then the Lavrentiev phenomenon does not occur.

In a joint work with Pierre Bousquet and Giulia Treu [2], we make a step forward in favor of the above conjecture and take into account a class of domains more general than Lispchitz, with a minimum set of assumptions (in particular no growth conditions). Actually, for a wide class of autonomous Lagrangeans $f(u, \nabla u)$ containing the case of those that are convex in both variables and some non autonomous and non convex cases, we prove the non occurrence of the Lavrentiev gap: if $I(u) := \displaystyle\int_\Omega f(u, \nabla u)$ is finite for some $u \in W^{1,1}_\phi(\Omega)$, with $\phi$ Lipschitz on $\partial\Omega$, then the value $I(u)$ can be approximated via a sequence $I(u_k)$ where the $(u_k)_k$ are Lipschitz and $u_k = \phi$ on $\partial\Omega$.

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# Controllability of some nonlinear systems with drift via generalized curvature properties

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**Keywords :** geometric control theory, Clarke's generalized gradient, curvature

We discuss the problem of local attainability for finite-dimensional nonlinear control systems with quite general assumption on the target set. Special emphasis is given to control-affine systems with a possibly nontrivial drift term. To this end, we provide some sufficient conditions ensuring local attainability, which involve geometric properties both of the target itself (such as a notion of generalized curvature), and of the Lie algebra associated to the control system. The main technique used is a convenient representation formula for the power expansion of the distance function along the trajectories, made at points sufficiently near to the target set.

# Approximate controllability of the Schrödinger equation via adiabatic techniques

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In this talk I will present a constructive method to control the bilinear Schrödinger equation with three inputs. The method is based on adiabatic techniques and works if the spectrum of the Hamiltonian, pararameterized by the control inputs, admits conical eigenvalue intersections. The results allow to attain a prescribed probability density with an error that can be estimated in terms of the final time.

# A relaxation result for state constrained inclusions in infinite dimension

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**Keywords** : Relaxation, differential inclusions, state constraints.

We provide sufficient conditions for a relaxation theorem concerning the semilinear differential inclusion $x' \in Ax + F(t,x)$, under state constraints of the form $x(t) \in K$. Here $A$ is the infinitesimal generator of a strongly continuous semigroup on an infinite dimensional separable Banach space $X$, $F : [0,1] \times X \to X$ is a set-valued map and $K$ is a closed subset of $X$. Some applications to PDE control problems are presented.

# Quantitative compactness estimates for Hamilton-Jacobi equations

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**Keywords** : Hamilton-Jacobi equations, Hopf-Lax semigroup, Kolmogorov entropy, semiconcave functions

Consider a first-order Hamilton-Jacobi equation

$$u_t(t,x) + H(\nabla u(t,x)) = 0, \qquad x \in \mathbb{R}^N, \quad t > 0\,, \tag{0.1}$$

with a strictly convex and coercive Hamiltonian $H : \mathbb{R}^N \to \mathbb{R}$. For every $\overline{u} \in W^{1,1}(\mathbb{R}^N, \mathbb{R})$, let $S_t\overline{u} \doteq u(t,\cdot)$ denote the unique viscosity solution of (**??**) with initial data $u(0,\cdot) = \overline{u}$. Having in mind the analysis recently developed for solutions to conservation laws [2-4], inspired by a question posed by Lax, we are interested in studying the compactifying effect of the operator $S_t$, at any fixed time $t > 0$, w.r.t the $W^{1,1}$-topology. Namely, we wish to estimate the Kolmogorov $\varepsilon$-entropy in $W^{1,1}$ of the image of bounded sets of initial data through the map $S_t$. We recall that, given a metric space $(X, d)$, and a totally bounded subset $K$ of $X$, we let $N_\varepsilon(K \mid X)$ denote the minimal number of sets in a cover of $K$ by subsets of $X$ having diameter $\leq 2\varepsilon$, and define the Kolmogorov $\varepsilon$-entropy of $K$ as $H_\varepsilon(K \mid X) \doteq \log_2 N_\varepsilon(K \mid X)$. Entropy numbers play a central roles in various areas of information theory and statistics as well as of ergodic and learning theory. In the present setting, as suggested by Lax, this concept could provide a measure of the order of "resolution" of a numerical method for (**??**).

Our main result in [1] shows that, for every fixed $L, M > 0$, letting $C_{[L,M]}$ denote the set of Lipschitz functions $u : \mathbb{R}^N \to \mathbb{R}$ with Lipschitz constant $L$ and with support contained in $[-M, M]^N$, there holds

$$H_\varepsilon\big(S_T(C_{[L,M]}) \mid W^{1,1}(\mathbb{R}^N, \mathbb{R})\big) \approx (1/\varepsilon^N). \tag{0.2}$$

Relying on fine properties of monotone operators we derive upper estimates on the $\varepsilon$-entropy of classes of semiconcave functions, which in turn yield upper estimates on $H_\varepsilon(S_T(C_{[L,M]}))$. Instead, lower bounds on $H_\varepsilon(S_T(C_{[L,M]}))$ are established in two steps. We first introduce a class of semiconcave functions $\mathcal{SF}$ defined as combinations of suitable bump functions, and with a combinatorial argument we provide an optimal lower

estimate on the $\varepsilon$-entropy of such a class. Next, we prove a controllability result showing that any element of $\mathcal{SF}$ can be obtained, at any given time $T > 0$, as the value $u(t, \cdot)$ of a viscosity solution of (**??**), with initial data in $C_{[L,M]}$. (*Joint work with Fabio Ancona and Piermarco Cannarsa*).

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# State-local optimality in the minimum time problem

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**Keywords** : Pontryagin extremals, minimum time problem

We consider the minimum time problem for a control affine system of the form

$$\begin{gathered} T \to \min, \\ \dot\xi(t) = f_0(\xi(t)) + \sum_{s=1}^{m} u_s(t) f_s(\xi(t)), \\ \xi(0) = N_0, \qquad \xi(T) = N_f, \\ |u_s(t)| \le 1 \quad s = 1, 2, \dots, m \text{ a.e. } t \in [0, T]. \end{gathered}$$

For such a control system we consider the following problem: given a Pontryagin extremal of the problem, i.e. a reference admissible triplet $\left(\widehat{T}, \widehat{\xi}, \widehat{u}\right)$ satisfying Pontryagin Maximum Principle, we are interested in finding sufficient conditions for the extremal trajectory $\widehat{\xi}$ to be a *state-local optimal trajectory* according to the following definition:

*The trajectory $\widehat{\xi}$ is a state-local minimiser if there are neighborhoods $\mathcal{U}$ of its range $\widehat{\xi}([0,\widehat{T}])$, $\mathcal{U}_0$ of $\widehat{\xi}(0)$ and $\mathcal{U}_f$ of $\widehat{\xi}(\widehat{T})$ such that $\widehat{\xi}$ is a minimiser among the admissible trajectories whose range is in $\mathcal{U}$, whose initial point is in $N_0 \cap \mathcal{U}_0$ and whose final point is in $N_f \cap \mathcal{U}_f$.*

The previous definition is to be compared to the better known notion of *(time, state)-local optimality*:

*The trajectory $\widehat{\xi}\colon [0,\widehat{T}] \to M$ is a (time, state)-local minimiser if there exist a neighborhood $\mathcal{U}$ of its graph in $\mathbf{R} \times M$ and $\varepsilon > 0$ such that $\widehat{\xi}$ is a minimiser among the admissible trajectories whose graphs are in $\mathcal{U}$ and whose final time is greater than $\widehat{T} - \varepsilon$.*

The problem of state-local optimality of Pontryagin extremals has been addressed via Hamiltonian methods in several different situations i.e. for different end-point conditions and different structure of the reference control $\widehat{u}$, see [3, 1, 4, 5, 2].

A related problem that can be studied by Hamiltonian methods is the problem of structural stability i.e. if $\widehat{\xi}$ is a (time, state)- or a state-local optimal trajectory and the associated optimal control $\widehat{u}$ has a certain structure, what happens when the data of the problem are perturbed? Given

that in concrete control problems the data are never known exactly, one would like to see if the structure of the optimal control is preserved under such perturbations, see [6, 7].

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# Modeling different degrees of rationality in pedestrian crowds motion

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**Keywords** : crowd models, optimal feedback, mean field games

In the last decade an increasing attention has been devoted to models describing the behavior of pedestrian crowds in 2d domains. Typically, the phenomenon is described through a continuity equation

$$\partial_t \rho + \mathrm{div}(\rho\, v) = 0\,, \qquad (t,x) \in [0,+\infty[\,\times \mathbf{R}^2\,,$$

for the density of pedestrian $\rho(t,x)$, representing the conservation of the total "mass", with a suitable flow velocity $v = v(\rho) \in \mathbf{R}^2$ which encodes into the model the fact that pedestrians are "active particles" who can make choices.

In this talk, we present a family of models introduced in [1, 2], where the flow velocity is chosen as the superposition of two separate effects: a non–local *interaction term* in integral form, that accounts for the reaction of each individual to the presence of other walkers, and a *feedback control term*, that is chosen by each pedestrian by solving a suitable finite dimensional optimal control problem.
In order to describe different pedestrian behaviors, we will formulate different control problems, depending on the amount of informations that walkers use in their planning. This allows to handle at once several different situations: from pedestrians which just ignore the presence of the others (leading to a control problem which is independent from the continuity equation) to pedestrians that can forecast the long–term effects of their choices (leading to a fully coupled mean field game).

Theoretical and numerical results will be presented to highlight the different behaviors of pedestrians and to suggest that environment design could be exploited to get a "good" behavior even in the case of incomplete information and limited predictive capabilities.

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# Representation formulas for limit values in optimal control

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A classical problem in optimal control problem concerns a payoff which is an average of a given function of the control and of the trajectory. We investigate the existence of a limit value when the averaging parameter converges. Typical cases of such limit values are Cesaro mean when the horizon tends to infinity and Abel mean when the discount factor tends to zero. Our main goal is to obtain the existence of a limit in situations less restrictive than the ergodic case where the limit value is constant (i.e . it does not depend of the initial condition). We give a representation formula of the possible limit which appears to be independent of the chosen averaging procedure. We discuss several conditions showing the existence of a limit value.

# Control and Dynamics

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We address several problems in dynamics with a geometric control approach. We will focus on two types of problems related with generic properties of geodesic flows, namely the closing of geodesics and the perturbation of Poincar maps along geodesics. We will show that some of these problems are indeed problems of controllability and can be solved by classical control methods. However, part of these problems are not so easy to tackle.

# Fréchet Generalized Trajectories and Minimizers for Optimal Control Problems of Low Coercivity

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We address consecutively several problems related to generalized minimizers of the optimal control problems with the functionals of low growth.

First is introducing a class of so called Fréchet generalized controls for a multi-input control-affine system with non-commuting controlled vector fields. For each control of the class one is able to define a unique generalized trajectory, and the topology, introduced in the spaces of the generalized controls and trajectories, guarantees continuity of input-to-trajectory map.

On the other side the class of generalized controls is broad enough to settle the problem of proving existence of generalized minimizers of Lagrange variational problem with functionals of linear growth.

Besides we explore possibility of Lavrentiev-type gap between the infima of the functionals in the spaces of ordinary and generalized controls.

# On the Local Structure of Optimal Controls in the Context of Biomedical Applications

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**Keywords** : local structure of optimal control, chattering, biomedical applications

In the 1980s and 1990s, using geometric methods in optimal control numerous results about the local structure of time-optimal controls near a point (local synthesis) were established in low dimensions. For example, in the research of H. Sussmann a complete solution to the time-optimal control problem in the plane for real-analytic systems has been given. These results have far greater reach than just applying to the time-optimal control problem in the sense that they also suggest the structure of local syntheses near specific generic situations for general control-affine problems in higher dimension like, for example, when optimal singular arcs saturate. In this talk, we highlight some of these applications to mathematical models for cancer treatments.

In such models the controls typically denote concentrations, respectively dose rates of some therapeutic agent and it is of practical significance whether or not optimal therapy protocols contain singular arcs (administration at reduced doses versus maximum tolerated doses). Dose rates relate to concentrations by means of pharmacokinetic models. These are described by typically a 1- or 2-dimensional time-invariant linear system that is injected into the overall nonlinear dynamics. As a result, optimality properties of a singular arc are preserved when concentrations are replaced with the dose rates as controls, but the intrinsic order of the singular control increases by the dimension of the pharmacokinetic model. Thus chattering concatenations arise naturally for such models. The local structure of extremals (optimal controls) near such a junction point will be discussed.

# Homogenization and singular limits for Hamilton–Jacobi Bellman equations

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**Keywords** : homogenization, singular limit, control problems, Hamilton Jacobi Bellman equations

We present an unified approach of periodic homogenization and singular limit in a nonperiodic setting for a family of Hamilton–Jacobi–Bellman equations associated to control problems. The aim is to provide a qualitative analysis by means of tools coming from weak KAM theory and to give a detailed description of the limit problem using invariant measures of a flow related to the controlled dynamics.

# Control of the multi-input Schrödinger equation

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We present a sufficient condition for approximate controllability of the bilinear discrete-spectrum Schrödinger equation in the multi-input case. The controllability result extends to simultaneous controllability, approximate controllability in $H^s$, and tracking in modulus. The sufficient condition allows severe degeneracies in the spectrum of the uncontrolled Hamiltonian, as illustrated by an application to the model of a rotating polar linear molecule, driven by three orthogonal external fields.

# Generalized characteristic flow for eikonal equation

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We consider the time dependent eikonal equation

$$\begin{cases} u_t(t,x) + \frac{1}{2}|\nabla u(t,x)|^2 = 0 & \text{a.e. } (t,x) \in (0,+\infty) \times R^n \\ u(0,x) = u_0(x) & \text{a.e. } x \in R^n, \end{cases} \tag{0.1}$$

whose solution is given by the Hopf formula. We show that the singularities (i.e., the points of nondifferentiability) of the solution propagate forward in time along generalized characteristics, as described in the following result.

**Theorem** (P. Cannarsa, M. Mazzola, C.S.) Let $u$ be the solution of (0.1) and let $\Sigma \subset (0,+\infty) \times R^n$ denote the set of points where $u$ is not differentiable. Let $\nabla^+ u(t,x)$ denote the superdifferential of $u$ at $(t,x)$ with respect to the $x$ variable. Then, for any $(t_0,x_0) \in \Sigma$, there exists a Lipschitz continuous arc $\gamma : [t_0,\infty) \to R^n$ satisfying

$$\begin{cases} \dot\gamma(t) \in \nabla^+ u(t,x) & \text{a.e. } t \in (0,+\infty) \\ \gamma(t_0) = x_0, \end{cases}$$

and such that $(t,\gamma(t)) \in \Sigma$, for all $t \in [t_0,\infty)$.

We also give generalizations to the same equation on a riemannian manifold $M$ instead of $R^n$. The novelty of the above result lies in the fact that the generalized characteristic $\gamma$ remains singular for all times $t \in [t_0,\infty)$ and not only in some right neighborhood of $t_0$. This can be regarded as the evolutive counterpart of the result recently obtained in collaboration with P. Albano, P. Cannarsa and K.T. Nguyen for the distance function from the boundary of an open set, which satisfies a stationary eikonal equation.

# A very weak regularity result for minimizers of real analytic subriemannian metrics

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Using various results on subanaytic stratifications, we prove that, for a real analytic subriemannian structure, every length-minimizer parametrized by arc-length is real analytic on an open dense subset of its interval of definition. This is a stronger version of our previous result —which, however, is valid for a much larger class of real analytic optimal control problems— according to which for every optimal trajectory there exists a—possibly different—optimal trajectory which is real analytic on an open dense subset of its interval of definition and has the same initial and terminal points.

# Pointwise second order necessary conditions in optimal control under closed control constraints

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**Keywords** : Second order conditions, Mayer problem, singular optimal controls

When the optimal control is singular, the classical Legendre-Clebsch condition is no more a useful necessary optimality condition for the Mayer problem in optimal control. Alternative pointwise necessary optimality conditions, known as Goh and generalized Legendre-Clebsch conditions, were proved by Goh in [3], in the case of an open control constraint. When the control set is a closed subset of $\mathcal{R}^m$, new techniques are necessary even when dealing with points that are in the interior of it. We show a generalization of the Goh and the generalized Legendre-Clebsch conditions to particular cases of closed control constraints. These results are contained in [1, 2].

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# Maximal monotone relations and incrementally port-Hamiltonian systems

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**Keywords** : Maximal monotone relations, compositions, Hamiltonian systems, passivity, convexity

In this talk we discuss the newly introduced class of incrementally port-Hamiltonian systems. Incrementally port-Hamiltonian systems extend port-Hamiltonian systems by replacing the composition of the Dirac structure and energy-dissipating relation, as underlying the definition of port-Hamiltonian systems, by a maximal monotone relation. After introducing and motivating by physical examples this new class of systems, we study their compositions, and show that incrementally port-Hamiltonian systems are closed under composition. The proof of this relies on the result that compositions of maximal monotone relations are again maximal monotone. Furthermore, we discuss relationships with recently introduced notions of incremental and differential passivity. In particular, we show how incrementally port-Hamiltonian systems with convex lower-bounded Hamiltonians are maximal equilibrium independent passive. We recall the close connection of maximal monotone relations with convexity, and show how this leads to an optimization perspective, and in particular to a generalization of recently obtained results on asymptotic behavior of interconnections of incrementally port-Hamiltonian systems.

# Regularity Properties of the Hamiltonian for State Constrained Optimal Control Problems

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**Keywords** : Optimal Control, State Constraints, Differential Inclusions

It is well known that the Hamiltonian, evaluated along the optimal trajectory and costate arc, is constant. In this talk we observe that this is just one manifestation of the fact that the Hamiltonian inherits certain regularity properties of the data (with respect to time). Special attention is given to the regularity properties of the Hamiltonian, when the data is merely of bounded variation. In this case, New estimates are given on the cumulative variation of the Hamiltonian, in terms of the cumulative variation of the data. The estimates lead to improved conditions for non-degeneracy of the state constrained Maximum Principle, and also for the Lipschitz continuity of minimizers.

# Some new results on sub-Riemannian geodesics

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**Keywords** : Sub-Riemannian geodesics, abnormal extremals, Carnot groups

We present some result on sub-Riemannian geodesics recently obtained in collaboration with E. Le Donne, G. P. Leonardi, and R. Monti. A sub-Riemannian manifold is a manifold $M$ endowed with a distinguished subbundle $HM$ of the tangent bundle $TM$ and with a metric on $HM$. A distance on $M$ can be defined on minimizing the length among curves which are tangent to $HM$. One of the main open problems in the field is the regularity of length minimizers, that is not trivial due to the presence of the so called abnormal curves. We provide a characterization of abnormal curves in Carnot groups showing that they are contained in certain algebraic varieties; applications to the problem of geodesics' regularity will be discussed.

# Another look at the strong invariance property

*Peter R. Wolenski*, Louisiana State University

We survey the strong invariance property for control problems, and demonstrate its relevance in Hamilton-Jacobi theory. We will use a modified form of its classical statement with Lipschitz dynamics to prove a more general result for stratified systems.