Conjunctive Use of Drinking Water Sources with Multiple Providers

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Contribution

Optimal management of drinking water involves conjunctive use of different sources, one of which is flow and the other stock. This is more complex when multiple providers use the same surface water source (i.e. river) to supply distinct groups of customers. In this paper we consider the interaction between two profit maximizer providers (upstream and downstream) and specifically we determine the optimal conjunctive use at every catchment point.

Model - 2 players - degenerate differential game

Upstream provider (1) with constant demand d_1 Downstream provider (2) with constant demand d_2

- each one has his own aquifer → 2 AQUIFERS $u_i(t)$ groundwater extraction rate (m^3/ sec)

P_{gi} constant unit profit

- each one abstracts from the **same river** → 1 RIVER

F surface flow (m^3/ sec)
mLF minimum Life Flow(m^3/ sec)
P_{si} constant unit profit

Upstream provider

$$\max_{u_1 \geq 0} \int_0^T e^{-rt} \left(\overbrace{P_{g1}u_1(t)}^{Groundwater\ \ Profit} + \overbrace{P_{s1}(d_1 - u_1(t))}^{Surface\ \ Profit} - \overbrace{C_{e1}(x_1(t), u_1(t)) \cdot u_1(t)}^{Environmental\ \ Costs} \right) dt$$

$$\text{s t} \quad \dot{x_1}(t) = R_1 - u_1(t)$$

$$x_1(0) = x_{10}, \quad x_1(t) \geq 0$$

$$u_1(t) \in [\underbrace{\max\{0, d_1 - (F - mLF)\}}_{\underline{u}_1}, \underbrace{\min\{d_1, u_1max\}}_{\overline{u}_1}]$$

Feasibility condition: $u_1max > d_1 - (F - mLF)$

Downstream provider

$$\max_{u_2 \geq 0} \int_0^T e^{-rt} \left(P_{g2}u_2(t) + P_{s2}(d_2 - u_2(t)) - C_{e2}(x_2(t), u_2(t)) \cdot u_2(t)\right) dt$$

$$\text{s t} \quad \dot{x_2}(t) = R_2 - u_2(t)$$

$$x_2(0) = x_{20}, \quad x_2(t) \geq 0$$

$$u_2(t) \in [\underbrace{\max\{0, d_1 + d_2 - (F - mLF) - u_1^*(t)\}}_{\underline{u}_2(u_1^*(t))}, \underbrace{\min\{d_2, u_2max\}}_{\overline{u}_2}]$$

Optimal control problem with mixed constrained

Environmental costs not depending on the extraction rate $C_{ei}(x_i)$

$\frac{\partial C_{ei}(x_i,u_i)}{\partial u_i} = 0$ **bang-bang solution** For all $t \in [0, T]$,

$$u_i^*(t) = \begin{cases} \underline{u}_i, & x_{i0} < \frac{\alpha_i}{(P_{gi} - P_{si}) - \lambda_i(0)}, \ \textit{Low aquifer volume} \\ \overline{u}_i, & x_{i0} > \frac{\alpha_i}{(P_{gi} - P_{si}) - \lambda_i(0)}, \ \textit{High aquifer volume} \end{cases} \qquad \dot{\lambda}_i(t) = \frac{\partial C_{ei}(x_i)}{\partial x_i} u_i(t) + r\lambda_i(t), \qquad \lambda_i(T) = 0$$

Environmental costs depending on the extraction rate $C_e(x_i, u_i) = \beta_i u_i / x_i$

$$\hat{u}_i(t) = \frac{(P_{gi} - P_{si}) - \lambda_i(t)}{2\beta_i} x_i(t) \quad \textit{feedback solution}$$

$$\lambda_i(t) = (P_{gi} - P_{si}) + 2\beta_i r + 2\sqrt{\beta_i r}\sqrt{\beta_i r + (P_{gi} - P_{si})} \tanh\left(\frac{\sqrt{r}(t - T)\sqrt{\beta_i r + (P_{gi} - P_{si})}}{2\sqrt{\beta_i}} - \tanh^{-1}\left(\frac{2\beta_i r + (P_{gi} - P_{si})}{2\sqrt{\beta_i}\sqrt{r}\sqrt{\beta_i r + (P_{gi} - P_{si})}} \right) \right)$$

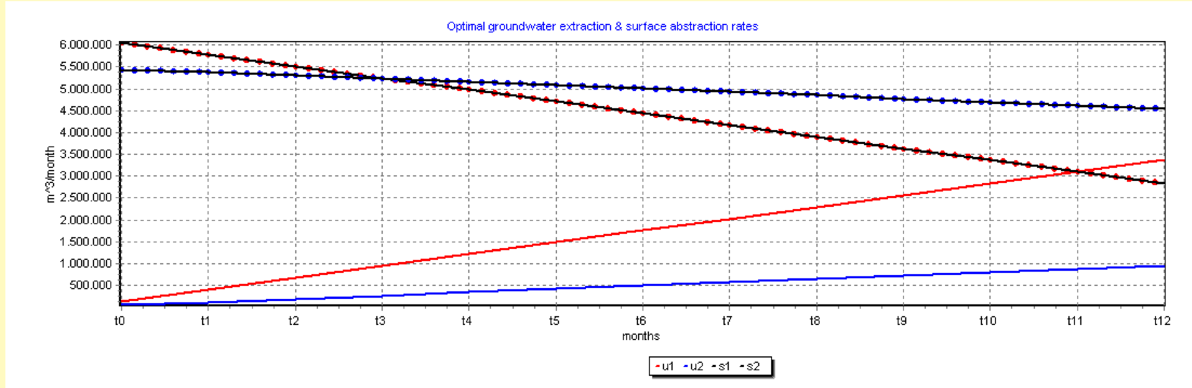
$$u_i^*(t) = \max\{\underline{u}_i, min\{\overline{u}_i, \hat{u}_i(t)\}\}$$

- Optimal extraction rate depends on the water volume at a given time and on the instant itself
- Being the water volume fixed, the more I approximate to the final time, the more I can abstract. I have no more time left so that I can exploit the aquifer (no condition on final water volume)
- $x_i(t) = 0 \Rightarrow u_i^*(t) = 0$ (empty aquifer ⇒ no extraction)

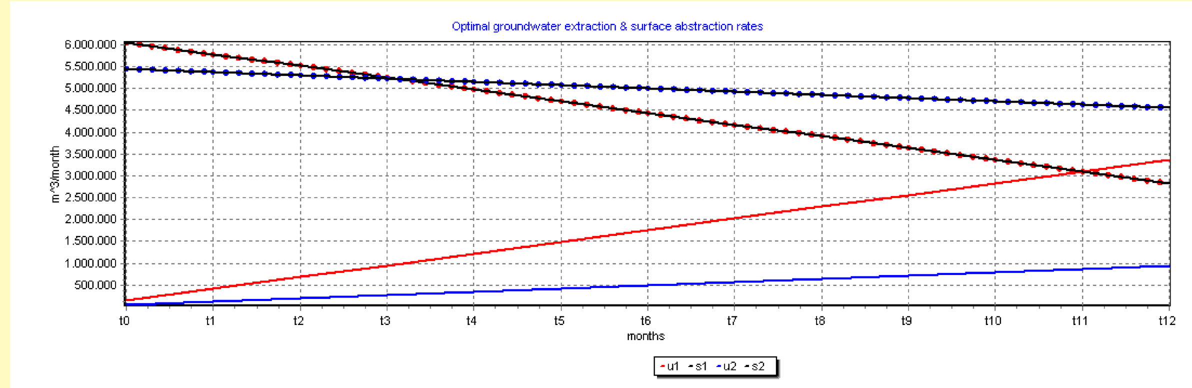
Numerical simulations: Non-cooperative vs Cooperative management strategies
(G.A.M.S. Euler method)

No First mover advantage: $P_{g1} > P_{s1}, \quad P_{g2} > P_{s2}$

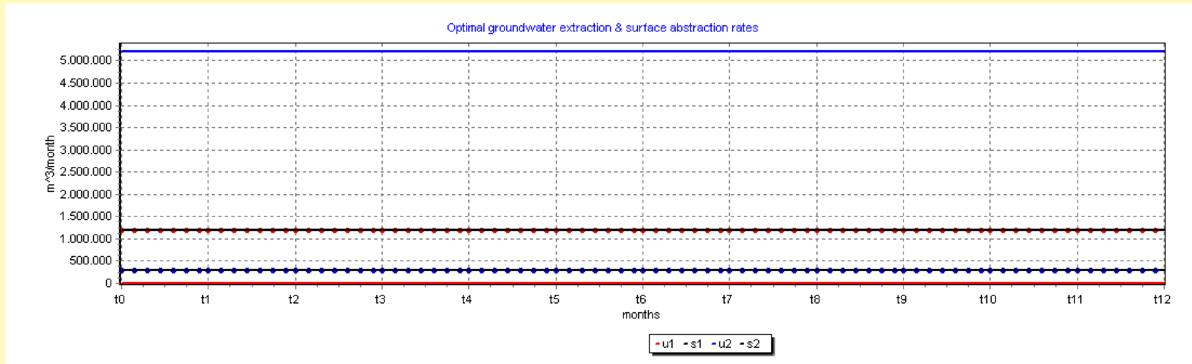
Non cooperative case

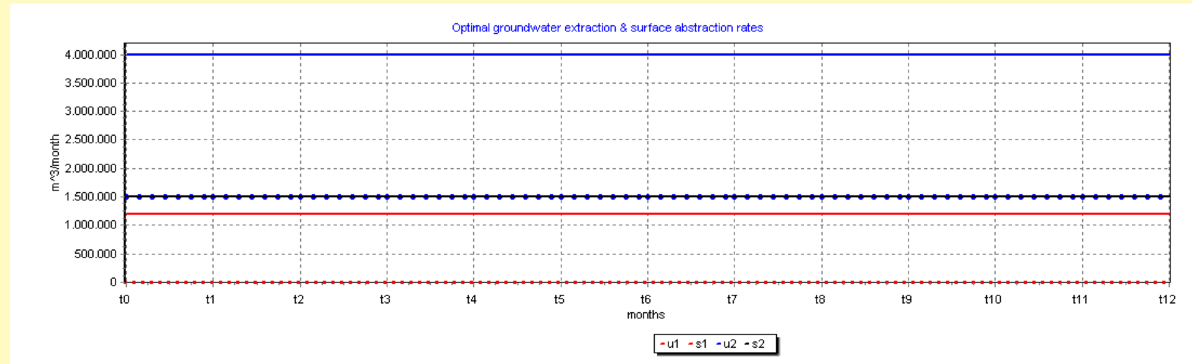


Cooperative case

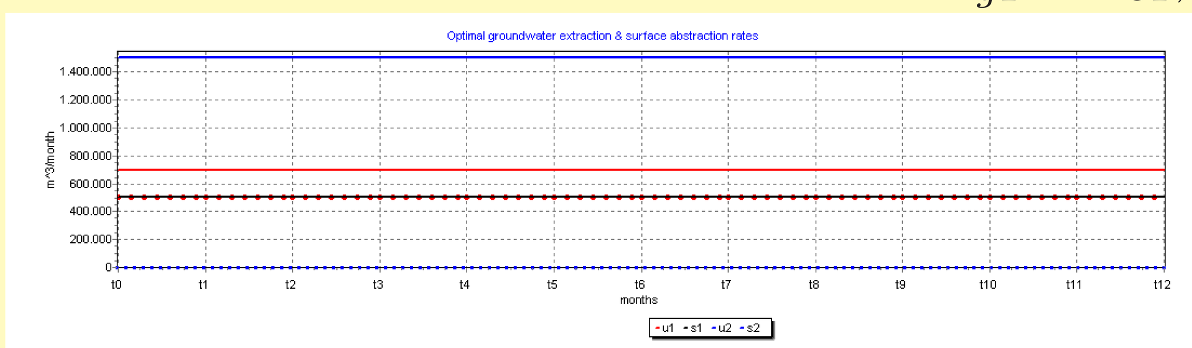


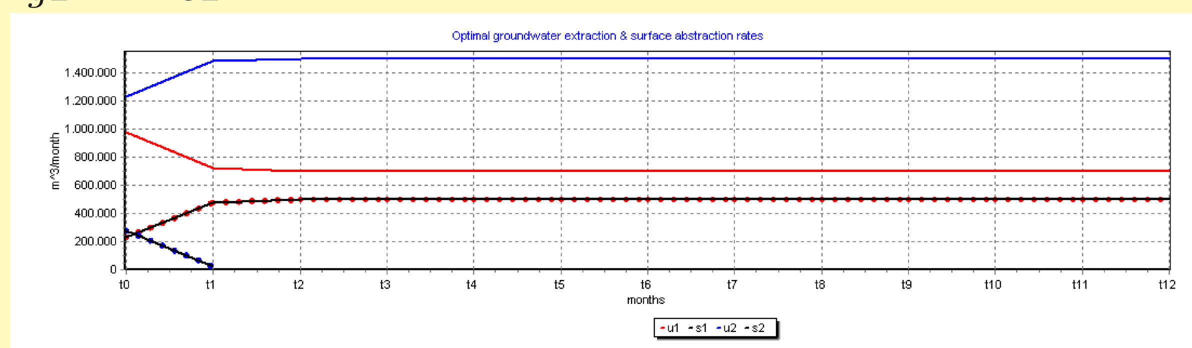
First mover advantage: $P_{g1} < P_{s1}, \quad P_{g2} << P_{s2}$





General case: $P_{g1} < P_{s1}, \quad P_{g2} > P_{s2}$





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