Controllability properties of dynamical systems with hysteresis

Marta Zoppello joint work with F. Bagagiolo

"Hysteresis and controllability of affine driftless systems" Submitted

Control of state constrained dynamical systems
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Motivations

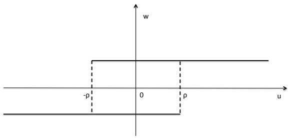
Many dynamical systems present delay phenomena:

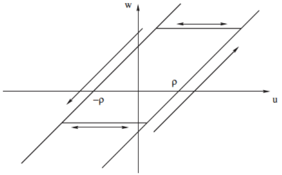
- Gear systems,
- Hydraulic controlled valves,
- Systems governed by a magnetic field for example magnetic micro-swimmers

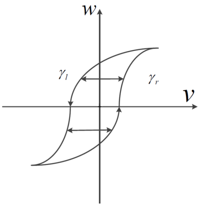
$\Rightarrow$ MEMORY EFFECT

Hysteresis

One way of representing mathematically this memory effect is the use the so called hysteresis operators

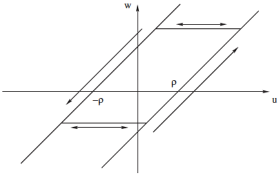






Properties of the Play operator

$$\mathcal{F} : C^0([0,T]) \times \mathcal{B} \to C^0([0,T])$$



a) Causality: $u|_{[0,t]} = v|_{[0,t]} \Rightarrow \mathcal{F}[u, w_0](t) = \mathcal{F}[v, w_0](t)$

b) Rate independence: $\mathcal{F}[u \circ \phi, w_0] = \mathcal{F}[u, w_0] \circ \phi$
$\forall \phi$ continuous non decreasing

c) Lipschitz continuity:
$||\mathcal{F}[u, w_0^1] - \mathcal{F}[v, w_0^2]||_{C^0([0,T])} \leq L(||u - v||_{C^0([0,T])} + ||w_0^1 - w_0^2||_B)$

d) semigroup property: $\mathcal{F}[u, w_0](t) = \mathcal{F}[u|_{[\tau,t]}, \mathcal{F}[u, w_0](\tau)](t - \tau)$

$$\dot{z} = \sum_{i=1}^{m} \mathbf{g}_i(z)u_i$$

Hysteresis Play operator

↙ ↘

On the controls

On the state

$$\dot{z} = \sum_{i=1}^{m} \mathbf{g}_i(z)\mathcal{F}[u_i, w_0]$$

$$\dot{z} = \sum_{i=1}^{m} \mathbf{g}_i(\mathcal{F}[z, w_0])u_i$$

Hysteresis applied on the controls

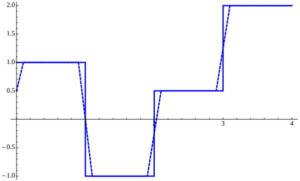
$$\dot{z} = \sum_{i=1}^{m} \mathbf{g}_i(z) u_i \qquad (1)$$

$$\dot{z} = \sum_{i=1}^{m} \mathbf{g}_i(z) \mathcal{F}[v_i, u_0] \qquad (2)$$

Theorem 1: Approximating sequence

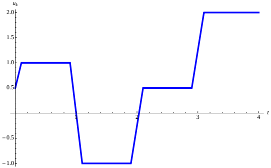
Let us suppose that the system (1) is controllable in time T and let $\bar{\mathbf{u}}$ be the piecewise constant control which steers the system between two fixed configurations in time T, then we are always able to find a sequence of continuous functions $\mathbf{v}_k = (v_{i_k})_{i=1}^{m}$, such that $\mathbf{u}_k = \mathcal{F}[\mathbf{v}_k, \bar{\mathbf{u}}_0]$ converges to $\bar{\mathbf{u}}$ in $L^1([0, T])$.

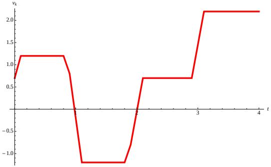
- Find a sequence $u_k \in C_0$



s.t.

$$\lim_{k\to\infty} u_i^k = \bar{u}_i \qquad \text{in } L^1([0,T])\ \forall i = 1 \cdots m$$





Lemma

The play operator is surjective on the set of the ***ziggurat functions***

Theorem 2:

The trajectory of the system (2) with the controls $\mathbf{v}_k$ defined in Theorem 1 converges to the trajectory of the non hysteretic system (1) with controls $\bar{\mathbf{u}}$.

Proof.

$$||\mathbf{z}_k - \mathbf{z}||_\infty \leq C_k + mMLt||\mathbf{z}_k - \mathbf{z}||_\infty$$

where $C_k = mt||\mathbf{g}_i||_\infty \int_0^t |\mathcal{F}[v_k^i(s)] - \bar{u}^i(s)|\, ds \to 0$ for the convergence of u_k^i to $\bar{u}^i$ in L^1. The last inequality for the Gronwal lemma implies that

$$||\mathbf{z}_k - \mathbf{z}||_\infty \leq C_k e^{mMLt} \to 0$$

□

Hysteresis acting on the state

$$\dot{z} = \sum_{i=1}^{m} \mathbf{g}_i(\mathcal{F}[z, w_0])u_i \qquad (3)$$

Questions

- In which cases we are able to obtain controllability results?
- Classical Lie algebra conditions are still applicable?
- Which kind of techniques are applicable?

An example

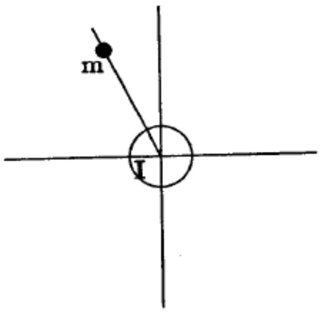


Figure: The system of the Heisenberg flywheel

Consider the following control system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 1 \\ f(x) \end{pmatrix} u_2 \tag{4}$$

and its hysteretic version

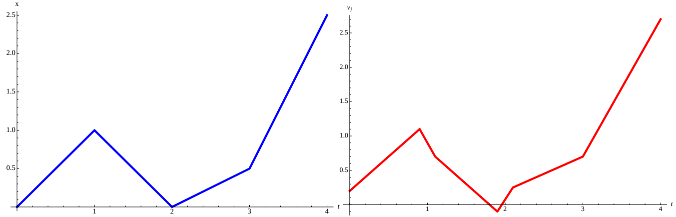
$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 1 \\ f(\mathcal{F}[x, w_0]) \end{pmatrix} u_2 \tag{5}$$

Idea

1. Approximate the linear trajectory generated by u_1

Theorem 3:

The play operator has dense image in the space of piecewise linear continuous functions



2. Use it to reach the final y_B and z_B
3. Adjust the last coordinate

Proposition

Given $\bar{x}(t)$ the piecewise linear continuous trajectory of the non hysteretic system we are able to find a sequence $v^j(t)$ such that $\mathcal{F}[v^j, w_0](t) \to \bar{x}(t)$ in L^∞ as $j \to \infty$.

Theorem

For any initial and final configurations A and B and for any suitable w_0, there always exists a sequence of piecewise constant controls (u_1^j, u_2^j) and a final time T^* such that the solution $(x^j(t), y^j(t), z^j(t))$ of system (5) starting from A is such that

$$x^j(T^*) = x_B \quad y^j(T^*) = y_B \quad z^j(T^*) \to z_B \quad \text{as } j \to \infty$$

Idea for generalization

The same result can be reached iterating the procedure if one has the following kind of system
$\mathbf{z} = (x_1 \cdots x_m, y_{m+1}, \cdots y_{2m-1})$

$$
\begin{aligned}
&\mathbf{g}_1(\mathbf{z}) = \partial_{x_1} \\
&\mathbf{g}_2(\mathbf{z}) = \partial_{x_2} + f_2(\mathcal{F}[x_1, w_0])\partial_{y_{m+1}} \\
&\vdots \\
&\mathbf{g}_m(\mathbf{z}) = \partial_{x_m} + f_m(\mathcal{F}[x_1, w_0], \cdots, \mathcal{F}[x_{m-1}, w_0])\partial y_{2m-1}
\end{aligned}
\tag{6}
$$

Sketch of the proof

- Adjust the last coordinate y_{2m-1} using an appropriate sequence of controls
- Put $u_m \equiv 0$ and use another sequence of controls $u_1^j, \cdots, u_{m-1}^j$ to adjust the coordinate y_{2m-2}.
- Repeat the procedure

Perspectives and open problems

- Are the last kind of systems wide enough? Are they all Carnot groups?
- Is it possible to have some “Chow-like” theorem?
- Use other hysteresis operators
- this techniques are applicable to the magnetic micro-swimmer system?

THANK YOU FOR YOUR ATTENTION