Possibility Theory for Reasoning About Uncertain Soft Constraints

Maria Silvia Pini, Francesca Rossi, and Brent Venable

Department of Pure and Applied Mathematics, University of Padova, Italy
{mpini, frossi, kvenable}@math.unipd.it

Abstract. Preferences and uncertainty occur in many real-life problems. The theory of possibility is one non-probabilistic way of dealing with uncertainty, which allows for easy integration with fuzzy preferences. In this paper we consider an existing technique to perform such an integration and, while following the same basic idea, we propose various alternative semantics which allow us to observe both the preference level and the robustness w.r.t. uncertainty of the complete instantiations. We then extend this technique to other classes of soft constraints, proving that certain desirable properties still hold.

1 Introduction

Preferences and uncertainty occur in many real-life problems. In this paper we are concerned with the coexistence of such concepts in the same problem. In particular, we consider uncertainty that comes from lack of data or imprecise knowledge and scenarios where probabilistic estimates are not available.

The theory of possibility [9, 14] is one non-probabilistic way of dealing with uncertainty, which allows for easy integration with fuzzy preferences [6]. In fact, both possibilities and fuzzy preferences are values between 0 and 1 associated to events and express the level of plausibility that the event will occur, or its preference.

In our context, we will describe a real-life problem as set of variables with finite domains and a set of soft constraints among subsets of the variables. A variable will be said to be uncertain if we cannot decide its value. In this case, we will associate a possibility degree to each value in its domain, which will tell how plausible it is that the variable will get that value.

Soft constraints allow to express preferences over the instantiations of the variables of the constraints. In particular, fuzzy preferences are values between 0 and 1, which are combined using the *min* operator, and are ordered in such a way that higher values denote better preferences.

In this paper we consider an existing technique to integrate fuzzy preferences and uncertainty, which uses possibility theory [6]. This technique allows one to handle uncertainty within a fuzzy optimization engine. However, we claim that the integration provided by this technique is too tight since the resulting ordering over complete assignments does not allow one to discriminate be-

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tween solutions which are highly preferred but assume unlikely events and solutions which are not preferred but robust with respect to uncertainty. This is due to the fact that a single value, which summarizes the contributions of both the uncertain variables and the fuzzy preferences, is associated to each solution.

While following the same basic idea of translating uncertainty into fuzzy constraints, we propose various alternative semantics which allow us to observe separately the preference level and the robustness of the complete instantiations. More precisely, each solution will be associated to a pair of values between 0 and 1: one value will refer to the preference level, while the other one will refer to the robustness of the solution w.r.t. the uncertain variables. In this way, given a solution and the pair of values associated to it, we can see how preferred it is according to the constraints, by looking at the first value of its pair, and also how robust it is, by looking at the second value of its pair.

The desired ordering over such pairs will then be used to order the solutions. Thus, by choosing different orderings, we can reason in a more or less pessimistic way, giving more or less importance to the preferences w.r.t. the robustness of the problem. In this way, we define a class of different semantics.

2 Soft Constraints

Soft constraints [2] are a very general formalism to describe quantitative preferences. In general, a soft constraint is just pair $\langle def, con\rangle$, where *con* is the set of variables of the constraint (that is, its scope), and def is a function from the Cartesian product of the domains of the variables in *con* to a preference set, say A. Therefore def defines the constraint, by associating a level of preference from A to each assignment of values to the variables of the constraint.

Set A can be totally or partially ordered, and its ordering, denoted by $\leq$, can be used to order the assignments of values to variables: assignments corresponding to higher preferences are more preferred. Moreover, a combination operation $\times$ should be defined over A, to combine different constraints and generate the preference level of an assignment of values to variables which range over the scopes of several constraints. More precisely, A should have properties similar to a semiring. We will therefore say that a soft constraints is defined *over semiring* A. For more details on semiring-based soft constraints, see [2].

A soft constraint problem is usually denoted by a tuple $\langle S, V, C\rangle$ where S is a semiring, V is a set of variables, and C is a set of soft constraints over S whose scopes are subsets of these variables. An optimal solution of a soft constraint problem is an assignment of its variables which is optimal according to the ordering associated to the semiring.

This general description of soft constraints instantiates to several classes of concrete constraints:

- *Fuzzy constraints*: when $A = [0, 1]$, $\leq$ is derived by the *max* operator, and the combination operator is *min*. This means that a fuzzy constraint associates an element between 0 and 1 to each instantiation of its variables, that values closer to 1 denote a higher preference, and that the preferences of two or more constraints are combined by taking their minimum value.

- *Hard constraints*: they can also be described by this framework, by just choosing $A = \{true, false\}$, $\leq$ derived by logical *or* (thus 1 is better than 0), and combination is logical *and*.
- *Weighted constraints*: they are soft constraints where each assignment of values to variables has a weight, and the goal is to minimize the sum of the weights: this can be cast by choosing A as the set of possible weights, by deriving the ordering by the *min* operator, and by using the *sum* as the combination operator.

The concept of fuzzy constraint, as defined above, was originally based on the notion of fuzzy set [14, 8, 11]. A fuzzy set A is a subset of a referential set U whose boundaries are gradual. More formally: the *membership function* μ_A of a fuzzy set A assigns to each element $u \in U$ its degree of membership $\mu_A(u)$ usually taking values in $[0, 1]$. If $\mu_A(u) = 1$, it means that u belongs to A, while $\mu_A(u) = 0$ means that u does not belong to A. If $\mu_A(u)$ is between 0 and 1, then it means that $u \in A$ with degree $\mu_A(u)$.

The *complement* of a fuzzy set A in U is denoted A^C and its membership function is $\mu_{A^C} = 1 - \mu_A$. The *union* and *intersection* of fuzzy sets are obtained by respectively taking the maximum and the minimum of membership degrees of each element of U in each of the fuzzy sets.

Fuzzy constraints use the notion of fuzzy sets to describe the level of preference of a certain assignment of values to variables. More precisely, a *soft fuzzy constraint* [6] C on variables $\{x_1, \dots, x_n\}$ is associated with a *fuzzy* relation R, i.e. a fuzzy subset of $D_1 \times \cdots \times D_n$ of values that more or less satisfy C. A membership function μ_R is associated with relation R and specifies for each tuple $(d_1, \dots, d_n) \in D_1 \times \cdots \times D_n$ the level of satisfaction $\mu_R(d_1, \dots, d_n)$ in a set L, which is totally ordered (e.g. [0,1]). In particular, $\mu_R(d_1, \dots, d_n) = 1$ if tuple $(d_1, \dots, d_n)$ totally satisfies C, $\mu_R(d_1, \dots, d_n) = 0$ if it totally violates C, and $0 < \mu_R(d_1, \dots, d_n) < 1$ if it partially satisfies C. Moreover, $\mu_R(d_1, \dots, d_n) > \mu_R(d'_1, \dots, d'_n)$ means that tuple $(d_1, \dots, d_n)$ is better than tuple $(d'_1, \dots, d'_n)$.

In the following we will use two operations on fuzzy constraints [6]: projection and combination. The *projection* of a fuzzy constraint, represented by fuzzy relation R on variables $\{x_1, \dots, x_k\} \subseteq V(R) = \{x_1, \dots, x_n\}$, is a fuzzy relation $R^{\downarrow\{x_1,\dots,x_k\}}$ defined on $\{x_1, \dots, x_k\}$ such that: $\mu_{R^{\downarrow\{x_1,\dots,x_k\}}}(d_1, \dots, d_k) = sup_{\{d=(d_1,\dots,d_n) | d^{\downarrow\{x_1,\dots,x_k\}}=(d_1,\dots,d_k)\}}\ \mu_R(d)$. The *conjunctive combination* of two fuzzy constraints, represented by fuzzy relations R_i and R_j, is a fuzzy relation $R_i \otimes R_j$ defined on variables $V(R_i) \cup V(R_j)$ such that: $\mu_{R_i \otimes R_j}(d_1, \dots, d_k) = min(\mu_{R_i}(d_1, \dots, d_k)^{\downarrow V(R_i)}, \mu_{R_j}(d_1, \dots, d_k)^{\downarrow V(R_j)})$ where $\mu_{R_i \otimes R_j}(d_1, \dots, d_k)$ evaluates to what extent $(d_1, \dots, d_k)$ satisfies both C_i and C_j.

3 Possibility Theory

A *possibility distribution* [14] is the membership function of a fuzzy set A attached to a single-valued variable x. It is denoted $\pi_x = \mu_A$ and represents the set of more or less plausible, mutually exclusive values of x. A possibility distribution is similar to a probability density. However, $\pi_x(u) = 1$ only means that $x = u$ is a plausible situation, which cannot be excluded. Thus, a degree of possibility can be viewed as an upper bound of a degree of probability.

Possibility theory encodes incomplete knowledge while probability accounts for random and observed phenomena. In particular, the complete ignorance about x is expressed by $\pi_x(u) = 1$, for all $u \in U$, because in this case all values u are plausible for x and so it is impossible to exclude any of them. Whereas, $\pi_x(\bar{u}) = 1$ for a specific value $\bar{u}$ and $\pi_x(u) = 0$ otherwise, expresses the complete knowledge about x, because in this case only the value $\bar{u}$ is plausible for x.

The *possibility* of an event "$x \in E$", denoted by $\Pi(x \in E)$, is $\Pi(x \in E) = sup_u min(\pi_x(u), \mu_E(u)) = sup_{u\in E}\pi_x(u)$.

If an event has possibility equal to 1, it means that it is totally possible. However, it could also not happen. Therefore it means that we are completely ignorant about its occurrence. On the contrary, having a possibility equal to 0 means that the event for sure will not happen.

The dual measure of *necessity* of "$x \in E$", denoted by $N(x \in E)$, evaluates the extent to which "$x \in E$" is *certainly* true, $N(x \in E) = inf_u max(c(\pi_x(u)), \mu_E(u)) = inf_{u\notin E}(c(\pi_x(u))) = 1 - \Pi(x \in E^C)$. where c is the order reversing map such that $c(p) = 1 - p$ and E^C is the complement of E in U.

$N(x \in E) = 1$ when it is certain that $x \in E$. On the contrary, having necessity equal to 0 means that the event is not necessary at all, although it may happen. In fact, $N(x \in E) = 0$ iff $P(x \in E^C) = 1$.

For example, if we have a possibility distribution π, attached to a variable x with domain $D_x = \{5, 6, 7, 8\}$, such that $\pi(5) = 0.9$, $\pi(6) = 0.4$, $\pi(7) = 0.7$, $\pi(8) = 0.5$, then, if $A = \{5, 6\}$ is a subset of D_x, the possibility degree of the event $x \in A$ is $\Pi(A) = sup_{d\in A}\pi(d) = sup\{0.9, 0.4\} = 0.9$, whereas the necessity degree of the same event, $x \in A$, is $N(A) = inf_{d\notin A}c(\pi(d)) = inf\{c(\pi(7)), c(\pi(8))\} = inf\{c(0.7), c(0.5)\} = inf\{0.3, 0.5\} = 0.3$. Computing $N(A)$ using the formula $N(A) = 1 - \Pi(\bar{A})$ is the same, in fact, $N(A) = 1 - \Pi(\bar{A}) = 1 - sup_{d\in\bar{A}}\pi(d) = 1 - sup\{0.7, 0.5\} = 1 - 0.7 = 0.3$.

4 Uncertainty in Soft Constraints

Whereas in usual soft constraint problems all the variables are assumed to be controllable, that is, their value can be decided according to the constraints which relate them to other variables, in many real-world problems uncertain parameters must be used. Such parameters are associated with variables which are not under the user's direct control and thus cannot be assigned. Only Nature will assign them.

Formally, we can define an uncertain soft constraint problem as a tuple $\langle S, V_c, V_u, C\rangle$, where S is a semiring, V_c is the set of controllable variables, V_u is the set of uncontrollable variables, and C is the set of soft constraints. The soft constraints in C may involve any subset of variables of $V_c \cup V_u$.

While in a classical soft constraint problem we can decide how to assign the variables to make the assignment optimal, in the presence of uncertain parameters we must assign values to the controllable variables guessing what Nature will do with the uncontrollable variables. So, in this paper an optimal solution for an uncertain soft constraint problem is an assignment of values to the variables in V_c such that, *whatever* Nature will decide for the variables in V_u, the overall assignment will be optimal. This is a pessimistic view and other definitions of solutions can be considered [1].

For example, we could be satisfied with finding an assignment of values to the variables in V_c such that, *for at least* one assignment decided by Nature for the variables in V_u, the overall assignment will be optimal. This definition follows an optimistic view. Other definitions can be between these two extremes.

Moreover, the uncontrollable variables can be equipped with additional information on the likelyhood of their values. Such information can be given in several ways, depending on the amount and precision of knowledge we have. In this paper for expressing such information we will consider possibility distribution. This information can be used to infer new soft constraints over the controllable variables, expressing the compatibility of the controllable parts of the problem with the uncertain parameters, and can be used to change the notion of optimal solution.

5 Unifying Fuzzy Preferences and Uncertainty via Possibility Theory

Possibility theory [14] can be used to code some information about the uncertain variables in an uncertain soft constraint problem. In this section we will describe an existing approach for uncertain fuzzy soft constraints and later we will show how to modify it and extend it also to other classes of soft constraints.

In [6] it is shown how it is possible to replace a fuzzy constraint involving at least one uncontrollable variable with a fuzzy constraint over controllable variables only. Consider a fuzzy constraint C, represented by the fuzzy relation R, which relates a set of controllable variables $X = \{x_1, \ldots, x_n\}$ to a set of uncertain parameters $Z = \{z_1, \ldots, z_k\}$ with domains $A_1, \ldots, A_k$. The knowledge of the uncertain parameters is modeled with the possibility distribution π_Z defined on $A_Z = A_1 \times \cdots \times A_k$. The constraint C is considered satisfied by the assignment $d = (d_1, \ldots, d_n) \in D_1 \times \cdots \times D_n$ if, *whatever the values of* $z = (z_1, \ldots, z_k)$*,* d *is compatible with* z, i.e., the set of possible values for z is included in $T = (R \otimes \{(d_1, \ldots, d_n)\})^{\downarrow Z}$. Obviously $\mu_T(a) = \mu_R(a, d)$ and $\mu'(d) = \mu'_R(d) = N(d\, satisfies\, C) = N(z \in T) = inf_{a \in A_Z} max(\mu_T(a), c(\pi_Z(a)))$ $= c(sup_{a \in A_Z} min(c(\mu_T(a)), \pi_Z(a)))$. If C is a hard constraint, then the formula above still applies, and becomes the following one: $N(d\, satisfies\, C) = inf_{a \notin T = (R \cap \{d\})^{\downarrow D_Z}} c(\pi_Z(a))$.

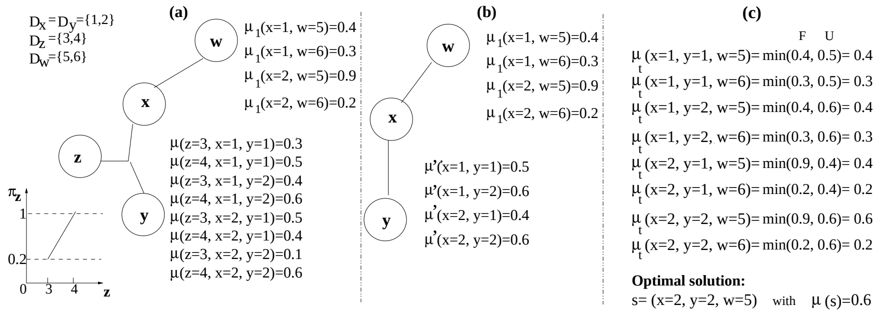

Fig. 1. An example of application of algorithm DFP

Notice that, when C is a soft constraint, μ' is computed by applying the *min* operator between preferences and possibilities. This can be done since their scales are equal assuming the *commensurability* between preferences and possibilities.

Summarizing, the method proposed in [6], which we call **Algorithm DFP** (by the name of the authors), for managing uncertainty in a fuzzy CSP, is the following:

1. It starts from an uncertain fuzzy CSP, say P.
2. P is reduced to a fuzzy constraint problem P': all the constraints which link uncertain parameters to decision variables are replaced by fuzzy constraints only among the decision variables. The new preference levels of the decision variables in such new constraints are computed by applying the specific procedure given above in this section.
3. P' has only fuzzy constraints, therefore it can be solved by applying the usual method for solving fuzzy CSPs, i.e. using the *min* operator to combine the constraints and choosing the complete assignments with the highest preference.

An application of algorithm DFP to an uncertain fuzzy CSP is shown in Figure 1. Part (a) shows a fuzzy CSP with uncertainty. There are three decision variables (X, Y, W), one uncertain variable (Z), and two constraints: one, C_{XYZ}, among X, Y and Z with function μ and another one, C_{XW}, between X and W with function μ_1. The constraint C_{XYZ} has membership function μ. The possibility distribution π_Z describes the plausibility of Z. Part (b) shows the fuzzy constraint problem on variables X and Y obtained by the one in part (a). Part (c) shows the complete assignments of the problem in part (b) with their preference degrees defined by μ_t.

In [6] the following property is given.

Property 1: $\mu'_R(d) \geq \alpha$ means that if it is taken for granted that the actual value of z has plausibility strictly greater than $c(\alpha)$, then it is sure that the decision d satisfies C at least at level α (i.e., $\mu'_R(d) \geq \alpha$ means that if $\pi_Z(a) > c(\alpha) \Rightarrow \mu_R(d, a) \geq \alpha$, where a is the actual value of z).

6 Derived Properties

Property 1 continues to hold also if the uncertain parameters are provided with a *probability distribution*, $prob_Z$, and not a possibility one. We have proved this recalling that a possibility is an upper bound to a probability. In this case, Property 1 becomes: $\mu'_R(d) \geq \alpha$ means that if $prob_Z(a) > c(\alpha) \Rightarrow \mu_R(d,a) \geq \alpha$, where a is the actual value of z.

Moreover we have proved the following two properties:

Property 1.1: Once the possibilities of uncertain parameters, defined by π_Z are fixed, if we consider an assignment d to the decision variables $X_1, \ldots, X_k$ and an assignment d' such that, $\mu(d,a) \leq \mu(d',a)$ $\forall a$, then the new preference incorporating also uncertainty μ' is such that $\mu'(d) \leq \mu'(d')$.

Property 1.2: Once the preferences $\mu(d,a)$ are fixed, where d is an assignment to the decision variables $X_1, \ldots, X_k$ and a is the value of uncertain parameters, if π_1 and π_2 are two possibility distributions on uncertain parameters such that $\pi_1(a) \geq \pi_2(a)$ $\forall a$, then the new preferences incorporating also uncertainty are such that $\mu'_1(d) \leq \mu'_2(d)$, where μ'_1 is the preference obtained considering possibility distribution π_1 and μ'_2 considering possibility distribution π_2.

7 Separation and Projection

By using algorithm DFP, the preference of a complete assignment is the minimum value among all the preferences of the constraints, both the original fuzzy constraints and those obtained via the transformation which eliminates the uncontrollable variables. By looking again at Figure 1, we can see that the overall preference is $min(F,U)$, where F is the minimum of the preferences in the initially given fuzzy constraints only on decision variables, and U is the minimum of the preferences of the new fuzzy constraints. This means, for example, that a low overall preference may be caused from a low preference in some of the new fuzzy constraints (when U is less than F), that is, a low compatibility with the uncertain events, or also from a low preference on some fuzzy constraint initially given only on decision variables (when F is less than U). In oder words, some information is lost by passing from F and U to $min(F,U)$.

In other words, according to algorithm DFP, an assignment d associated with the pair of preferences $\langle F,U \rangle$ is compared with another one d' associated with the pair $\langle F',U' \rangle$ by just comparing $min(F,U)$ and $min(F',U')$: d is better than d' iff $min(F,U) > min(F',U')$. Consider the following situations:

- $F = F'$ and $U > U'$: in this case, one would like to say that d is better than d'. However, this is the case only if $F > U'$, but not if $U' > F$, in which case d and d' are equally preferred. The same reasoning holds also in the dual case when $U = U'$.
- $F = U'$ and $U = F'$: in this case, d and d' are equally preferred, independently of the ordering of the two values. This means that the same

importance is given to both components of the pair, and there is no way to distinguish or consider differently the preference derived from the initial fuzzy constraints among decision variables (that is, F) and the measure of certainty of the problem (that is, U) w.r.t. uncertain parameters.

This problem is caused by the fact that, by using the *min* operator, one forgets about all the other elements, which are higher than the minimum. This is usually called the "drowning effect" [10].

To avoid this problem, it is important to keep separate these two components (F and U), rather than applying the *min* operator over them. Other approaches have gone in this direction. For example, in [1] there are two components which are indeed computed separately; however, solutions are then ordered only via the minimum or the maximum between the two values.

However, just keeping the two components separate will not give the desired ordering among the solutions. In fact, by replacing a fuzzy constraint c between decision variables X and uncertain parameters Y with a new constraint c' over X, and by computing the pair of preferences $\langle F, U \rangle$, it may happen that F is greater than all the preferences appearing in the constraint c. Thus the overall preference for a solution may be high even if this solution has a very low compatibility with all the values of the uncertain variables.

This can be solved by performing, for each constraint c involving both decision and uncertain variables (X and Z), a projection over the decision variables. This will create a new constraint c'' over X where, for each assignment of values to its variables, the preference is computed by assuming the best in the uncertain parameters. Since preferences are combined via the *min* operator, this new constraint will force the overall preference to be no higher than its best preference. Given an assignment to decision variables, we denote with P the minimum preference over these new *projection constraints*. Such value P, combined with F given by the initial constraints, defines the new preference value F_P.

We have proved that $min(F, U) \leq P$, which implies that projections would be redundant in algorithm DFP, since it computes as final preference $min(F, U)$.

We therefore propose the following algorithm, which we will call **algorithm SP** (from *separation* and *projection*), to handle uncertain fuzzy constraint problems:

1. It starts from an uncertain fuzzy CSP with fuzzy constraints C.
2. All the constraints which link uncertain parameters to decision variables are replaced by fuzzy constraints only among the decision variables. Let us call C_u such new constraints.
3. It computes the projection constraints, say C_p.
4. For each complete assignment, it computes its overall preference as a pair $\langle F_P, U \rangle$, where $F_P = min(F, P)$ and F, P, and U are, respectively,the minimum preference over the constraints in C, C_p, and C_u.

Algorithm SP differs from algorithm DFP for points 3 and 4.

Let us consider the following example, where we have a complete assignment d with preference given by the pair $\langle F = 0.3, U = 0.9 \rangle$ and another one d' with

$\langle F = 0.9, U = 0.3 \rangle$. According to algorithm DFP, d is considered equal to d' since d and d' have the same preference $min(F, U) = 0.3$. We will show that our method distinguishes among them. Since d is associated with the pair $\langle 0.3, 0.9 \rangle$ then the best preference that could be obtained is $F = 0.3$. Moreover there is at least a value $\bar{a}$ for the uncertain parameters such that the tuple $(d, \bar{a})$ has preference 0.3, since in this case $P \geq min(F, U) = 0.3$ implies $F_P = F$. In addition certainty degree $U = 0.9$ implies if $\pi_Z(a) > 0.1$ then $\mu(d, a) \geq 0.9$, where a is the actual value of z. Thus the preference of (d, a) is < 0.3 only if $\pi_Z(a) \leq 0.1$. This means we have a high certainty to obtain as final preference $F_P = 0.3$.

On the contrary, the fact $F = 0.9$ for d' doesn't imply that such preference is obtained by any pair (a, d'). In fact, assume that P is 0.4. Then for any pair (a, d') the preference is ≤ 0.4. This is an example of how the information in F can be misleading when the components of the preferences are kept separate and thus needs to be corrected combining it with P. Moreover $U = 0.3$ means that if $\pi_Z(a) > 0.7$ then $\mu(d, a) \geq 0.3$, where a is the actual value of z. Hence we can have a high possibility ≤ 0.7 to have a preference strictly less than 0.3, i.e., d' may have preference 0.9 if $P \geq 0.9$, but there is a high possibility (≤ 0.7) to have a preference < 0.3.

As it can be seen for the reasoning above d and d' differ on both preference and robustness. This is why we believe it is reasonable to define distinct semantics ordering d and d' differently. In particular, if for example we assume that $P = 0.9$ for d':

- We should prefer d over d' if we want be $safe$, because assignment d is more cautious than d', showing a low preference similar to the one that can really happen.
- On the other hand, we should prefer d' over d if we want be *risky*, because assignment d' is more risky than d, showing the best preference that we can obtain, if we are very lucky, i.e., with a low possibility.
- We should not choose between d' and d if we want be *diplomatic*, i.e., we want to have a high preference with a high certainty.

In Figure 2 (a) there is the new fuzzy CSP obtained from the one in Figure 1 (a) after applying the *projection* of the ternary constraint C_{XYZ} on the decision variables X and Y, and the usual step 2 of the algorithm. In Figure 2 (b) all the complete assignments to decision variables of the FCSP. Each assignment, d, is associated with a tuple of three preference values: the first one (P) is the preference obtained by the projection constraints, the second one (F) is the preference given by the initial fuzzy constraint C_{XW}, and the third one is the value obtained by the uncertain parameter Z, which represents the *certainty* that d satisfies C_{XYZ}.

Given an assignment d to the decision variables and the pair $\langle F_P, U \rangle$ computed as described above, F_P tells us how much d is preferred by the constraints, while U represents to what extent it is impossible to have a possible value of the uncertain parameters violating the constraints involving uncertain parameters. This means that $1 - U$ gives an idea of the risk of hitting a value of uncertain

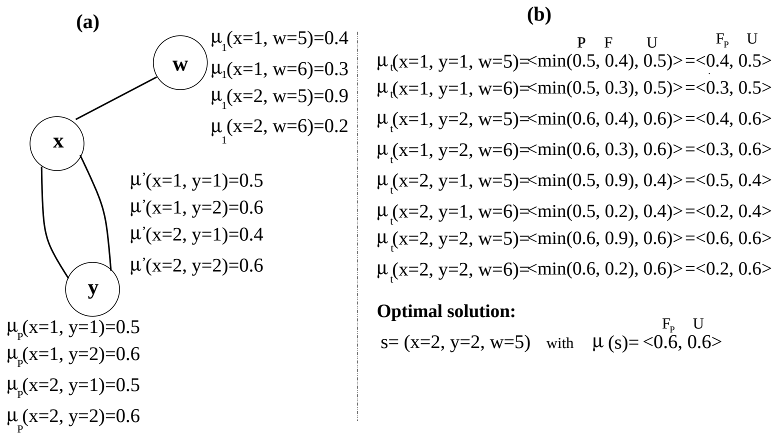

Fig. 2. Result of algorithm SP for the uncertain fuzzy CSP in Figure 1 (a)

parameter that is inconsistent with d, hence U can be seen as a measure of the *certainty* of d.

U is computed as in [6], so Properties 1, 1.1 and 1.2 still hold. We recall that these properties state that U can increase only in two cases: either when the possibilities of the uncertain parameters remain fixed and the preferences of the constraints involving them increase, or when preferences are fixed but possibilities decrease.

8 Three New Semantics

Consider two solutions d and d' and the corresponding pairs of values $\langle F_P(d), U(d)\rangle = \langle a_1, b_1\rangle$ and $\langle F_P(d'), U(d')\rangle = \langle a_2, b_2\rangle$.

The first semantics we propose, that we will call *Risky*, can be seen as a Lex ordering on pairs $\langle a_i, b_i\rangle$, with the first component as the most important feature. Hence

- if $a_1 > a_2$ then $\langle a_1, b_1\rangle >_R \langle a_2, b_2\rangle$ (and the opposite for $a_2 > a_1$);
- if $a_1 = a_2$ then
  - if $b_1 > b_2$ then $\langle a_1, b_1\rangle >_R \langle a_2, b_2\rangle$ (and the opposite for $b_2 > b_1$);
  - if $b_1 = b_2$ then $\langle a_1, b_1\rangle = \langle a_2, b_2\rangle$.

Informally, the idea is to give more importance to the preference level that can be reached in the best case (a higher a_i) considering less important a high risk of being inconsistent (a low certainty b_i).

The second semantics, called *Safe*, follows the opposite attitude with the respect to the previous one: it can be seen as a Lex ordering on pairs $\langle a_i, b_i\rangle$, with the second component as most important feature. Hence:

Table 1. Dubois et al. semantics compared to Risky, Safe and Diplomatic

Dubois et al.	Risky	Safe	Dipl.
$=$	$<,>,=$	$<,>,=$	$<,>,=,\bowtie$
$>$	$<,>$	$<,>$	$>,\bowtie$
$<$	$<,>$	$<,>$	$<,\bowtie$

- if $b_1 > b_2$ then $\langle a_1, b_1 \rangle >_S \langle a_2, b_2 \rangle$ (and the opposite for $b_2 > b_1$);
- if $b_1 = b_2$ then
  - if $a_1 > a_2$ then $\langle a_1, b_1 \rangle >_S \langle a_2, b_2 \rangle$ (and the opposite for $a_2 > a_1$);
  - if $a_1 = a_2$ then $\langle a_1, b_1 \rangle = \langle a_2, b_2 \rangle$.

Informally, the idea is to give more importance to the certainty level that can be reached (a higher b_i) considering less important having a high preference (a high a_i).

Our third semantics, called *Diplomatic*, aims at giving the same importance to the two aspects of a solution: preference and certainty. In order to do that, it is obtained via the Pareto ordering on pairs $\langle a_i, b_i \rangle$, where both components have the same importance. Hence:

- if $a_1 \leq a_2$ and $b_1 \leq b_2$ then $\langle a_1, b_1 \rangle \leq_D \langle a_2, b_2 \rangle$ (and the opposite for $a_2 \leq a_1$ and $b_2 \leq b_1$);
- if $a_1 = a_2$ and $b_1 = b_2$ then $\langle a_1, b_1 \rangle = \langle a_2, b_2 \rangle$;
- else $\langle a_1, b_1 \rangle \bowtie \langle a_2, b_2 \rangle$.

In this definition $\bowtie$ stands for incomparability. The idea is that a pair is to be preferred to another only if it wins both on preference and certainty, leaving incomparable all the pairs that have one component higher and the other lower. Contrarily to the diplomatic semantics, the other two semantics produce a total order over the solutions.

Figure 2 (b) shows a solution of the FCSP in Figure 1 which is optimal according to all semantics.

Let us now consider an example that explains the differences between our semantics and the approach of [6]. Suppose we have two complete assignments, d_1 and d_2, with preference resp. 0.3 and 0.5, and certainty resp. 0.5 and 0.3. Then the method of [6] would say that they are equally good, since it would just consider the minimum of each pair, that is, 0.3. On the other hand, for our semantics we have the following ordering: $\langle 0.3, 0.5 \rangle <_R \langle 0.5, 0.3 \rangle$ according to Risky; $\langle 0.3, 0.5 \rangle >_S \langle 0.5, 0.3 \rangle$ according to Safe; $\langle 0.3, 0.5 \rangle \bowtie \langle 0.5, 0.3 \rangle$ according to Diplomatic.

In general, the comparison among the orders induced by our three semantics and the one of the method in [6] can be seen in Table 1.

9 Conclusion and Future Work

We defined a new way to deal with preference and uncertainty which assumes commensurability but does not mix preferences and compatibility with uncertain events. This allows us to obtain a solution ordering which better reflects the desirability and the robustness of a solution. Other approaches which do not mix these two aspects [12, 5, 3, 4] do not assume commensurability and thus cannot compare directly preferred assignments and uncertain events.

We plan to develop a solver that can handle problems with several classes of soft constraints, together with uncertainty expressed via possibility or probability distributions. The solver will be able to generate orderings according the three semantics proposed in this paper as well as others that we will define by following different optimistic or pessimistic approaches.

We plan also to extend the results of this paper to other classes of soft constraints (such as probabilistic and weighted) and also to probabilistic uncertainty.

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