

Uncertainty in soft constraint problems

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Abstract

Preferences and uncertainty occur in many real-life problems. The theory of possibility is one way of dealing with uncertainty, which allows for easy integration with fuzzy preferences. In this paper we consider an existing technique to perform such an integration and, while following the same basic idea, we propose to generalize it to other classes of preferences and to probabilistic uncertainty while maintaining certain desirable properties.

1 Introduction

Preferences and uncertainty occur in many real-life problems. In this paper we are concerned with the coexistence of such concepts in the same problem. In particular, we consider uncertainty that comes from imprecise knowledge.

The theory of possibility [4, 9] is one non-probabilistic way of dealing with uncertainty, which allows for easy integration with fuzzy preferences [2]. In fact, both possibilities and fuzzy preferences are values between 0 and 1 associated to events and express the level of plausibility that the event will occur, or its preference.

In our context, we will describe a real-life problem as set of variables with finite domains and a set of soft constraints among subsets of the variables. A variable will be said to be uncertain if we cannot decide its value. In this case, we will associate a possibility degree to each value in its domain, telling how plausible it is that the variable will get that value.

Soft constraints [1] allow to express preferences over the instantiations of the variables of the constraints. In particular, fuzzy preferences are values between 0 and 1, which are combined using the *min* operator, and are ordered in such a way that higher values denote better preferences. Probabilistic preferences represent probabilities, hence they are values between 0 and 1 and they are combined via multiplication: the goal is to maximize the product of the probabilities. Weighted constraints use instead preferences rep-

resenting costs which are combined by summing them: the goal is to minimize the sum of the weights.

In this paper we consider existing techniques to integrate fuzzy preferences and uncertainty, which use possibility theory [2, 8]. The approach in [8] allows one to handle uncertainty within a fuzzy optimization engine, and at the same time to observe separately the preference level and the robustness of the complete instantiations. This approach has certain desirable properties, that we describe formally.

We then generalize the approach, and its properties, in order to use it also for other classes of soft constraints, not necessarily fuzzy, and to probabilistic uncertainty, by pointing out a sufficient condition for the properties to hold. This allows us to handle the coexistence of preferences and uncertainty in a more general setting.

2 Soft constraints

Soft constraints [1] are a very general formalism to describe quantitative preferences. In general, a soft constraint is just pair $\langle def, con \rangle$, where *con* is the set of variables of the constraint (that is, its scope), and *def* is a function from the Cartesian product of the domains of the variables in *con* to a preference set, say *A*. Therefore *def* defines the constraint, by associating a level of preference from *A* to each assignment of values to the variables of the constraint.

Set *A* can be totally or partially ordered, and its ordering, denoted by \leq , can be used to order the assignments of values to variables: assignments corresponding to higher preferences are more preferred. Moreover, a combination operation \times should be defined over *A*, to combine different constraints and generate the preference level of an assignment of values to variables which range over the scopes of several constraints. More precisely, *A* should have properties similar to a semiring. We will therefore say that a soft constraint is defined over semiring *A*. For more details on semiring-based soft constraints, see [1].

A soft constraint problem is usually denoted by a tuple $\langle S, V, C \rangle$ where *S* is a semiring, *V* is a set of variables, and *C* is a set of soft constraints over *S* whose scopes are subsets

of these variables. An optimal solution of a soft constraint problem is an assignment of its variables which is optimal according to the ordering associated to the semiring.

This general description of soft constraints instantiates to several classes of concrete constraints:

- *Fuzzy constraints*: when $A = [0, 1]$, \leq is derived by the *max* operator, and the combination operator is *min*. This means that a fuzzy constraint associates an element between 0 and 1 to each instantiation of its variables, that values closer to 1 denote a higher preference, and that the preferences of two or more constraints are combined by taking their minimum value.
- *Hard constraints*: when $A = \{true, false\}$, \leq derived by logical *or* (thus 1 is better than 0), and combination is logical *and*.
- *Weighted constraints*: they are soft constraints where each assignment of values to variables has a weight, and the goal is to minimize the sum of the weights: this can be cast by choosing A as the set of possible weights, by deriving the ordering by the *min* operator, and by using the *sum* as the combination operator.
- *Probabilistic constraints*: they are soft constraints where each assignment is associated to a probability, which informally represents the chance for the assignment to satisfy the constraint in the real problem. Constraints are then combined by multiplying the associated probabilities, and *max* is used to induce the ordering over preferences: the best solutions have the highest probability.

The concept of fuzzy constraint, as defined above, was originally based on the notion of fuzzy set [3, 6, 9]. A fuzzy set A is a subset of a referential set U whose boundaries are gradual. More formally: the *membership function* μ_A of a fuzzy set A assigns to each element $u \in U$ its degree of membership $\mu_A(u)$ usually taking values in $[0, 1]$. If $\mu_A(u) = 1$, it means that u belongs to A , while $\mu_A(u) = 0$ means that u does not belong to A . If $\mu_A(u)$ is between 0 and 1, then it means that $u \in A$ with degree $\mu_A(u)$.

Fuzzy constraints use the notion of fuzzy sets to describe the level of preference of a certain assignment of values to variables. More precisely, a *soft fuzzy constraint* [2] C on variables $\{x_1, \dots, x_n\}$ is associated with a *fuzzy relation* R , i.e. a fuzzy subset of $D_1 \times \dots \times D_n$ of values that more or less satisfy C . A membership function μ_R is associated with relation R and specifies for each tuple $(d_1, \dots, d_n) \in D_1 \times \dots \times D_n$ the level of satisfaction $\mu_R(d_1, \dots, d_n)$ in a set L , which is totally ordered (e.g. $[0, 1]$). In particular, $\mu_R(d_1, \dots, d_n) = 1$ if tuple (d_1, \dots, d_n) totally satisfies C , $\mu_R(d_1, \dots, d_n) = 0$ if it totally violates C , and

$0 < \mu_R(d_1, \dots, d_n) < 1$ if it partially satisfies C . Moreover, $\mu_R(d_1, \dots, d_n) > \mu_R(d'_1, \dots, d'_n)$ means that tuple (d_1, \dots, d_n) is better than tuple (d'_1, \dots, d'_n) .

In the following we will use two operations on fuzzy constraints [2]: projection and combination. The *projection* of a fuzzy constraint, represented by fuzzy relation R on variables $\{x_1, \dots, x_k\} \subseteq V(R) = \{x_1, \dots, x_n\}$, is a fuzzy relation $R^{\downarrow\{x_1, \dots, x_k\}}$ defined on $\{x_1, \dots, x_k\}$ such that: $\mu_{R^{\downarrow\{x_1, \dots, x_k\}}}(d_1, \dots, d_k) = \sup_{\{d=(d_1, \dots, d_n) \mid d^{\downarrow\{x_1, \dots, x_k\}}=(d_1, \dots, d_k)\}} \mu_R(d)$. The *conjunctive combination* of two fuzzy constraints, represented by fuzzy relations R_i and R_j , is a fuzzy relation $R_i \otimes R_j$ defined on variables $V(R_i) \cup V(R_j)$ such that: $\mu_{R_i \otimes R_j}(d_1, \dots, d_k) = \min(\mu_{R_i}(d_1, \dots, d_k)^{\downarrow V(R_i)}, \mu_{R_j}(d_1, \dots, d_k)^{\downarrow V(R_j)})$ where $\mu_{R_i \otimes R_j}(d_1, \dots, d_k)$ evaluates to what extent (d_1, \dots, d_k) satisfies both C_i and C_j .

3 Possibility theory

A *possibility distribution* [9] is the membership function of a fuzzy set A attached to a single-valued variable x . It is denoted $\pi_x = \mu_A$ and represents the set of more or less plausible, mutually exclusive values of x . A possibility distribution is similar to a probability density. However, $\pi_x(u) = 1$ only means that $x = u$ is a plausible situation, which cannot be excluded. Thus, a degree of possibility can be viewed as an upper bound of a degree of probability.

The *possibility* of an event " $x \in E$ ", $E \subseteq U$, denoted by $\Pi(x \in E)$, is formally $\Pi(x \in E) = \sup_u \min(\pi_x(u), \mu_E(u)) = \sup_{u \in E} \pi_x(u)$. If an event has possibility equal to 1, it means that it is totally possible, but it could also not happen and so we are completely ignorant about its occurrence. On the contrary, having a possibility equal to 0 means that the event will not happen.

The dual measure of *necessity* of " $x \in E$ ", denoted by $N(x \in E)$, evaluates the extent to which " $x \in E$ " is *certainly* true, i.e. to what extent the proposition " $x \in E$ " is implied by the item of information " $x \in A$ ": $N(x \in E) = \inf_u \max(c(\pi_x(u)), \mu_E(u)) = \inf_{u \notin E} (c(\pi_x(u))) = 1 - \Pi(x \in E^C)$, where c is the order reversing map such that $c(p) = 1 - p$ and E^C is the complement of E in U . $N(x \in E) = 1$ when it is certain that $x \in E$. On the contrary, having necessity equal to 0 means that the event is not necessary at all, although it may happen. In fact, $N(x \in E) = 0$ iff $\Pi(x \in E^C) = 1$.

4 Uncertainty in soft constraints

Whereas in usual soft constraint problems all the variables are assumed to be controllable, that is, their value can be decided according to the constraints which relate them

to other variables, in many real-world problems uncertain parameters must be used. Such parameters are associated with variables which are not under the user's direct control and thus cannot be assigned. Only Nature will assign them.

Formally, we can define an *uncertain soft constraint problem* as a tuple $\langle S, V_c, V_u, C \rangle$, where S is a semiring, V_c is the set of controllable variables, V_u is the set of uncontrollable variables, and C is the set of soft constraints. The soft constraints in C may involve any subset of variables of $V_c \cup V_u$. While in a classical soft constraint problem we can decide how to assign the variables to make the assignment optimal, in the presence of uncertain parameters we must assign values to the controllable variables guessing what Nature will do with the uncontrollable variables. So, in this paper an optimal solution for an uncertain soft constraint problem is an assignment of values to the variables in V_c such that, *whatever* Nature will decide for the variables in V_u , the overall assignment will be optimal. This means that we can make the assumption that the values of the uncontrollable variables are never observable, i.e., they are decided upon without observing the values of the uncontrollable variables. This is a pessimistic view and other definitions of solutions can be considered. Moreover, the uncontrollable variables can be equipped with additional information on the likelihood of their values. In this paper we will consider two ways of expressing such information: possibilities and probabilities.

The next section describes two existing approaches [2, 8] for integrating fuzzy constraints and uncertainty given by possibilities. In both, the original problem is replaced by another one without uncontrollable variables and with new soft constraints depending on the possibilistic distributions. In [8] the two sets of constraints are kept separate, thus allowing for a fine discrimination between preferences and robustness to uncertainty.

5 Unifying fuzzy preferences and uncertainty via possibility theory

In [2] it is shown how it is possible to replace a fuzzy constraint involving at least one uncontrollable variable with a fuzzy constraint over controllable variables only. Consider a fuzzy constraint C , represented by the fuzzy relation R , which relates a set of controllable variables $X = \{x_1, \dots, x_n\}$ to a set of uncertain parameters $Z = \{z_1, \dots, z_k\}$ with domains A_1, \dots, A_k . The knowledge of the uncertain parameters is modeled with the possibility distribution π_Z defined on $A_Z = A_1 \times \dots \times A_k$. The constraint C is considered satisfied by the assignment $d = (d_1, \dots, d_n) \in D_1 \times \dots \times D_n$ if, *whatever the values of* Z , $z = (z_1, \dots, z_k)$, d is *compatible with* z , i.e., the set of possible values for z is included in $T = (R \otimes \{(d_1, \dots, d_n)\})^{\downarrow z}$. Therefore $\mu_T(a) =$

$$\mu_R(a, d) \text{ and } \mu'(d) = \mu'_R(d) = N(d \text{ satisfies } C) = N(z \in T) = \inf_{a \in A_Z} \max(\mu_T(a), c(\pi_Z(a))) = c(\sup_{a \in A_Z} \min(c(\mu_T(a)), \pi_Z(a))).$$

If C is a hard constraint, then the formula above still applies, and becomes the following one: $N(d \text{ satisfies } C) = \inf_{a \notin T = (R \cap \{d\})^{\downarrow z}} c(\pi_Z(a))$.

The method proposed in [2], which we call *Algorithm DFP* (by the name of the authors), for managing uncertainty in a fuzzy CSP, starts from an uncertain fuzzy CSP, say $ufcsp$, that is then reduced to a fuzzy constraint problem fcp : all the constraints which link uncertain parameters to decision variables are replaced by fuzzy constraints only among the decision variables. The new preference levels of the decision variables in such new constraints are computed by applying the specific procedure defining μ' . fcp has only fuzzy constraints, hence it can be solved by applying the usual method for solving fuzzy CSPs, i.e. using the *min* operator to combine the constraints and choosing the complete assignments with the highest preference.

In [2] the following property is given:

Property 1. $\mu'(d) \geq \alpha$ iff, when $\pi_Z(a) > c(\alpha)$ then $\mu_R(d, a) \geq \alpha$, where a is the actual value of Z .

Moreover, from the definition of μ' , the following two properties can also be proved [8].

Property 2. Given the possibilities of uncertain parameters, defined by π_Z , an assignment d to the decision variables X_1, \dots, X_k , and two preference functions μ_1 and μ_2 such that $\mu_1(d, a) \leq \mu_2(d, a)$ for all a assignments to Z , then $\mu'_1(d) \leq \mu'_2(d)$.

Property 3. Given a preference function μ , an assignment d to the decision variables X_1, \dots, X_k , and two possibility distributions π_1 and π_2 on Z , such that $\pi_1(a) \geq \pi_2(a)$ for all a , then $\mu'_1(d) \leq \mu'_2(d)$, where μ'_i is the preference function obtained considering π_i , $i=1,2$.

By using algorithm DFP, the overall preference for a solution is $\min(F, U)$, where F is the minimum of the preferences in the initial fuzzy constraints only on decision variables, and U is the minimum of the preferences of the new fuzzy constraints. This means that a low overall preference may be caused from a low preference in some of the new fuzzy constraints (when U is less than F), that is, a low compatibility with the uncertain events, or also from a low preference on some fuzzy constraint initially given only on decision variables (when F is less than U).

In [8] these two components (F and U) are kept separate, rather than combined with *min*, by performing, for each constraint involving both decision and uncertain variables (X and Z), a projection over the decision variables, that is a new constraint over X where, for each assignment of values to its variables, the preference is computed by assuming the best in the uncertain parameters. Since preferences are combined via the *min* operator, this new constraint will force the overall preference to be no higher than its best

preference. Given an assignment to decision variables, we denote with P the minimum preference over these new *projection constraints*. The value P , combined with preference F given by the initial constraints only on controllable variables, defines the new preference F_P .

The algorithm presented in [8], called *algorithm SP* (from *separation* and *projection*) starts from an uncertain fuzzy CSP with fuzzy constraints C , and then all the constraints which link uncertain parameters to decision variables are replaced by fuzzy constraints, say C_u , only among the decision variables. After that, it computes the projection constraints, say C_p , and for each complete assignment, it computes its overall preference as a pair $\langle F_P, U \rangle$, where $F_P = \min(F, P)$ and F , P , and U are, respectively, the minimum preference over C , C_p , and C_u .

Given an assignment d to the decision variables, the pair $\langle F_P, U \rangle$ represents two degrees of satisfaction: a sure one and an expected one. More precisely, F_P tells us how much d is preferred by the constraints, while $1 - U$ gives an idea of the risk of hitting a value of uncertain parameter that is inconsistent with d , hence U can be seen as a measure of the *certainty* (or *robustness*) of d . Notice that, since U is computed as in [2], Properties 1, 2 and 3 still hold.

Let us consider the following example, where we have two complete assignments: d with $F = 0.3$, $P = 0.9$, and $U = 0.9$, and d' with $F = 0.9$, $P = 0.9$, and $U = 0.3$. The algorithm DFP, considers d equally preferred to d' since d and d' have the same preference $\min(F, U) = 0.3$. However, d and d' differ on both preference and robustness.

The approach based on algorithm **SP** preserves such a difference, by defining various semantics exploiting both elements of the pair $\langle F_P, U \rangle$ to deduce a solution ordering.

Given two solutions d and d' and the corresponding pairs of values $\langle F_P(d), U(d) \rangle = \langle a_1, b_1 \rangle$ and $\langle F_P(d'), U(d') \rangle = \langle a_2, b_2 \rangle$, in [8] three semantics are presented for ordering them: *Risky*, a Lex ordering on pairs $\langle a_i, b_i \rangle$, with the first component as the most important feature, that gives more importance to the preference level that can be reached in the best case (a higher a_i) considering less important a high risk of being inconsistent (a low certainty b_i), *Safe*, that follows the opposite attitude with the respect to the previous one, that can be seen as a Lex ordering on pairs $\langle a_i, b_i \rangle$, with the second component as most important feature, and *Diplomatic* that aims at giving the same importance to the preference and the certainty, i.e., that a pair is to be preferred to another only if it wins both on preference and certainty, leaving incomparable all the pairs that have one component higher and the other lower.

6 A generalized approach

The methods described in the previous sections can handle only uncertainty in fuzzy constraints. In the following of

the paper we will extend the method in [8] to other classes of soft constraints and to probabilistic uncertainty. In particular, we will consider the combinations of fuzzy, probabilistic and weighted constraints with possibilistic uncertainty.

To do this, we now redefine **SP** for generic soft constraints and generic uncertainty. Then we will define the extensions above as instances of this general framework.

We recall that in Fuzzy CSPs with uncertainty the preferences of a new constraints C , obtained removing the uncertain parameters, is computed via the formula $\mu'(d) = \inf_{a \in A_Z} \max(\mu_T(a), c(\pi_Z(a)))$, where $T = (R \otimes \{(d_1, \dots, d_n)\})^{\downarrow Z}$, R is the fuzzy relation defining the constraint C , c is the order reversing map such that $c(p) = 1 - p$ and π_Z is the possibilistic distribution.

Let us consider any semiring $S = \{A, +, \times, 0, 1\}$, where \leq_S is the semiring ordering on A (we denote incomparability with \bowtie_S). The formula above can be generalized to deal with any semiring as follows:

$$\mu'(d) = \inf_{a \in A_Z} (\mu_T(a) + c(\pi_Z(a)))$$

where $+$ refers to the one of the semiring operations, \inf is one of the bottom elements of A_Z (i.e., an element such that $\forall a' \in A_Z$ with $a' \neq a$ then $a' >_S a$ or $a' \bowtie_S a$), $[0, 1] \subseteq A$, π_Z is a possibilistic distribution and c is a is an order-reversing map w.r.t. semiring S , that is bijection from $[0, 1]$ to $[0, 1]$ such that, for each $a_1, a_2 \in [0, 1]$, $a_1 \leq a_2$ if and only if $c(a_1) \geq_S c(a_2)$ and $c(c(a)) = a$ for all a .

Notice that, by generalizing the formula, the set of values for the possibilities remains $[0, 1]$. When working with other classes of soft constraints rather than fuzzy CSPs, then $\mu_T(a) = \mu_R(d, a)$, where μ_R associates a preference from set A to an assignment. In particular, μ_R is the preference function of the soft constraint R .

The algorithm **SP** [8], can be generalized as follows: it starts from an uncertain soft CSP $\langle S, V_c, V_u, C \rangle$, then all the constraints which link variables in V_u to variables in V_c are replaced by soft constraints, say C_u , defined by μ' , only among variables in V_c . After that, all the constraints which link variables in V_u to variables in V_c are used to compute their projection constraints, say C_p , over variables in V_c . Finally, for each assignment of the variables in V_c , it computes its overall preference as the pair $\langle F_P, U \rangle$, where $F_P = F \times P$ and F , P , and U are, resp., the preference of the assignment over the constraints in C , C_p , and C_u .

We recall that, given a soft constraint R , defined on variables X and Z its projection over X is defined by $\mu_P(d) = \sum_{a \in A_Z} \mu_R(d, a)$. We have proved that properties 1, 2, 3 hold for generic soft constraints where the set A of the semiring is *totally ordered*.

General Property 1: $\mu'(d) \geq_S \alpha$ if and only if, when $\pi_Z(a) > c(\alpha)$, then $\mu_R(d, a) \geq_S \alpha$.

If A is *partially ordered*, this property doesn't hold. However, two slightly weaker properties can be proved.

Weak general Property 1 ($\not\leq_S$): $\mu'(d) \not\leq_S \alpha$ if and only if, when $\pi_Z(a) \geq c(\alpha)$, then $\mu_R(d, a) \not\leq_S \alpha$.

Weak general Property 1 ($\not\leq_S$): $\mu'(d) \not\leq_S \alpha$ if and only if, when $\pi_Z(a) > c(\alpha)$, then $\mu_R(d, a) \not\leq_S \alpha$.

It can be proved that properties 2, 3 hold for problems with generic soft constraints and uncertainty described by π_Z that can be a possibilistic distribution.

General Property 2: Given π_Z and two preference functions μ_1 and μ_2 such that $\mu_1(d, a) \leq_S \mu_2(d, a)$ for all a assignments to Z , then $\mu'_1(d) \leq_S \mu'_2(d)$, where μ'_i is the preference function obtained considering μ_i , $i=1,2$.

General Property 3: Given a preference function μ and two distributions π_1 and π_2 on Z , such that $\pi_1(a) \geq \pi_2(a)$ for all a , then $\mu'_1(d) \leq_S \mu'_2(d)$, where μ'_i is the preference function obtained considering π_i , $i=1,2$.

If A is *partially ordered*, general property 2 and 3 assume a slightly weaker form. The only change in the statement is that $\mu'_1(d) \leq_S \mu'_2(d)$ is replaced with $\mu'_1(d) \not\leq_S \mu'_2(d)$.

Since these properties hold in general, then we can safely and effectively handle problems with many kinds of soft constraints as well as uncertain possibilistic variables.

The generic semantics are defined like those ones in [8], except that \leq is replaced by \leq_S .

7 Uncertain Probabilistic CSPs

In several real-life scenarios, fuzzy constraints are not the ideal setting. Probabilistic CSPs (PCSPs) [7] model those situations where each constraint c has a certain probability $p(c)$, independent from the probability of the other constraints, to be part of the given real problem. The probability levels on constraints give then, to each instantiation of all the variables, a probability that it is a solution of the real problem. Constraints are combined by multiplying the associated probabilities and the aim is to get those instantiations with the maximum probability. The semiring corresponding to PCSPs is $S = \{[0, 1], max, \times, 0, 1\}$.

Uncertain PCSPs are PCSPs where there are not only controllable variables but also uncontrollable variables. We assume here that the values in the domains of the uncontrollable variables are defined by a possibility distribution.

To make sure that the desired properties 1, 2, and 3 hold in this setting, we just need to check whether the assumptions we made are met: $[0, 1] \subseteq A$ and c is a bijection from $[0,1]$ to $[0,1]$ such that, for each $a_1, a_2 \in [0, 1]$, $a_1 \leq a_2$ if and only if $c(a_1) \geq_S c(a_2)$ and $c(c(a)) = a$ for all a .

The first assumption is trivially true since $A = [0, 1]$. As for the second one, we consider $c(x) = 1 - x$ for all x , which satisfies the order-reversing property. In fact, given $a_1, a_2 \in [0, 1]$, with $a_1 \leq a_2$ then $1 - a_1 \geq_S 1 - a_2$, since $max(1 - a_1, 1 - a_2) = 1 - a_1$. In this setting, we have

$$\mu'(d) = \min_{a \in A_Z} \max(\mu_T(a), 1 - \pi_Z(a)).$$

Figure 1 shows a soft CSP with uncertainty. There are three decision variables (X, Y, W), one uncertain variable (Z), and two constraints: C_{XYZ} with function μ and C_{XW} with function μ_1 . The possibility distribution π_Z describes the plausibility of Z .

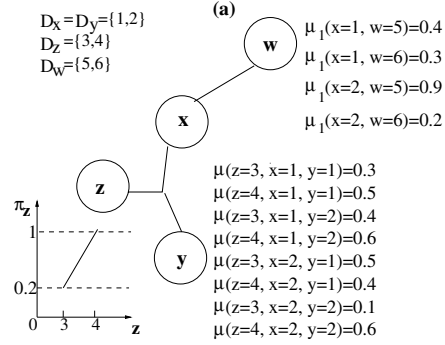


Figure 1. An uncertain soft CSP.

Figure 2 shows the result of applying SP to the problem in Figure 1, seen as an uncertain PCSP. In this case, the algorithm is instantiated with $\times_S = \times$, $+_S = max$, and $c(p) = 1 - p$. In particular, Figure 2 (a) shows the resulting probabilistic CSP obtained by the algorithm, while Figure 2 (b) shows all complete assignments, together with the associated pair.

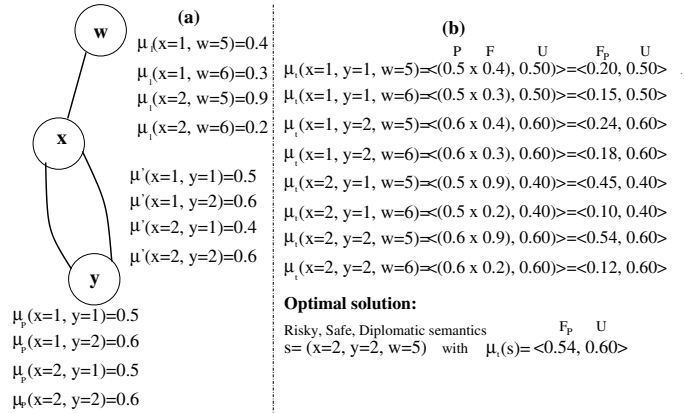


Figure 2. Result of algorithm SP for the uncertain soft CSP in Figure 1, seen as an uncertain PCSP.

8 Uncertain Weighted CSPs

In several situations where neither fuzzy nor probabilistic constraints are ideal, weighted constraints can be useful to model preferences. For example, when dealing with costs

which are naturally combined by a sum. In this setting, preferences are penalties (or costs) to be added, and the best solutions are those with the smallest preference. Thus operators $+$ and \min are the instantiation of the operators \times and $+$ of the general case. Therefore, the semiring to be used is $S = \{\mathcal{R}^+, \min, +, +\infty, 0\}$.

Uncertain Weighted CSPs are Weighted CSPs where there are both controllable and uncontrollable variables. We assume here that the values in the domains of the uncontrollable variables are defined by a possibility distribution.

The three desired property hold if we choose c as the identity map. In fact, $[0, 1] \subseteq \mathcal{R}^+$. Moreover, c is an order-reversing map w.r.t semiring S . In fact, given $a_1, a_2 \in [0, 1]$ such that $a_1 \leq a_2$, we have $a_1 \geq_S a_2$ since $\min(a_1, a_2) = a_1$. The instantiated formula for μ' is then

$$\mu'(d) = \max_{a \in A_z} \min(\mu_T(a), \pi_Z(a)).$$

Figure 3 shows how algorithm **SP** works on weighted CSP with possibilistic uncertainty, as the one in Figure 1 (where preferences are interpreted as costs). Figure 2 (a) shows the resulting weighted CSP, while Figure 2 (b) shows all complete assignments, together with the associated pair. In this problem the optimal solution obtained using the *Risky* semantics is different from the one obtained using *Safe*, whereas *Diplomatic* considers them both optimal.

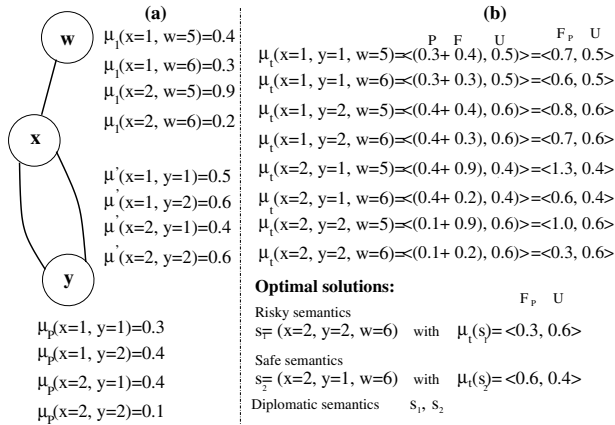


Figure 3. Result of algorithm SP for the uncertain soft CSP in Figure 1, seen as an uncertain weighted CSP.

9 Probabilistic and possibilistic uncertainty

In the previous sections we have considered different classes of soft constraints with uncertainty described via a possibilistic distribution. Here we assume that some uncontrollable variables are defined by a possibilistic distribution, while for others by a probabilistic distribution.

In this case, we could get to the usual setting by replacing probabilities with possibilities. In [5] it is presented a way to do this. Thus we could use such a method to obtain only one kind of distribution and then use our approach.

However, transforming a probability into a possibility distribution we loose information, and solutions have a lower robustness. In fact, using property 3, it is possible to see that, if we use possibilities, which are higher than probabilities, we get a smaller robustness value. Thus we can say that the robustness value obtained in this way is a lower bound to the certainty that the values of the decision variables are compatible with the uncertain variables.

10 Future work

We plan to develop a solver that can handle problems with several classes of soft constraints, together with uncertainty expressed via possibility or probability distributions. The solver will be able to generate orderings according the three semantics proposed in this paper as well as others that we will define by following different optimistic or pessimistic approaches.

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